Homework 2: Predicate logic.
Due: 9:30am, 5th Sept.
Example answers highlighted in yellow.

1. Let \( H(x) \) stand for “\( x \) is happy” and \( R(x) \) “\( x \) is rich” where the domain of discourse of \( x \) is people. Translate each of the following propositions into an unambiguous English sentence. Try to make the sentence as simple as possible (with no negations at the beginning).
   a. \( \neg \forall x (\neg H(x)) \)
      i. “Not everyone is not happy.”
      ii. Or simplifying, \( \exists x (H(x)) \).
      iii. “There is a person that is happy.”
   b. \( \neg \exists x (H(x) \land R(x)) \)
   c. \( \neg \forall x (H(x) \rightarrow R(x)) \)
   d. \( \exists x (R(x) \leftrightarrow H(x)) \)
   e. \( \exists y \forall x (H(x) \leftrightarrow H(y)) \)

2. Express the following propositions into predicate logic. Make up predicates as you need. State what each predicate means. Also state the domain of discourse for that predicate.
   a. “Anyone who completes all homework assignments will pass this course.”
      i. Let \( \text{Completed}(x, y) \) stand for “\( x \) completed assignment \( y \),” and let \( \text{Pass}(x) \) stand for “\( x \) passes the course,” where the domain of discourse for \( x \) is people and the domain of discourse of \( y \) are assignments. Then, \( \forall x \forall y (\text{Completed}(x, y) \rightarrow (\text{Pass}(x))) \).
   b. “Not everyone likes to do the homework.”
   c. “There is one class that all of my friends have taken.”
   d. “No student failed Logic, but at least one student failed History.”
   e. “Everyone danced with each other.”
   f. “If you danced with Jill, you danced with everyone.”
   g. “There is exactly one Venezuelan.”

3. Let \( \text{Listens}(x, y) \) stand for “\( x \) listens to \( y \),” where the domain of discourse for \( x \) consists of your friends and the domain of discourse for \( y \) consists of music genres. Translate each of the following propositions into an unambiguous English sentence (Do they each mean something different?):
   a. \( \forall x \forall y \text{Listens}(x, y) \)
      i. All of my friends listens to all music genres.
   b. \( \exists x \exists y \text{Listens}(x, y) \)
   c. \( \forall x \exists y \text{Listens}(x, y) \)
   d. \( \exists x \forall y \text{Listens}(x, y) \)
4. Decide whether each of the following arguments is valid. If it is valid argument, give a formal proof (i.e., justify which laws of logic need to be applied to each of the premises, in a sequence of arguments, to arrive at the conclusion). If the argument is invalid (i.e., you can’t find a sequence of laws that leads from the premises to the conclusion), show that it is invalid by finding an appropriate assignment of truth values to the propositional variables.

a. \( p \rightarrow q, \ q \rightarrow s, \ s \therefore p \)
   i. \( p \rightarrow q \) premise
   ii. \( q \rightarrow s \) premise
   iii. \( \neg q \lor s \) definition of \( \rightarrow \)
   iv. \( s \) premise
   v. We are stuck, because we do not know anything about \( \neg q \) given that \( s \) is true.
   vi. We move to try to find a set of truth values that show that the argument could be invalid.
   vii. Let \( p \) be False, and \( q \) be False. \( p \rightarrow q \) is True, as the premise.
   viii. Let \( s \) be True, as in the last premise.
   ix. Then \( q \rightarrow s \) is True, as the second premise.
   x. But \( p \) is not True, as the conclusions would want us to believe.
   xi. Therefore the argument is not valid, as shown by the case when \( p=\text{False}, \ q=\text{False}, \) and \( s=\text{True}, \) where all of the premises are True, but the conclusion does not follow.

b. \( \neg p \rightarrow t, \ q \rightarrow s, \ r \rightarrow q, \ \neg (q \lor t) \therefore p \)

c. \( q \rightarrow t, \ p \rightarrow (t \rightarrow s), \ p \therefore q \rightarrow s \)

d. \( p, \ s \rightarrow r, \ q \lor r, \ q \rightarrow \neg p \therefore \neg s \)