Unit 1 Review: Logic and Deduction

1. Let $R =$ it’s raining, $L =$ Logic is fun, and $B =$ Batman exists.

   a. Translate the following propositions into the most natural equivalent statements in English.
      
      i. $(R \lor B) \rightarrow L$
      ii. $(L \land \neg R) \lor (\neg L \land R)$
      iii. $(R \lor L) \leftrightarrow B$
      iv. $B \rightarrow (\neg L \lor R)$
      v. $(R \land B \land L) \land (L \land R \land \neg B)$
      vi. $(R \land \neg B) \land (R \lor B)$
      vii. $(B \lor \neg B) \leftrightarrow (L \land \neg R)$
      viii. $L \lor (R \land \neg R) \rightarrow \neg B$
      ix. $(L \oplus R) \rightarrow B$
      x. $(L \land \neg R) \land (L \lor R)$

   b. Translate the following statements into propositional logic.
      
      i. Batman does not exist and it is raining, but logic is fun.
      ii. Logic is fun only if it is raining.
      iii. In order for Batman to exist, it is necessary and sufficient for it to be raining or for logic to be fun, or both.
      iv. Exactly one of the following statements is true: Batman exists, it is not raining, and logic is fun.
      v. Whenever it is raining, Batman exists or logic is fun, but not both.
      vi. For Batman to exist, it is sufficient but not necessary for logic to be fun.
      vii. Whether or not Batman exists, logic is fun.
      viii. Logic is not fun if Batman exists or if it is raining.
      ix. If Batman exists, then it must be raining in order for logic to be fun.
      x. Batman exists if it is raining and logic is fun.

2. Construct truth tables for the following propositions.
   
   i. $p \rightarrow (\neg q \land p)$
   ii. $(p \land q) \leftrightarrow (\neg r \land q)$
   iii. $(p \lor r \land q) \rightarrow (q \leftrightarrow \neg p)$
   iv. $(p \land r) \lor (\neg p \rightarrow \neg r)$
   v. $((p \land r) \lor \neg q) \leftrightarrow (p \land q)$
3. Give the converse and the contrapositive of each of the following English sentences.
   i. If you save money, you can go to school.
   ii. If you tweet while you are crossing the street, you will get hit by a car.
   iii. If you cannot swim, you will not sink to the bottom of the pool.
   iv. Babies can crawl if they can walk.
   v. Socrates is mortal if he is human and all human are mortal.

4. Determine whether each of the following propositions is a tautology, a contradiction, or neither (a contingency). If it is a contingency, provide two interpretations that demonstrate this is the case.
   i. \((p\land q)\leftrightarrow(\neg r \lor q)\)
   ii. \((p\land r)\lor(p\rightarrow\neg r)\)
   iii. \((p\land q)\leftrightarrow(\neg r \lor q)\)
   iv. \(r\land c\lor((a\lor b)\land\neg c)\)
   v. \((q\to p)\land(\neg s \rightarrow \neg q)\land(\neg s \lor s)\land q\)

5. Let Loves(x,y) mean “x loves y,” Student(x) mean “x is a student,” Friend(x,y) mean “x is a friend of y,” Bicycle(x) mean “x is a bicycle,” Owns(x,y) mean “x owns y,” and Sister (x,y) mean “x is a sister of y.”
   a. Translate the following propositions into the most natural equivalent statements in English.
      i. Student(Bill) \(\land\) \(\exists x (\text{Bicycle}(x) \land \text{Owns}(Bill,x))\) \(\land\) \(\forall y (y\neq Bill) \leftrightarrow \neg \text{Friend}(y,Bill)\)
      ii. \(\forall x (\text{Student}(x) \to \exists y (\text{Bicycle}(y) \land \text{Owns}(x,y) \land \text{Loves}(x,y)))\)
      iii. \(\forall y (\text{Bicycle}(y) \to \exists x (\text{Student}(x) \land \text{Owns}(x,y)))\)
      iv. \(\forall x \exists y ((\text{Student}(x) \land \text{Bicycle}(y) \land \text{Owns}(x,y)) \to \exists w \text{Sister}(x,w))\)
      v. \(\exists x \exists y \exists z \exists a (\text{Student}(x) \land \text{Friend}(x,y) \land \text{Sister}(y,z) \land \text{Bicycle}(a) \land \text{Owns}(z,a))\)
   b. Translate the following statements into predicate logic.
      i. Every student has a sister or a bicycle (or both).
      ii. If a student is a friend of all of his or her sisters, then that student owns a bicycle.
      iii. There is at least one student who is loved by everyone but is a friend of only himself/herself.
      iv. Every student has at least one friend and one bicycle.
      v. Bill loves Sally and is a friend of everyone who owns a bicycle.

6. For each of the following, prove that the conclusion follows logically from the premises.
   i. Premises: \(p \lor \neg q\), \(\neg r \to (p \lor q)\), \(r \lor p\) Prove: \(q \lor r\)
   ii. Premises: \((\neg q \land \neg r) \to p\), \(\neg r \to (p \lor q)\), \((r \land p) \to \neg q\) Prove: \(q\)
   iii. Premises: \(p \land \neg q\), \(q \lor r\) Prove: \(\neg p \lor q \lor r\)
   iv. Premises: \(p \lor \neg q\), \(p \to q\), \(p\) Prove: \(p\)
   v. Premises: \(p \lor (\neg q \land r)\), \(r \to (q \land \neg p)\), \(q\) Prove: \(\neg r \land q\)
vi. Premises: \( p \land (n \leftrightarrow s) \), \( (p \lor r) \rightarrow (q \land m) \)  Prove: \( p \land q \)

vii. Premises: \( s \rightarrow t \), \( p \land q \), \( (p \lor r) \rightarrow s \)  Prove: \( t \)

viii. Premises: \( (m \rightarrow n) \rightarrow (q \rightarrow m) \), \( m \lor q \), \( (m \lor q) \rightarrow (m \rightarrow n) \)  Prove: \( n \lor m \)

ix. Premises: \( (\neg w \lor y) \leftrightarrow (z \lor a) \), \( w \rightarrow \neg x \), \( x \), \( \neg a \)  Prove: \( z \)

x. Premises: \( D \rightarrow E \), \( E \rightarrow F \)  Prove: \( (D \land A) \rightarrow (F \land E) \)

7. Transform each informal argument into predicate logic. Then give a formal proof.
   i. All monkeys who play chess drive well. Therefore, if there is a monkey that plays chess, then there is a monkey that can drive well.
   ii. All superheroes who wear a cape get stuck in sliding doors. Therefore, if all superheroes wear a cape, then at least some superheroes get stuck in sliding doors.
   iii. Everyone who is sane can do logic. No lunatics are fit to serve on a jury. None of your friends can do logic. Therefore, if a person is fit to serve on a jury, then he/she is not your friend.
   iv. Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised. Therefore babies cannot manage crocodiles.