Homework 2: Predicate logic.  
Due: 9:30am, 5th Sept.  
Answers highlighted in yellow.

1. Let $H(x)$ stand for “$x$ is happy” and $R(x)$ “$x$ is rich” where the domain of discourse of $x$ is people.  Translate each of the following propositions into an unambiguous English sentence.  Try to make the sentence as simple as possible (with no negations at the beginning).

   a. $\neg \forall x (\neg H(x))$
      i. “Not everyone is not happy.”
      ii. Or simplifying, $\exists x (H(x))$.
      iii. “There is a person that is happy.”

   b. $\neg \exists x (H(x) \land R(x))$
      i. “There does not exist one person that is happy and rich.”
      ii. Or simplifying, $\forall x (\neg H(x) \lor \neg R(x))$.
      iii. “Everyone is either not happy or not rich.”

   c. $\neg \forall x (H(x) \rightarrow R(x))$
      i. “Not everyone who is happy is rich.”
      ii. Or simplifying, $\exists x (H(x) \land \neg R(x))$.
      iii. “There exist some who are happy but not rich.”

   d. $\exists x (R(x) \leftrightarrow H(x))$
      i. “There are some that are happy if and only if they are rich.”

   e. $\exists y \ \forall x (H(x) \leftrightarrow H(y))$
      i. “There are some that are happy if and only if everyone is happy.”

2. Express the following propositions into predicate logic. Make up predicates as you need.  State what each predicate means.  Also state the domain of discourse for that predicate.

   a. “Anyone who completes all homework assignments will pass this course.”
      i. Let $Completed(x, y)$ stand for “$x$ completed assignment $y$,” and let $Pass(x)$ stand for “$x$ passes the course,” where the domain of discourse for $x$ is people and the domain of discourse of $y$ are assignments. Then, $\forall x \forall y (Completed(x, y) \rightarrow (Pass(x))$.

   b. “Not everyone likes to do the homework.”
      i. Let $LikesHomework(x)$ stand for “$x$ likes to do the homework,” where the domain of discourse of $x$ is the students. Then, $\neg \forall x (LikesHomework(x))$ or $\exists x (\neg LikesHomework(x))$.

   c. “There is one class that all of my friends have taken.”
      i. Let $TookClass(x, y)$ stand for “$x$ took class $y$,” where the domain of discourse of $x$ is friends and the domain of discourse of $y$ are courses. Then, $\exists y \forall x (TookClass(x, y))$. 

d. “No student failed Logic, but at least one student failed History.”
   i. Let Student(x) stand for “x is a student” and let Failed(x,y) stand for “x failed course y,” where the domain of discourse for x is people and the domain of discourse for y is courses. Then, (\(\forall x\) Student(x) \(\rightarrow\) Failed(x, Chemistry)) \(\land\) (\(\exists x\) Student(x) \(\rightarrow\) Failed(x, History)).

e. “Everyone danced with each other.”
   i. Let Danced(x,y) stand for “x danced with y,” where the domain of discourse for x and y is people at the party. Then, \(\forall x\forall y\)(Danced(x,y)).

f. “If you danced with Jill, you danced with everyone.”
   i. Let Danced(x,y) stand for “x danced with y,” where the domain of discourse for x and y is people at the party. Then, \(\forall x\forall y\)(Danced(x,Jill) \(\rightarrow\) Danced(x,y)).

g. “There is exactly one Venezuelan.”
   i. Let Ven(x) stand for “x is Venezuelan,” where the domain of discourse is people. Then, either of the following two compound propositions are valid:
      1. \(\exists x\) (Ven(x) \(\land\) (\(\forall y\) (Ven(y) \(\rightarrow\) (y = x))))
      2. (\(\exists x\)Ven(x) \(\land\) (\(\forall y\forall z\)((Ven(y)\land Ven(z)) \(\rightarrow\) (y = z))))

3. Let Listens(x,y) stand for “x listens to y,” where the domain of discourse for x consists of your friends and the domain of discourse for y consists of music genres. Translate each of the following propositions into an unambiguous English sentence (Do they each mean something different?):
   a. \(\forall x\forall y\) Listens(x,y)
      i. All of my friends listens to all music genres.
   b. \(\exists x\exists y\) Listens(x,y)
      i. There is a friend who listens to at least one music genre.
   c. \(\forall x\exists y\) Listens(x,y)
      i. All of my friends listen to at least one music genre.
   d. \(\exists x\forall y\) Listens(x,y)
      i. At least one of my friends listens to all music genres.
   e. \(\exists y\forall x\) Listens(x,y)
      i. There is a music genre that all my friends listen to.
   f. \(\forall y\exists x\) Listens(x,y)
      i. For each music genre, there is a friend who listens to it.
   g. Yes, all 6 of those mean something different.

4. Decide whether each of the following arguments is valid. If it is valid argument, give a formal proof (i.e., justify which laws of logic need to be applied to each of the premises, in a sequence of arguments, to arrive at the conclusion). If the argument is invalid (i.e., you can’t find a sequence of laws that leads from the premises to the conclusion), show that it is invalid by finding an appropriate assignment of truth values to the propositional variables.
   a. \(p \rightarrow q\) , \(q \rightarrow s\) , \(s \therefore p\)
i. \( p \rightarrow q \) premise

ii. \( q \rightarrow s \) premise

iii. \( \neg q \lor s \) definition of \( \rightarrow \)

iv. \( s \) premise

v. We are stuck, because we do not know anything about \( \neg q \) given that \( s \) is true.

vi. We move to try to find a set of truth values that show that the argument could be invalid.

vii. Let \( p \) be False, and \( q \) be False. \( p \rightarrow q \) is True, as the premise.

viii. Let \( s \) be True, as in the last premise.

ix. Then \( q \rightarrow s \) is True, as the second premise.

x. But \( p \) is not True, as the conclusions would want us to believe.

xi. Therefore the argument is not valid, as shown by the case when \( p=\text{False}, q=\text{False}, \) and \( s=\text{True}, \) where all of the premises are True, but the conclusion does not follow.

b. \( (\neg p) \rightarrow t, q \rightarrow s, r \rightarrow q, \neg(q \lor t) \therefore p \)

i. \( (\neg p) \rightarrow t \) premise

ii. \( \neg(q \lor t) \) premise

iii. \( \neg q \land \neg t \) DeMorgan’s law on ii

iv. \( \neg t \) Identity law on iii

v. \( p \) from iv and i (modus tollens)

c. \( q \rightarrow t, p \rightarrow (t \rightarrow s), p \therefore q \rightarrow s \)

i. \( p \rightarrow (t \rightarrow s) \) premise

ii. \( p \) premise

iii. \( t \rightarrow s \) from i and ii modus ponens

iv. \( q \rightarrow t \) premise

v. \( q \rightarrow s \) (law of syllogism)

d. \( p, s \rightarrow r, q \lor r, q \rightarrow \neg p \therefore \neg s \)

i. \( q \rightarrow \neg p \) premise

ii. \( p \) premise

iii. \( \neg q \) from x and y (modus tollens)

iv. \( q \lor r \) premise

v. \( r \) (contradiction..)

vi. \( \neg r \)

vii. \( s \rightarrow r \) premise

viii. \( \neg s \) from x and y (modus tollens)

ix. In the case where \( p \) is true, \( s \) is true, \( r \) is true, and \( q \) is false, the four premises are true, but the conclusion \( \neg s \) is false.