ECB Mini-Course: Monetary-Fiscal Policy Interactions

Lecture 4. Generalizing Policy Interactions (A)

Eric M. Leeper

Indiana University

September 2010
THE MESSAGES

• Draws heavily from “Generalizing the Taylor Principle,” with Troy Davig (AER, June 2007)

• We do see policy rules—or regimes—change
  • to study the implications of recurring changes, need to model them coherently

• Before studying monetary-fiscal interactions when policy regimes can change, need some preliminary analysis when only MP can switch

• This allows simple analytical derivations that build intuition and understanding

• Many of our inferences are monetary policy effects change in subtle ways once we allow recurring regime change

• Subsequent work will allow both monetary and fiscal regime to undergo recurring change
Simplifying Policy

- Monetary policy is complex
- For descriptive & prescriptive reasons, seek to simplify
- Most successful simplification due to Taylor

\[ i_t = \bar{i} + \alpha(\pi_t - \pi^*) + \gamma x_t + \varepsilon_t \]

- Taylor principle: \( \alpha > 1 \)
  - necessary & sufficient for unique bounded eqm (w/ bounded shocks)
- Unique & stable eqm necessary for good policy
  - rules out arbitrarily large fluctuations
The Taylor Rule & Principle

- Central banks can stabilize economy by adjusting nominal interest rate more than one-for-one with inflation
  - approximates Federal Reserve behavior since 1982
  - nearly optimal in workhorse class of monetary models
  - used by central banks as a benchmark
- Maintains two key assumptions
  - fiscal policy is perpetually passive
  - policy rule permanent & agents believe change impossible
- Here we relax this second assumption
  - rule evolves according to a Markov chain
  - consider two conventional monetary models
Generalizing the Taylor Rule & Principle

- $\alpha(s_t), \gamma(s_t) s_t \sim$ Markov chain
- $s_t$: "rule," "regime," "state"
- $s_t$ exogenous (for now)
- Can believe actual policy rule time invariant
  - but Taylor rule is a gross simplification of reality
  - paper shows that a particular form of non-linearity can change predictions of models
In the Fisherian Model . . .

- Derive long-run Taylor principle
  - imposes much weaker conditions on MP for uniqueness
  - departures from short-run Taylor principle can be substantial—but brief—or modest—and prolonged
  - the more “hawkish” one regime is, the more “dovish” the other can be and still deliver uniqueness
  - “expectations formation effects”—beliefs about possible future regimes affect current eqm, increasing volatility even in a regime that satisfies TP
In the New-Keynesian Model . . .

- Derive long-run Taylor principle: dramatically expands region of determinacy
- Inference that inflation of the 70’s due to failure to obey TP does not hold up when expectations embed possibility of regime change
- Occasional large departures from TP—due to worries about financial instability or economic weakness—can have quantitatively important impacts even in a regime that satisfies TP
- Misleading inferences can arise from dividing data into regime-specific periods to interpret estimates as arising from distinct fixed regimes
WHY REGIME CHANGE?

- Evidence that monetary policy regime changed
- Institutional or policy reforms
  - adoption of inflation targeting by over 20 countries
  - Fed’s “just trust us” approach
- Logical consistency
  - if regime has changed, regime can change
  - expectations depend on prob. distn. over possible regimes
- Recurring: in US, no legislated change installed Volcker or Greenspan
  - confluence of economic/political conditions allowed US to dodge a bullet and get Bernanke (coulda’ been a FOG)
A Modeling Choice

- Because Taylor rule a gross simplification, deviations occur
  - can be large and serially correlated
  - are systematic responses to state of economy
- How should we model these deviations?
  - shuffled into the $\varepsilon$’s?
  - time-varying feedback coefficients, $\alpha_t$ & $\gamma_t$?
- $\varepsilon$’s affect conditional expectations
- $\alpha_t$ & $\gamma_t$ affect expectations *functions*
- A substantive choice
MODEL OF INFLATION DETERMINATION

• A simple Fisherian economy

\[ i_t = E_t \pi_{t+1} + r_t \]
\[ r_t = \rho r_{t-1} + \nu_t, \quad \nu \text{ bounded support} \]
\[ i_t = \alpha(s_t) \pi_t, \quad s_t \text{ Markov; } s_t = 1, 2 \]

\[ p_{ij} = P[s_t = j \mid s_{t-1} = i] \]

\[ \alpha(s_t) = \begin{cases} 
\alpha_1 & \text{for } s_t = 1 \\
\alpha_2 & \text{for } s_t = 2 
\end{cases} \]

• a monetary policy regime: realization of \( \alpha(s_t) \)
• a monetary policy process: collection \( (\alpha_1, \alpha_2, p_{11}, p_{22}) \)
• policy is active if \( \alpha_i > 1 \); passive if \( \alpha_i < 1 \)
Determinacy: Definition

- Seek generalization of Taylor principle
  - necessary & sufficient condition for existence of unique bounded eqm
- Why boundedness?
  - consistent w/ standard definition under fixed regime
  - corresponds to locally unique eqm
    - can analyze small perturbations
  - considering log-linearized models
    - boundedness ensures approximations are good
**Determinacy: Formalism**

Model: \( \alpha(s_t)\pi_t = E_t\pi_{t+1} + r_t \)

- Let \( \Omega_t^{-s} = \{r_t, r_{t-1}, \ldots, s_{t-1}, s_{t-2}, \ldots\} \) and \( \Omega_t = \Omega_t^{-s} \cup \{s_t\} \)
- Integrating over \( s_t \), for \( s_t = 1 \) and \( s_t = 2 \)

\[
E_t\pi_{t+1} = E[\pi_{t+1} \mid s_t = i, \Omega_t^{-s}]
= p_{i1}E[\pi_{1t+1} \mid \Omega_t^{-s}] + p_{i2}E[\pi_{2t+1} \mid \Omega_t^{-s}]
\]

where \( \pi_{it} = \pi_t(s_t = i, r_t) \), the solution when \( s_t = i \)

- The system is

\[
\begin{bmatrix}
\alpha_1 & 0 \\
0 & \alpha_2 \\
\end{bmatrix}
\begin{bmatrix}
\pi_{1t} \\
\pi_{2t} \\
\end{bmatrix}
= \begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22} \\
\end{bmatrix}
\begin{bmatrix}
E_t\pi_{1t+1} \\
E_t\pi_{2t+1} \\
\end{bmatrix}
+ \begin{bmatrix}
r_t \\
r_t \\
\end{bmatrix}
\]

where \( E_t\pi_{it+1} \) denotes \( E[\pi_{it+1} \mid \Omega_t^{-s}] \)
Determinacy: Formalism (con’t)

- Write system as

\[ \pi_t = ME_t\pi_{t+1} + \alpha^{-1}r_t \]

- MSV solution: \( \pi_t \) function only of \((r_t, s_t)\)
- Define \( x_t = \pi_t - \pi_t^{MSV}(r_t, s_t) \)
- Bounded soln for \( \{x_t\} \iff \text{bounded soln for } \{\pi_t\} \)
- We study: \( x_t = ME_t x_{t+1} \)
- Proof of determinacy shows that under certain conditions on the policy process, \( x_t = 0 \) is the only solution
Prop. 1 When $\alpha_i > 0$, a unique bounded solution exists iff all the eigenvalues of $M$ lie inside the unit circle.

Sufficiency: the usual proof in linear RE models

- Intuition: boundedness requires that $\lim_{n \to \infty} M^n = 0$, so $x_t = 0$ the only solution.
- Delivered by eigenvalue condition.
Determinacy: Formalism (con’t)

- Necessity: Suppose $\lambda_1 \geq 1, \lambda_2 < 1$
  - diagonalize $M$, let $y_t = V^{-1}x_t$, then
    \[
    \begin{bmatrix}
    y_{1t} \\
    y_{2t}
    \end{bmatrix} =
    \begin{bmatrix}
    \lambda_1 & 0 \\
    0 & \lambda_2
    \end{bmatrix}
    \begin{bmatrix}
    E_t y_{1t+1} \\
    E_t y_{2t+1}
    \end{bmatrix}
    \]

  bounded solutions $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}$, so
  \[
  \begin{bmatrix}
  x_{1t} \\
  x_{2t}
  \end{bmatrix} =
  \begin{bmatrix}
  \gamma v_{11} \lambda_1^{-t} \\
  \gamma v_{21} \lambda_1^{-t}
  \end{bmatrix}
  \]

- also exist bounded sunspot solutions:
  
  $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}, y_{2t+1} = 0, E_t \phi_{t+1} = 0$, bounded

- multiple eq & sunspots possible w/ more stringent det defn
LONG-RUN TAYLOR PRINCIPLE

• **Prop. 2** Given $\alpha_i > p_{ii}$ for $i = 1, 2$, the following statements are equivalent:

  (A) All the eigenvalues of $M$ lie inside the unit circle.

  (B) $\alpha_i > 1$, for some $i = 1, 2$, and the long-run Taylor principle (LRTP)

  \[
  (1 - \alpha_2) p_{11} + (1 - \alpha_1) p_{22} + \alpha_1 \alpha_2 > 1
  \]

  is satisfied.

• **Premise** $\alpha_i > p_{ii}$ all $i$ unfamiliar
  - fixed regime: MP always obeys TP
  - LRTP is hyperbola w/ asymptotes $\alpha_1 = p_{11}$ & $\alpha_2 = p_{22}$
  - restricts $\alpha$’s to economically interesting portion of hyperbola
**A Range of Policies Deliver Uniqueness**

\[ \alpha_1 > 1: \ p_{11}(1 - \alpha_2) + p_{22}(1 - \alpha_1) + \alpha_1 \alpha_2 > 1 \]

- Some policy processes that deliver unique equilibria
  \[ \alpha_1 \to \infty \Rightarrow \alpha_2 > p_{22} \]
  or
  \[ p_{11} = 1 \Rightarrow \text{need } \alpha_1 > 1 \text{ and } \alpha_2 > p_{22} \]

- more active is one regime, more passive the other can be
  \[ p_{22} \to 1 \text{ OK if } \alpha_2 \approx 1 (\text{but } < 1) \]
- ergodic prob of passive regime can be \( \approx 1 \) (but \( < 1 \))
  \[ p_{11} = p_{22} = 0 \text{ need } \alpha_2 > 1/\alpha_1 \]
- more active in one regime, less active in the other
- Figure illustrates these points
**Determinacy Region: Fisherian Model**

- **Case 1:** \( p_{11} = 0.95 ; p_{22} = 0.95 \)
  - Region: \( \alpha_1 \) and \( \alpha_2 \) coordinates

- **Case 2:** \( p_{11} = 0.8 ; p_{22} = 0.95 \)
  - Region: \( \alpha_1 \) and \( \alpha_2 \) coordinates

- **Case 3:** \( p_{11} = 0.95 ; p_{22} = 0 \)
  - Region: \( \alpha_1 \) and \( \alpha_2 \) coordinates

- **Case 4:** \( p_{11} = 0 ; p_{22} = 0 \)
  - Region: \( \alpha_1 \) and \( \alpha_2 \) coordinates
FISHERIAN MODEL: SOLUTION

• Define state as \((r_t, s_t)\) & find MSV solutions

  • posit regime-dependent rules:

    \[\pi_t = a(s_t = i)r_t\]

    \[a(s_t) = \begin{cases} 
    a_1 \text{ for } s_t = 1 \\
    a_2 \text{ for } s_t = 2 
  \end{cases}\]

  • expectations functions:

    \[E[\pi_{t+1} | s_t = 1, r_t] = [p_{11}a_1 + (1 - p_{11})a_2]\rho r_t\]

    \[E[\pi_{t+1} | s_t = 2, r_t] = [(1 - p_{22})a_1 + p_{22}a_2]\rho r_t\]

  • solve simple \(2 \times 2\) system to get \(a_1\) and \(a_2\)
SOLUTION

• Solutions are:

\[ a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

and

\[ a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

\[ p_{12} = 1 - p_{11}, \ p_{21} = 1 - p_{22} \ & \text{“fixed-regime” coefficients} \]

\[ a_i^F = \frac{1}{\alpha_i - \rho p_{ii}}, \quad i = 1, 2 \]

• \( \alpha_1 > \alpha_2 \iff a_1 < a_2 \)
**Expectations-Formation Effects**

- Solutions are:

\[
a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right)
\]

and

\[
a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right)
\]

- Expectations-formation effects from regime 2 to regime 1
  - through \( p_{12} a_2^F \)
  - large if \( p_{12} \) large, \( p_{22} \) large, \( \alpha_2 \) small
Special Case

- Real interest rate serially uncorrelated \((\rho = 0)\), solution is
  \[ a_1 = \frac{1}{\alpha_1} \]
  \[ a_2 = \frac{1}{\alpha_2} \]

- Looks like fixed-regime solution, BUT
  - determinacy in FR: \(\alpha_i > 1\) all \(i\)
  - switching allows determinacy w/ some \(\alpha_i < 1\)
  - if \(p_{22} < \alpha_2 < 1\), regime 2 *amplifies* shocks
  - possible to fit volatile data with determinate eqm?
A New-Keynesian Model

• Bare-bones model with nominal rigidities
  • from class in wide use for monetary policy analysis
  • general insights extend to more complex models now confronting data

• With recurring regime change and rational expectations:
  • How does the Taylor principle change?
  • How do impacts of demand and supply shocks change?

• Expectations-formation effects can be large
A New-Keynesian Model

- Consumption-Euler equation and AS relations

\[ x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u^D_t \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u^S_t \]

- Disturbances: bounded, autoregressive, mutually uncorrelated

\[ u^D_t = \rho_D u^D_{t-1} + \varepsilon^D_t \]
\[ u^S_t = \rho_S u^S_{t-1} + \varepsilon^S_t \]

- A Taylor rule for \( s_t = 1, 2 \)

\[ i_t = \alpha(s_t) \pi_t + \gamma(s_t) x_t \]
**New-Keynesian Model: Determinacy**

- Let $\pi_{it} = \pi_t(s_t = i)$ & $x_{it} = x_t(s_t = i)$, $i = 1, 2$
- Define forecast errors
  \[
  \eta_{\pi t+1}^\pi = \pi_{1t+1} - E_t \pi_{1t+1} \\
  \eta_{x t+1}^x = x_{1t+1} - E_t x_{1t+1} \\
  \eta_{\pi t+1}^\pi = \pi_{2t+1} - E_t \pi_{2t+1} \\
  \eta_{x t+1}^x = x_{2t+1} - E_t x_{2t+1}
  \]
- Model is
  \[
  AY_t = BY_{t-1} + A\eta_t + Cu_t
  \]
- Unique bounded eqm requires the 4 generalized eigenvalues of $(B, A)$ to lie inside unit circle
- Derive *long-run Taylor principle*
New-Keynesian Model: Determinacy

- Set $\gamma(s_t) = 0$
- Intertemporal margins interact w/ expected policy to affect determinacy
- Determinacy regions expand w/ parameters that reduce ability to substitute away from future policy
  - increase degree of stickiness ($\kappa$)
  - reduce intertemporal elasticity of substitution ($\sigma$)
Determinacy Regions Expand

\[ p_{11} = 0.95 \quad p_{22} = 0.95 \]

\[ p_{11} = 0.8 \quad p_{22} = 0.95 \]

\[ p_{11} = 0.95 \quad p_{22} = 0 \]

\[ p_{11} = 0 \quad p_{22} = 0 \]
Det. Regions & Private Parameters

- $p_{11} = 0.9, p_{22} = 0.9, \omega = 0.01$
- $p_{11} = 0.9, p_{22} = 0.9, \omega = 0.99$
- $p_{11} = 0.9, p_{22} = 0.9, \sigma = 0.01$
- $p_{11} = 0.9, p_{22} = 0.9, \sigma = 10$
New-Keynesian Model: Solutions

- MSV solution is straightforward to compute
- Easiest to consider numerical examples
- For inflation, intuition from fixed regimes carries through
  - more active MP process reduces inflation volatility
- For output, switching introduces non-monotonicity
  - more active MP process can raise or lower output volatility, depending on source of shock
A Return to the 1970s?

- Studies find Fed passive 1960-79; active since 1982
- Fears of reverting to 1970s behind calls for IT
- Fiscal policy may be an impetus for switching to passive MP
- Embed estimates of Lubik-Schorfheide in switching setup
  - compute set of \((p_{11}, p_{22})\) that deliver uniqueness
- Implications
  - inference that US switched from indeterminate to determinate eqm requires current state be absorbing
  - fixed regime badly mispredicts impacts of supply & demand shocks
**Determinacy Regions: L-S Estimates**

**LS:** $\alpha_1 = 2.19, \gamma_1 = .30, \alpha_2 = .89, \gamma_2 = .15$

Dark: high flexibility ($\sigma = 1.04, \kappa = 1.07$)

Light: low flexibility ($\sigma = 2.84, \kappa = .27$)
Financial Crises & Business Cycles

- MP shifts focus from inflation to other concerns
  - financial stability & job creation
  - shift can last few months or more than year
  - during Greenspan era: 2 market crashes, 2 foreign financial crises, 2 jobless recoveries
  - documented by Marshall and Rabanal
- Take normal times to be $\alpha_1 = 1.5$, $\gamma_1 = .25$, and persistent
  - other regime: $\gamma_2 = .5$, $\alpha_2$ and $p_{22}$ vary
  - a crude characterization of those events
- Spillovers from demand shocks can make inflation much more volatile and output much less volatile than if the active regime were permanent
## Financial Crises & Business Cycles

<table>
<thead>
<tr>
<th>Demand</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td><strong>Supply</strong></td>
</tr>
<tr>
<td>Inflation</td>
<td>Output</td>
</tr>
<tr>
<td>$p_{11} = .95$</td>
<td></td>
</tr>
<tr>
<td>$p_{22} = 0$</td>
<td>$\alpha_2 = .25$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2 = 0$</td>
</tr>
<tr>
<td>$p_{22} = .75$</td>
<td>$\alpha_2 = .25$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2 = 0$</td>
</tr>
</tbody>
</table>

**Standard Deviation Active Regime Relative to Fixed Regime**

Active and fixed regimes set $\alpha_1 = \alpha = 1.5$, $\gamma_1 = \gamma = .25$; $\gamma_2 = .5$
**Demand & Supply Shocks:**

**Lubik-Schorfheide Parameters**

\[ \alpha_1 = 2.19, \gamma_1 = .30, \alpha_2 = .89, \gamma_2 = .15, p_{11} = .95, p_{22} = .93 \]

Dashed: fixed regime; Solid: active, switching
Empirical Implications of Switching

- Commonplace for empirical work to split data into regime-dependent sub-periods
- Estimates then interpreted in fixed-regime theoretical model
- We simulate switching eqm, estimate correctly-specified (fixed-regime) identified VARs
  - assume econometrician knows when regime changed
- Estimated model

\[
x_t = \delta i_t + u_t^D + lags
\]
\[
\pi_t = \theta x_t + u_t^S + lags
\]
\[
i_t = \alpha \pi_t + \gamma x_t + u_t^{MP} + lags
\]
Empirical Implications of Switching

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\bar{\gamma}$</th>
<th>$\delta$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>2.182</td>
<td>0.30</td>
<td>-1.690</td>
<td>0.409</td>
</tr>
<tr>
<td>Regime 2</td>
<td>0.885</td>
<td>0.15</td>
<td>-0.750</td>
<td>1.675</td>
</tr>
<tr>
<td>Full Sample</td>
<td>1.375</td>
<td>0.225</td>
<td>-1.476</td>
<td>0.657</td>
</tr>
</tbody>
</table>

Estimates from an identified VAR using simulated data. Regime 1 is conditional on remaining in regime with $\alpha_1 = 2.19$. Regime 2 is conditional on remaining in regime with $\alpha_2 = 0.89$. Full sample is recurring changes from regime 1 to regime 2. $\alpha$ is the estimated response of monetary policy to inflation. $\bar{\gamma}$ is the policy response to output, held fixed in estimation.
Summary

- A broader perspective on Taylor principle and range of unique bounded equilibria it supports
- Endowing conventional models with empirically relevant MP switching processes
  - drastically alters conditions for a unique bounded eqm
  - generates important expectations-formation effects
- Developed a two-step solution method to get determinacy conditions and solutions
- Conventional models extremely sensitive to deviation from usual assumption that policy is permanent
- The possibility of regime change should be the default assumption in theoretical models
Many potential applications
  - any purely forward-looking model
  - exchange rate determination: switch between fixed & floating
  - term structure: policy switching
  - technology: switch between high- and low-growth periods
  - terms of trade: persistent & transitory changes

Need to develop methods to allow analytical solutions with endogenous state variables