EABCN Training School: Monetary-Fiscal Policy Interactions

Lecture 4. Generalizing Policy Interactions (A)

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September 2010
THE MESSAGES

• Draws heavily from “Generalizing the Taylor Principle,” with Troy Davig (AER, June 2007)
• We do see policy rules—or regimes—change
  • to study the implications of recurring changes, need to model them coherently
• Before studying monetary-fiscal interactions when policy regimes can change, need some preliminary analysis when only MP can switch
• This allows simple analytical derivations that build intuition and understanding
• Many of our inferences are monetary policy effects change in subtle ways once we allow recurring regime change
• Subsequent work will allow both monetary and fiscal regime to undergo recurring change
Simplifying Policy

- Monetary policy is complex
- For descriptive & prescriptive reasons, seek to simplify
- Most successful simplification due to Taylor

\[ i_t = \bar{i} + \alpha (\pi_t - \pi^*) + \gamma x_t + \varepsilon_t \]

- **Taylor principle:** \( \alpha > 1 \)
  - necessary & sufficient for unique bounded eqm (w/ bounded shocks)
- Unique & stable eqm necessary for good policy
  - rules out arbitrarily large fluctuations
THE TAYLOR RULE & PRINCIPLE

- Central banks can stabilize economy by adjusting nominal interest rate more than one-for-one with inflation
  - approximates Federal Reserve behavior since 1982
  - nearly optimal in workhorse class of monetary models
  - used by central banks as a benchmark
- Maintains two key assumptions
  - fiscal policy is perpetually passive
  - policy rule permanent & agents believe change impossible
- Here we relax this second assumption
  - rule evolves according to a Markov chain
  - consider two conventional monetary models
GENERALIZING THE TAYLOR RULE & PRINCIPLE

- $\alpha(s_t), \gamma(s_t) s_t \sim$ Markov chain
- $s_t$: “rule,” “regime,” “state”
- $s_t$ exogenous (for now)
- Can believe actual policy rule time invariant
  - but Taylor rule is a gross simplification of reality
  - paper shows that a particular form of non-linearity can change predictions of models
In the Fisherian Model . . .

- Derive *long-run Taylor principle*
  - imposes much weaker conditions on MP for uniqueness
  - departures from short-run Taylor principle can be substantial—but brief—or modest—and prolonged
  - the more “hawkish” one regime is, the more “dovish” the other can be and still deliver uniqueness
  - “expectations formation effects”—beliefs about possible future regimes affect current eqm, increasing volatility even in a regime that satisfies **TP**
In the New-Keynesian Model . . .

- Derive long-run Taylor principle: dramatically expands region of determinacy
- Inference that inflation of the 70’s due to failure to obey TP does not hold up when expectations embed possibility of regime change
- Occasional large departures from TP—due to worries about financial instability or economic weakness—can have quantitatively important impacts even in a regime that satisfies TP
- Misleading inferences can arise from dividing data into regime-specific periods to interpret estimates as arising from distinct fixed regimes
**Why Regime Change?**

- Evidence that monetary policy regime changed
- Institutional or policy reforms
  - adoption of inflation targeting by over 20 countries
  - Fed’s “just trust us” approach
- Logical consistency
  - if regime *has* changed, regime *can* change
  - expectations depend on prob. distn. over possible regimes
- Recurring: in US, no legislated change installed Volcker or Greenspan
  - confluence of economic/political conditions allowed US to dodge a bullet and get Bernanke (coulda’ been a FOG)
A Modeling Choice

• Because Taylor rule a gross simplification, deviations occur
  • can be large and serially correlated
  • are systematic responses to state of economy
• How should we model these deviations?
  • shuffled into the ε’s?
  • time-varying feedback coefficients, \( \alpha_t \) & \( \gamma_t \)?
• ε’s affect conditional expectations
• \( \alpha_t \) & \( \gamma_t \) affect expectations functions
• A substantive choice
MODEL OF INFLATION DETERMINATION

• A simple Fisherian economy

\[ i_t = E_t \pi_{t+1} + r_t \]
\[ r_t = \rho r_{t-1} + \nu_t, \quad \nu \text{ bounded support} \]
\[ i_t = \alpha(s_t) \pi_t, \quad s_t \text{ Markov}; s_t = 1, 2 \]

\[ p_{ij} = P[s_t = j \mid s_{t-1} = i] \]

\[ \alpha(s_t) = \begin{cases} 
\alpha_1 & \text{for } s_t = 1 \\
\alpha_2 & \text{for } s_t = 2 
\end{cases} \]

• a monetary policy regime: realization of \( \alpha(s_t) \)
• a monetary policy process: collection \( (\alpha_1, \alpha_2, p_{11}, p_{22}) \)
• policy is active if \( \alpha_i > 1 \); passive if \( \alpha_i < 1 \)
Determinacy: Definition

• Seek generalization of Taylor principle
  • necessary & sufficient condition for existence of unique bounded eqm

• Why boundedness?
  • consistent w/ standard definition under fixed regime
  • corresponds to locally unique eqm
    • can analyze small perturbations
  • considering log-linearized models
    • boundedness ensures approximations are good
Determacy: Formalism

Model: \( \alpha(s_t) \pi_t = E_t \pi_{t+1} + r_t \)

• Let \( \Omega_t^{-s} = \{r_t, r_{t-1}, \ldots, s_{t-1}, s_{t-2}, \ldots\} \) and \( \Omega_t = \Omega_t^{-s} \cup \{s_t\} \)

• Integrating over \( s_t \), for \( s_t = 1 \) and \( s_t = 2 \)

\[
E_t \pi_{t+1} = E[\pi_{t+1} \mid s_t = i, \Omega_t^{-s}] = p_{i1} E[\pi_{1t+1} \mid \Omega_t^{-s}] + p_{i2} E[\pi_{2t+1} \mid \Omega_t^{-s}]
\]

where \( \pi_{it} = \pi_t(s_t = i, r_t) \), the solution when \( s_t = i \)

• The system is

\[
\begin{bmatrix}
\alpha_1 & 0 \\
0 & \alpha_2
\end{bmatrix}
\begin{bmatrix}
\pi_{1t} \\
\pi_{2t}
\end{bmatrix}
= \begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix}
\begin{bmatrix}
E_t \pi_{1t+1} \\
E_t \pi_{2t+1}
\end{bmatrix}
+ \begin{bmatrix}
r_t \\
r_t
\end{bmatrix}
\]

where \( E_t \pi_{it+1} \) denotes \( E[\pi_{it+1} \mid \Omega_t^{-s}] \)
**Determinacy: Formalism (con’t)**

- Write system as
  \[ \pi_t = M E_t \pi_{t+1} + \alpha^{-1} r_t \]

- MSV solution: \( \pi_t \) function only of \((r_t, s_t)\)
- Define \( x_t = \pi_t - \pi_t^{MSV}(r_t, s_t) \)
- Bounded soln for \( \{x_t\} \iff \text{bounded soln for } \{\pi_t\} \)
- We study: \( x_t = M E_t x_{t+1} \)
- Proof of determinacy shows that under certain conditions on the policy process, \( x_t = 0 \) is the only solution
Determinacy: Formalism (con’t)

- **Prop. 1** When $\alpha_i > 0$, a unique bounded solution exists iff all the eigenvalues of $M$ lie inside the unit circle.

- **Sufficiency**: the usual proof in linear RE models
  - Intuition: boundedness requires that $\lim_{n \to \infty} M^n = 0$, so $x_t = 0$ the only solution
  - Delivered by eigenvalue condition
Determinacy: Formalism (con’t)

• Necessity: Suppose $\lambda_1 \geq 1$, $\lambda_2 < 1$
  
  • diagonalize $M$, let $y_t = V^{-1}x_t$, then

  \[
  \begin{bmatrix}
  y_{1t} \\
  y_{2t}
  \end{bmatrix} =
  \begin{bmatrix}
  \lambda_1 & 0 \\
  0 & \lambda_2
  \end{bmatrix}
  \begin{bmatrix}
  E_{t}y_{1t+1} \\
  E_{t}y_{2t+1}
  \end{bmatrix}
  \]

  bounded solutions $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}$, so

  \[
  \begin{bmatrix}
  x_{1t} \\
  x_{2t}
  \end{bmatrix} =
  \begin{bmatrix}
  \gamma v_{11} \lambda_1^{-t} \\
  \gamma v_{21} \lambda_1^{-t}
  \end{bmatrix}
  \]

  • also exist bounded sunspot solutions:

    $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}$, $y_{2t+1} = 0$, $E_{t}\phi_{t+1} = 0$, bounded

  • multiple eq & sunspots possible w/ more stringent det defn
LONG-RUN TAYLOR PRINCIPLE

• Prop. 2 Given $\alpha_i > p_{ii}$ for $i = 1, 2$, the following statements are equivalent:

(A) All the eigenvalues of $M$ lie inside the unit circle.
(B) $\alpha_i > 1$, for some $i = 1, 2$, and the long-run Taylor principle (LRTP)

\[
(1 - \alpha_2)p_{11} + (1 - \alpha_1)p_{22} + \alpha_1 \alpha_2 > 1
\]

is satisfied.

• Premise $\alpha_i > p_{ii}$ all $i$ unfamiliar
  • fixed regime: MP always obeys TP
  • LRTP is hyperbola w/ asymptotes $\alpha_1 = p_{11}$ & $\alpha_2 = p_{22}$
  • restricts $\alpha$’s to economically interesting portion of hyperbola
A RANGE OF POLICIES DELIVER UNIQUENESS

$\alpha_1 > 1: p_{11}(1 - \alpha_2) + p_{22}(1 - \alpha_1) + \alpha_1\alpha_2 > 1$

- Some policy processes that deliver unique equilibria
  \[ \alpha_1 \to \infty \Rightarrow \alpha_2 > p_{22} \]
  or
  \[ p_{11} = 1 \Rightarrow \text{need } \alpha_1 > 1 \text{ and } \alpha_2 > p_{22} \]

- more active is one regime, more passive the other can be
  \[ p_{22} \to 1 \text{ OK if } \alpha_2 \approx 1 \text{ (but } < 1) \]
- ergodic prob of passive regime can be $\approx 1$ (but $< 1$)
  \[ p_{11} = p_{22} = 0 \text{ need } \alpha_2 > 1/\alpha_1 \]
- more active in one regime, less active in the other

- Figure illustrates these points
DETERMINACY REGION: FISHERIAN MODEL

\begin{align*}
\text{Region 1:} & \quad p_{11} = 0.95 ; p_{22} = 0.95 \\
\text{Region 2:} & \quad p_{11} = 0.8 ; p_{22} = 0.95 \\
\text{Region 3:} & \quad p_{11} = 0.95 ; p_{22} = 0 \\
\text{Region 4:} & \quad p_{11} = 0 ; p_{22} = 0
\end{align*}
Fisharian Model: Solution

- Define state as \((r_t, s_t)\) & find MSV solutions
  - posit regime-dependent rules:
    \[
    \pi_t = a(s_t = i) r_t
    \]
    \[
    a(s_t) = \begin{cases} 
    a_1 & \text{for } s_t = 1 \\
    a_2 & \text{for } s_t = 2 
    \end{cases}
    \]
  - expectations functions:
    \[
    E[\pi_{t+1} | s_t = 1, r_t] = [p_{11} a_1 + (1 - p_{11}) a_2] \rho r_t
    \]
    \[
    E[\pi_{t+1} | s_t = 2, r_t] = [(1 - p_{22}) a_1 + p_{22} a_2] \rho r_t
    \]
  - solve simple \(2 \times 2\) system to get \(a_1\) and \(a_2\)
**Solution**

- Solutions are:

  \[ a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

  and

  \[ a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

- \( p_{12} = 1 - p_{11}, \ p_{21} = 1 - p_{22} \) & “fixed-regime” coefficients

  \[ a_i^F = \frac{1}{\alpha_i - \rho p_{ii}}, \quad i = 1, 2 \]

- \( \alpha_1 > \alpha_2 \iff a_1 < a_2 \)
**Expectations-Formation Effects**

- Solutions are:

\[
a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right)
\]

and

\[
a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right)
\]

- Expectations-formation effects from regime 2 to regime 1
  - through \( p_{12} a_2^F \)
  - large if \( p_{12} \) large, \( p_{22} \) large, \( \alpha_2 \) small
Special Case

• Real interest rate serially uncorrelated ($\rho = 0$), solution is

$$a_1 = \frac{1}{\alpha_1}$$

and

$$a_2 = \frac{1}{\alpha_2}$$

• Looks like fixed-regime solution, BUT
  
  • determinacy in FR: $\alpha_i > 1$ all $i$
  • switching allows determinacy w/ some $\alpha_i < 1$
  • if $p_{22} < \alpha_2 < 1$, regime 2 amplifies shocks
  • possible to fit volatile data with determinate eqm?
A New-Keynesian Model

- Bare-bones model with nominal rigidities
  - from class in wide use for monetary policy analysis
  - general insights extend to more complex models now confronting data

- With recurring regime change and rational expectations:
  - How does the Taylor principle change?
  - How do impacts of demand and supply shocks change?

- Expectations-formation effects can be large
A New-Keynesian Model

- Consumption-Euler equation and AS relations

\[
x_t = \frac{E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u_t^D}{\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t^S}
\]

- Disturbances: bounded, autoregressive, mutually uncorrelated

\[
\begin{align*}
    u_t^D &= \rho_D u_{t-1}^D + \varepsilon_t^D \\
    u_t^S &= \rho_S u_{t-1}^S + \varepsilon_t^S
\end{align*}
\]

- A Taylor rule for \( s_t = 1, 2 \)

\[
i_t = \alpha(s_t)\pi_t + \gamma(s_t)x_t
\]
New-Keynesian Model: Determinacy

• Let \( \pi_{it} = \pi_t(s_t = i) \) & \( x_{it} = x_t(s_t = i) \), \( i = 1, 2 \)
• Define forecast errors
  \[
  \eta_{1t+1}^\pi = \pi_{1t+1} - E_t \pi_{1t+1} \quad \eta_{2t+1}^\pi = \pi_{2t+1} - E_t \pi_{2t+1} \\
  \eta_{1t+1}^x = x_{1t+1} - E_t x_{1t+1} \quad \eta_{2t+1}^x = x_{2t+1} - E_t x_{2t+1}
  \]
• Model is
  \[
  AY_t = BY_{t-1} + A\eta_t + Cu_t
  \]
• Unique bounded eqm requires the 4 generalized eigenvalues of \((B, A)\) to lie inside unit circle
• Derive long-run Taylor principle
New-Keynesian Model: Determinacy

- Set $\gamma(s_t) = 0$
- Intertemporal margins interact w/ expected policy to affect determinacy
- Determinacy regions expand w/ parameters that reduce ability to substitute away from future policy
  - increase degree of stickiness ($\kappa$)
  - reduce intertemporal elasticity of substitution ($\sigma$)
**Determinacy Regions Expand**

1. $p_{11} = 0.95, p_{22} = 0.95$
2. $p_{11} = 0.8, p_{22} = 0.95$
3. $p_{11} = 0.95, p_{22} = 0$
4. $p_{11} = 0, p_{22} = 0$
DET. REGIONS & PRIVATE PARAMETERS

\begin{align*}
p_{11} &= 0.9, \quad p_{22} = 0.9, \quad \omega = 0.01 \\
p_{11} &= 0.9, \quad p_{22} = 0.9, \quad \omega = 0.99 \\
p_{11} &= 0.9, \quad p_{22} = 0.9, \quad \sigma = 0.01 \\
p_{11} &= 0.9, \quad p_{22} = 0.9, \quad \sigma = 10
\end{align*}
NEW-KEYNESIAN MODEL: SOLUTIONS

- MSV solution is straightforward to compute
- Easiest to consider numerical examples
- For inflation, intuition from fixed regimes carries through
  - more active MP process reduces inflation volatility
- For output, switching introduces non-monotonicity
  - more active MP process can raise or lower output volatility, depending on source of shock
A Return to the 1970s?

- Studies find Fed passive 1960-79; active since 1982
- Fears of reverting to 1970s behind calls for IT
- Fiscal policy may be an impetus for switching to passive MP
- Embed estimates of Lubik-Schorfheide in switching setup
  - compute set of \((p_{11}, p_{22})\) that deliver uniqueness
- Implications
  - inference that US switched from indeterminate to determinate eqm requires current state be absorbing
  - fixed regime badly mispredicts impacts of supply & demand shocks
Determinacy Regions: L-S Estimates

LS: $\alpha_1 = 2.19, \gamma_1 = .30, \alpha_2 = .89, \gamma_2 = .15$
Dark: high flexibility ($\sigma = 1.04, \kappa = 1.07$)
Light: low flexibility ($\sigma = 2.84, \kappa = .27$)
FINANCIAL CRISSES & BUSINESS CYCLES

• MP shifts focus from inflation to other concerns
  • financial stability & job creation
  • shift can last few months or more than year
  • during Greenspan era: 2 market crashes, 2 foreign financial crises, 2 jobless recoveries
  • documented by Marshall and Rabanal

• Take normal times to be $\alpha_1 = 1.5$, $\gamma_1 = .25$, and persistent
  • other regime: $\gamma_2 = .5$, $\alpha_2$ and $p_{22}$ vary
  • a crude characterization of those events

• Spillovers from demand shocks can make inflation much more volatile and output much less volatile than if the active regime were permanent
### Financial Crises & Business Cycles

<table>
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<th>Demand</th>
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<th>Supply</th>
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<tr>
<td></td>
<td>Inflation</td>
<td>Output</td>
<td>Inflation</td>
<td>Output</td>
</tr>
<tr>
<td>( p_{22} = 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_2 = .25 )</td>
<td>1.060</td>
<td>1.011</td>
<td>1.092</td>
<td>.994</td>
</tr>
<tr>
<td>( \alpha_2 = 0 )</td>
<td>1.073</td>
<td>1.014</td>
<td>1.110</td>
<td>.992</td>
</tr>
<tr>
<td>( p_{22} = .75 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_2 = .25 )</td>
<td>1.268</td>
<td>.886</td>
<td>1.412</td>
<td>1.066</td>
</tr>
<tr>
<td>( \alpha_2 = 0 )</td>
<td>1.454</td>
<td>.807</td>
<td>1.653</td>
<td>1.104</td>
</tr>
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**Standard Deviation Active Regime Relative to Fixed Regime**

Active and fixed regimes set \( \alpha_1 = \alpha = 1.5, \gamma_1 = \gamma = .25; \gamma_2 = .5 \)
Empirical Implications of Switching

- Commonplace for empirical work to split data into regime-dependent sub-periods
- Estimates then interpreted in fixed-regime theoretical model
- We simulate switching eqm, estimate correctly-specified (fixed-regime) identified VARs
  - assume econometrician knows when regime changed
- Estimated model

\[
\begin{align*}
x_t & = \delta i_t + u_{t}^{D} + \text{lags} \\
\pi_t & = \theta x_t + u_{t}^{S} + \text{lags} \\
i_t & = \alpha \pi_t + \gamma x_t + u_{t}^{MP} + \text{lags}
\end{align*}
\]
**Empirical Implications of Switching**

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\bar{\gamma}$</th>
<th>$\delta$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>2.182</td>
<td>0.30</td>
<td>-1.690</td>
<td>0.409</td>
</tr>
<tr>
<td>Regime 2</td>
<td>0.885</td>
<td>0.15</td>
<td>-0.750</td>
<td>1.675</td>
</tr>
<tr>
<td>Full Sample</td>
<td>1.375</td>
<td>0.225</td>
<td>-1.476</td>
<td>0.657</td>
</tr>
</tbody>
</table>

Estimates from an identified VAR using simulated data.
Regime 1 is conditional on remaining in regime with $\alpha_1 = 2.19$.
Regime 2 is conditional on remaining in regime with $\alpha_2 = 0.89$.
Full sample is recurring changes from regime 1 to regime 2.
$\alpha$ is the estimated response of monetary policy to inflation.
$\bar{\gamma}$ is the policy response to output, held fixed in estimation.
Demand & Supply Shocks: Lubik-Schorfheide Parameters

\[ \alpha_1 = 2.19, \gamma_1 = .30, \alpha_2 = .89, \gamma_2 = .15, p_{11} = .95, p_{22} = .93 \]

Dashed: fixed regime; Solid: active, switching
SUMMARY

- A broader perspective on Taylor principle and range of unique bounded equilibria it supports
- Endowing conventional models with empirically relevant MP switching processes
  - drastically alters conditions for a unique bounded eqm
  - generates important expectations-formation effects
- Developed a two-step solution method to get determinacy conditions and solutions
- Conventional models extremely sensitive to deviation from usual assumption that policy is permanent
- The possibility of regime change should be the default assumption in theoretical models
Wrap Up

- Many potential applications
  - any purely forward-looking model
  - exchange rate determination: switch between fixed & floating
  - term structure: policy switching
  - technology: switch between high- and low-growth periods
  - terms of trade: persistent & transitory changes

- Need to develop methods to allow analytical solutions with endogenous state variables