1. Let Loves(x,y) mean “x loves y,” Student(x) mean “x is a student,” Friend(x,y) mean “x is a
friend of y,” Bicycle(x) mean “x is a bicycle,” Owns(x,y) mean “x owns y,” and Sister (x,y)
mean “x is a sister of y.”

   a. Translate the following propositions into the most natural equivalent statements in English.
      i. ∀x∃y Student(x) → Sister(y,x)
      ii. ∃x(∀z((x ≠ z) → Friend(x,z)) ∧ ∃y(Bicycle(y) ∧ Owns(x,y)))
      iii. ∀x∃y((Student(x) → Friend(x,y)) ∧ ∀z(Friend(x,z) → (y = z)))
      iv. ∀x∀y∃z((Student(x) ∧ Bicycle(z) ∧ Owns(x,z)) → Friend(x,y))
      v. ∀x∃y((Student(x) ∧ Bicycle(y) ∧ Owns(x,y)) → ∃w Sister(x,w))

   b. Translate the following statements into predicate logic.
      i. Peter has only one friend but at least two bicycles.
      ii. Every student has at least one friend if and only if they own a bicycle.
      iii. If something is a student, it cannot be a bicycle, and vice versa.
      iv. Any student that loves his or her bicycle has at least two friends.
      v. All bicycle-loving sisters are students.

2. For each of the following, prove that the conclusion follows logically from the premises.

   i. Premises: ¬f, (g ∧ h) → (f ∨ e), ¬f ∨ c → h, (¬f ∨ c) → h, (¬f ∨ b) → g Prove: e
   ii. Premises: p ∧ q, (p ∧ r) → (t ∧ u), r ∧ s Prove: t ∨ m
   iii. Premises: r ∨ t, r → s, ¬t, (p ∧ q) ↔ (r ∧ s) Prove: p
   iv. Premises: ¬q, p ∨ q, p → (r ∧ s), ¬u, r ↔ (t ∨ u) Prove: t
   v. Premises: ¬(p ∨ q) → ¬r, ¬p, r Prove: q
   vi. Premise: w → x, w → ¬y, y ∨ ¬x Prove: ¬w
   vii. Premise (p ∨ q) → r, ¬r ∨ s, p → ¬s Prove: ¬p
   viii. Premise: ¬x → ¬z, y → z Prove: y → (x ∨ w)
   ix. Premises: p ↔ (q ∨ r), ¬q Prove: p → (r ∨ s)
   x. Premises: q ↔ s, p ∨ q, p → r, s → t Prove: r ∨ t
3. For each of the following, prove that the conclusion follows logically from the premises.
   i. Premises: ∀x (p(x) → q(x))  Prove: ∃x p(x) → ∃x q(x)
   ii. Premises: ∀x (p(x) → q(x))  Prove: ∀x p(x) → ∃x q(x)
   iii. Premises: ∃x (p(x) → q(x))  Prove: ∃x p(x) → ∃x q(x)

4. Transform each informal argument into predicate logic. Then give a formal proof.
   i. Every dog either likes people or hates cats. Rover is a dog. Rover loves cats. Therefore, some dog likes people.
   ii. Every committee member is rich and famous. Some committee members are old. Therefore, some committee members are old and famous.
   iii. No human beings are quadrupeds. All men are human beings. Therefore, no man is a quadruped.
   iv. Some freshman like sophomores. No freshman likes any junior. Therefore, no sophomore is a junior.