The fiscal theory of the price level in an open economy

Betty C. Daniel*

Department of Economics, University at Albany – SUNY, Albany, NY 12222, USA

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Abstract

Recent work (Loyo, E., Going international with the fiscal theory of the price level. Manuscript, John F. Kennedy School of Government, Harvard University, 1998; Dupor, B., Exchange rates and the fiscal theory of the price level. Journal of Monetary Economics 45, 613–630) has shown that extension of the fiscal theory of the price level to an open economy yields indeterminate prices and exchange rates, calling into question the usefulness of the theory. This paper shows that by carefully tailoring policies in an independent fashion, governments can eliminate price and exchange rate indeterminacy. Specifically, governments are assumed to care about the welfare of their agents so that they never set policy to yield an intertemporal government budget surplus. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The fiscal theory of the price level has generated considerable recent interest (Leeper, 1991; Woodford, 1994, 1995; Sims, 1994, 1997; Cochrane, 1998a, b).

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*Tel.: +1-518-442-4747; fax: +1-518-442-4736.

E-mail address: b.daniel@albany.edu (B.C. Daniel).

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The theory states that the price level is determined to equate the real value of nominal government debt with the present value of primary government budget surpluses. It is interesting on several counts. It predicts price level determinacy, even in the absence of a monetary aggregate. More importantly for policy, it predicts price level stability in the presence of rapid and uneven financial innovation, and price level determinacy in the presence of monetary policy rules focused on interest rate pegging. However, several recent papers have shown that extending the fiscal theory to an open economy with multiple currencies results in price level indeterminacy (Loyo, 1998; Dupor, 2000). This calls into question the usefulness of the theory.

This paper shows that by carefully tailoring policy rules in an independent manner, governments can eliminate price and exchange rate indeterminacy. Specifically, when each government sets policy to avoid an intertemporal government budget surplus, price and exchange rate indeterminacy are eliminated. Moreover, we argue that any government concerned with the welfare of its residents would choose this type of “no-surplus” policy. We show that when all governments follow a “no-surplus” policy, the open economy fiscal theory of the price level becomes the exact analogue to the closed-economy fiscal theory.

The paper is organized as follows. Section 2 presents a closed-economy model, in which the standard results of price level determinacy with non-Ricardian fiscal policy and indeterminacy with Ricardian policy, are derived. Section 3 contains the open-economy results on indeterminacy. Section 4 describes the model with open-economy determinacy, and Section 5 contains conclusions.

2. Closed economy

In this section, we present the fiscal theory of the price level in the context of a simple, representative-agent, money-in-the-utility-function model. The closed-economy model serves as a benchmark for the subsequent open-economy model. The behavior of the representative agent is presented first. This is followed by a description of government policy, and finally by a discussion of equilibrium and the fiscal theory. All models assume perfect foresight and are in continuous time.

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1It is important to note that we are assuming that governments avoid intertemporal budget surpluses, not flow budget surpluses at all points in time.
2.1. Representative agent

2.1.1. Budget constraints

Let the economy be populated by a large number of identical representative agents with unit mass. Assuming that an agent’s assets consist of nominal interest-bearing bonds (\(B_t\)) and currency (\(M_t\)), where subscript \(t \geq 0\) denotes a point in time, and letting a dot denote a time derivative, an agent’s flow budget constraint can be expressed as

\[
\dot{B}_t + \dot{M}_t \leq i_t B_t + P_t \bar{y} - T_t - C_t,
\]

where \(i\) is the nominal interest rate on debt, \(\bar{y}\) is real endowment income, \(P_t\) is the price level, \(T_t\) is nominal taxes, and \(C_t\) is nominal consumption. In general, capital letters denote nominal values and small letters denote real values (the nominal interest rate is the exception). Real endowment income is assumed to be constant across time, and carries an overbar to denote constancy. Letting the real interest rate be given by

\[
r_t = i_t - \frac{\dot{P}_t}{P_t},
\]

an agent’s flow budget constraint can be expressed in real terms as

\[
\dot{b}_t + \dot{m}_t \leq r_t (b_t + m_t) + \bar{y} - i_t m_t - \tau_t - c_t.
\]

This shows that asset growth is bound by the excess of real endowment and interest income over real expenditures on money (\(i_t m_t\)), taxes (\(\tau_t\)), and consumption (\(c_t\)).

Additionally, we assume that each agent faces a borrowing constraint such that

\[
\lim_{T \to \infty} (b_T + m_T) \beta_T \geq 0,
\]

where

\[
\beta_T = e^{-\int_0^T r_t \, dt}.
\]

Integrating Eq. (2) over time and imposing Eq. (3), an agent’s intertemporal budget constraint, in real terms, is expressed as

\[
\int_{t=0}^{\infty} (\bar{y} - \tau_t - c_t - i_t m_t) \beta_t \, dt + b_0 + m_0 \geq 0.
\]

This requires that the present value of the agent’s total expenditure on consumption, taxes, and money not exceed the present value of endowment income plus initial wealth.

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\(^2\) This assumption allows use of the same notation for aggregate and per capita values.
2.1.2. Optimal behavior

The representative agent maximizes the present value of his utility from consumption and real money balances

\[
U = \int_{t=0}^{\infty} [u(c_t) + v(m_t)]e^{-\delta t} \, dt,
\]

where the functions \( u \) and \( v \) are assumed twice differentiable, strictly increasing, and strictly concave, with \( u'(0) = \infty \). Maximization of utility is subject to constraints given by Eqs. (2) and (3).

Optimality requires

\[
\frac{c_t u''(c_t)}{u'(c_t)} = \theta - r_t,
\]

\[
v'(m_t) = \left[ r_t + \frac{\dot{P}_t}{P_t} \right] u'(c_t),
\]

\[
\int_{t=0}^{\infty} (\bar{y} - \tau_t - c_t - i_t m_t)\beta_t \, dt + b_0 + m_0 = \lim_{T \to \infty} (b_T + m_T)\beta_T = 0.
\]

Eq. (7) is the standard consumption Euler equation. Eq. (8) can be interpreted as an implicit money demand equation, while Eq. (9) combines the transversality condition with the representative agent’s budget constraint, yielding the condition that his intertemporal budget constraint hold with equality.

2.2. Governments

Let real government spending be fixed \((\bar{g})\). Also, assume that the government issues nominal debt in the form of interest-bearing debt \((B^g_t)\) and money \((M^g_t)\). The superscript \( g \) is used to distinguish supplies of assets issued by the government from holdings of these assets by agents. This assures that the flow borrowing constraint, imposed to yield Eq. (3), is never binding. Of course, in equilibrium, they are identical.

The government’s flow budget constraint in real terms is given by

\[
\bar{b}_t^g + \bar{m}_t^g = r_i(b_t^g + m_t^g) + \bar{g} - \tau_t - i_t m_t^g.
\]

This requires that real debt service \([r_i(b_t^g + m_t^g)]\) plus real expenditures on goods \((\bar{g})\) be financed by tax and seigniorage revenue \((\tau_t + i_t m_t^g)\) together with the issuance of new debt.

Integrating the government’s flow budget constraint, given by Eq. (10), over time yields

\[
\lim_{T \to \infty} (b_T^g + m_T^g)\beta_T = \int_{t=0}^{\infty} (\bar{g} - \tau_t - i_t m_t^g)\beta_t \, dt + b_0^g + m_0^g.
\]

This gives \( i_t m_t \) the interpretation of seigniorage revenue.
The government’s intertemporal budget constraint holds if

\[
\lim_{T \to \infty} (b_T^g + m_T^g) \beta_T = \int_{t=0}^{\infty} (\bar{y} - \tau_t - i_t m_T^g) \beta_t \, dt + b_0^g + m_0^g = 0. \tag{12}
\]

Eq. (12) is the familiar requirement that present-value government surpluses be sufficient to repay initial debt.

The definition of government policy follows Kocherlakota and Phelan (1999).

**Definition 1.** A policy rule \( \Pi \) is a function that maps each positive price path \( P = (P_t)_{t \geq 0} \) into a policy variable path \( (\tau_t, i_t, b_t^g, m_t^g)_{t \geq 0} \), satisfying the government’s flow budget constraint (Eq. (10)).

Note that even though the government actually chooses nominal bonds and money along a given price path, the policy rule can be written in terms of real values, since the policy is chosen for a given price level.

Using the government’s intertemporal budget constraint, we can define Ricardian government policy, following Kocherlakota and Phelan (1999).

**Definition 2.** A policy rule is Ricardian if the rule implies

\[
\lim_{T \to \infty} (b_T^g + m_T^g) \beta_T = \int_{t=0}^{\infty} (\bar{y} - \tau_t - i_t m_T^g) \beta_t \, dt + b_0^g + m_0^g = 0
\]

for all \( P \). Otherwise, the policy rule is non-Ricardian.

### 2.3. Equilibrium

Now, consider equilibrium under Ricardian and non-Ricardian policy. An equilibrium is a path \((\tau, i, P, b^g, m^g, b, m, c) = (\tau_t, i_t, P_t, b_t^g, m_t^g, b_t, m_t, c_t)_{t \geq 0}\) such that (1) each agent’s optimality conditions, given by Eqs. (7)–(9), are satisfied, with the definitions given in Eqs. (1) and (4); (2) government policy is \((\tau, i, b^g, m^g) = \Pi(P)\); and (3) goods, money, and bond markets are all clear.

By Walras’ Law, we explicitly consider only goods and money markets. To begin, we prove the following well-known proposition.

**Proposition 1.** Market equilibrium requires intertemporal government budget balance, Eq. (12).

Consider consumption supply and demand. The supply of goods available for consumption is constant and equal to \( \bar{y} - \bar{g} \). Therefore, in equilibrium, consumption demand must also be constant. Using Eq. (7), this implies that \( r_t = \theta \), such that \( \beta_t = e^{-\theta t} \). Using Eq. (9), each agent’s consumption demand
can be expressed as

\[ c_t = \theta \left( b_t + m_t \right) + \int_{s=t}^{\infty} \left( \tilde{y} - \tau_s - i_s m_t \right) e^{-\theta(s-t)} \, ds \].

Assuming a unit mass of agents, aggregating across agents, imposing bond and money market equilibrium, and using the integral of the government’s flow budget constraint, Eq. (11), to substitute for \( \int_{s=t}^{\infty} i_s m_s e^{-\theta(s-t)} \, ds \), yields an expression for aggregate consumption demand as

\[ c_t = \theta \left[ \int_{s=t}^{\infty} (\tilde{y} - \bar{\gamma}) e^{-\theta(s-t)} \, ds + \lim_{T \to \infty} \left( b_T^e + m_T^e \right) e^{-\theta(T-t)} \right]. \]

Performing the integration and equating consumption demand and supply yields

\[ \lim_{T \to \infty} \left( b_T^e + m_T^e \right) e^{-\theta(T-t)} = 0 \]

as a necessary condition for equilibrium.

Therefore, in equilibrium, the government’s intertemporal budget constraint, given by Eq. (12), must hold. However, if government policy is Ricardian, then government policy ensures that Eq. (12) holds for any \( P \). Eq. (12) is an independent equilibrium condition only if policy is non-Ricardian.

The money market is in equilibrium when all money issued by the government is willingly held. Substituting the supply of real money and the equilibrium values for consumption and the real interest rate into Eq. (8), aggregated across agents, yields

\[ v'(m_T^e) = \left[ \theta + \frac{\dot{P}}{P} \right] u'(\bar{y}). \]

Now, consider the nature of equilibrium under Ricardian policy. Since Ricardian government policy assures that Eq. (12) holds for any price level, there is only one equation, given by (13), to determine both the initial value for price and its rate of change, with the path for policy variables determined by \( \Pi(P) \). It is well known that under many assumptions about functional forms, there are infinitely many equilibria. Specifically, with nothing to pin down initial conditions, there is an equilibrium corresponding to each feasible choice of \( P_0 \).

In contrast, consider the nature of equilibrium under non-Ricardian policy. Since government policy does not automatically assure intertemporal government budget balance, Eq. (12) becomes an independent equilibrium condition. There are now two independent equations, given by Eqs. (12) and (13), to determine the initial price level and its rate of change, with the path of policy variables \( (\tau, i, B^e, M) \) determined by \( \Pi(P) \). These two equations are sufficient to pin down both the initial condition and the time rate of change of price. If there is an equilibrium, and there might not be one under arbitrary non-Ricardian policy, it is unique. As emphasized by advocates of the fiscal theory of the price
level, non-Ricardian policy can be an equilibrium selection device, yielding determinacy.

3. Open economy

Now, consider what happens to the theory when it is extended to an open economy. To keep the presentation as simple as possible, assume a two-country, two-currency, one-good world. Agents and governments can issue nominal bonds denominated in either currency, and these bonds are perfect substitutes. With one world good, goods market equilibrium requires purchasing power parity ($P_t = E_t P_t^*$), where $E_t$ gives the exchange rate as the domestic-currency price of foreign currency, and a superscript asterisk denotes a foreign-currency variable. Additionally, since bonds denominated in each currency are perfect substitutes, bond market equilibrium requires interest rate parity, $i_t = i_t^* + \bar{E}_t/E_t$. The combination of purchasing power parity and interest rate parity imply real interest rate parity so that $r_t = r_t^*$. To simplify the presentation, these equilibrium relationships will be imposed in writing budget constraints.

3.1. Representative agents

3.1.1. Budget constraints

Let each economy be populated by a large number of representative agents with unit mass. Consider, first, a domestic representative agent’s flow budget constraint. We assume that a domestic agent can hold assets denominated in either the domestic or foreign currency. However, below, we assume that only own-currency money yields utility. Therefore, domestic agents will choose to hold only home money and foreign agents will choose only foreign money. Domestic holdings of these assets together with domestic income, taxes, and consumption, carry a superscript $h$ for home. In real terms, a domestic agent’s flow budget constraint is expressed as

$$b_t^h + b_t^{*h} + m_t^{h} \leq r_t (b_t^h + b_t^{*h} + m_t^{h}) + \bar{y}_t - i_t m_t^h - x_t - c_t^h. \quad (14)$$

Additionally, a domestic representative agent faces a borrowing constraint assuring that

$$\lim_{T \to \infty} (b_T^h + b_T^{*h} + m_T^{h})\beta_T \geq 0. \quad (15)$$

$^6$The assumption that bonds are perfect substitutes makes composition of each agent’s bond portfolio indeterminate, but this is insignificant for present purposes.

$^7$This simplification reduces the dimension of the model since it is not necessary to solve for the allocations of the two monies across countries. Allowing both monies to yield utility would complicate the presentation with extra equations and unknowns and change nothing substantive.
Integrating Eq. (14) over time and imposing Eq. (15) yields an expression for an agent’s intertemporal budget constraint as

$$\int_{t=0}^{\infty} (\tilde{y}^h - \tau^h - \tilde{c}_t^h - i_t m_t^h) \beta_t \, dt + b_0^h + b_0^* + m_0^0 \geq 0.$$  

A representative foreign agent faces analogous budget constraints. Letting superscript $f$ denote foreign, a foreign agent’s budget constraint can be expressed in real terms as

$$b^f_t + b^*_f + m^*_f \leq r_t (b^f_t + b^*_f + m^*_f) + \tilde{y}_t^f - \tilde{c}_t^f - \tau^f - c_t^f. \tag{16}$$

A foreign agent must also satisfy

$$\lim_{T \to \infty} (b_T^f + b_T^* + m_T^*) \beta_T \geq 0. \tag{17}$$

Together, Eqs. (16) and (17) yield a foreign agent’s intertemporal budget constraint as

$$\int_{t=0}^{\infty} (\tilde{y}_t^f - \tau_t^f - \tilde{c}_t^f - i_t^* m_t^* f) \beta_t \, dt + b_0^f + b_0^* + m_0^* \geq 0.$$  

3.1.2. Optimal behavior

Domestic and foreign agents are assumed to have identical preferences over consumption and own-currency money. Money denominated in the other country’s currency is assumed to yield no utility.\(^8\) Utility of a domestic representative agent is given by

$$U = \int_{t=0}^{\infty} [u(c^h_t) + v(m^h_t)] e^{-\theta_t} \, dt,$$

while utility for a foreign agent is expressed as

$$U^* = \int_{t=0}^{\infty} [u(c^f_t) + v(m^*_f)] e^{-\theta_t} \, dt.$$

The functions $u$ and $v$ retain properties from the previous section. Domestic agents maximize utility subject to Eqs. (14) and (15), while foreign agents’ maximization is constrained by Eqs. (16) and (17).

First-order conditions for domestic agents are given by Eqs. (7) and (8), repeated below with open-economy notation:

$$\frac{\hat{c}_t^h u''(c^h_t)}{u'(c^h_t)} = \theta - r_t, \tag{18}$$

$$v'(m^h_t) = \left[ r_t + \hat{P}_t P_t \right] u'(c^h_t). \tag{19}$$

\(^8\)See the previous footnote for the implications of this simplification.
The transversality condition combined with the budget constraint for a domestic agent implies
\[
\int_{t=0}^{\infty} (\bar{g}^h - \tau_t^h - c_t^h - i_t^h m_t^h) \beta_t \, dt + b_0^h + b_0^*^h + m_0^h = \lim_{T \to \infty} (b_T^h + b_T^*^h + m_T^h) \beta_T = 0. \tag{20}
\]

The first-order and transversality conditions for foreign agents are analogous.

3.2. Governments

Now, consider government policy in an open economy. Governments can issue interest-bearing debt denominated in either domestic or foreign currency, but they can issue only their own currency. Let \( b^g_h \) and \( b^*^g_h \) (\( b^g_t \) and \( b^*^g_t \)) denote the real value of nominal government bonds issued by the home (foreign) government in domestic and foreign currency, respectively. The domestic government’s flow budget constraint can be expressed in real terms as
\[
b_t^g + b_t^*^g + \tilde{m}_t^g = r_t(b_t^g + b_t^*^g + m_t^g) + \bar{g}^h - \tau_t^h - i_t m_t^g. \tag{21}
\]

Integrating Eq. (21) over time yields
\[
\lim_{T \to \infty} (b_{hT}^g + b_{hT}^*^g + m_{hT}^g) \beta_T = \int_{t=0}^{\infty} (\bar{g}^h - \tau_t^h - i_t m_t^g) \beta_t \, dt + b_{h0}^g + b_{h0}^*^g + m_{h0}^g. \tag{22}
\]

Intertemporal government budget balance for the domestic country requires
\[
\lim_{T \to \infty} (b_{fT}^g + b_{fT}^*^g + m_{fT}^g) \beta_T = \int_{t=0}^{\infty} (\bar{g}^f - \tau_t^f - i_t m_t^g) \beta_t \, dt + b_{f0}^g + b_{f0}^*^g + m_{f0}^g = 0. \tag{23}
\]

Letting subscript \( f \) denote foreign-issued debt, the foreign government’s flow and intertemporal budget constraints are given, respectively, by
\[
b_t^g + b_t^*^g + \tilde{m}_t^g = r_t(b_t^g + b_t^*^g + m_t^g) + \bar{g}^f - \tau_t^f - i_t^* m_t^g, \tag{24}
\]
\[
\lim_{T \to \infty} (b_{fT}^g + b_{fT}^*^g + m_{fT}^g) \beta_T = \int_{t=0}^{\infty} (\bar{g}^f - \tau_t^f - i_t^* m_t^g) \beta_t \, dt + b_{f0}^g + b_{f0}^*^g + m_{f0}^g = 0. \tag{25}
\]

Domestic government policy in an open economy is defined as follows:

**Definition 3.** A home policy rule \( \Pi^h \) is a function that maps each price path \((P, P^*) = (P_t, P_t^*)_{t \geq 0}\) into a policy variable path \((\tau_t^h, i_t, b_t^g, b_t^*^g, m_t^g) = (\tau_t^h, i_t, b_t^g, b_t^*^g, m_t^g)_{t \geq 0}\), satisfying the government’s flow budget constraint (Eq. (21)).
A foreign policy rule $\Pi^f$ is defined analogously. Ricardian policy for the domestic country can be redefined as

**Definition 4.** A domestic policy rule is Ricardian if the rule implies

$$
\lim_{T \to \infty} (b^g_{hT} + b^{*g}_{hT} + m^g_T) \beta_T
$$

$$
= \int_{t=0}^{\infty} (\tilde{g}^h - \tau^h_t - i_t m^g_t) \beta_t \, dt + b^g_{h0} + b^{*g}_{h0} + m^g_0 = 0
$$

for all $P, P^*$. Otherwise, the policy rule is non-Ricardian.

The definition for foreign Ricardian policy is completely analogous and is therefore not stated.

### 3.3. Equilibrium

Now, compare equilibrium under Ricardian and non-Ricardian policy rules. An equilibrium is a path $(\tau^h, \tau^f, i, i^*, P, P^*, E, a, a^*, c^h, c^f)$, where $a = (b^g_h, b^g_f, b^h, b^f, m^g, m^h)$ and $a^* = (b^{*g}_h, b^{*g}_f, b^{*h}, b^{*f}, m^{*g}, m^{*h})$, such that (1) domestic and foreign agents’ optimality conditions, given by Eqs. (18)–(20), and analogous equations for the foreign agent, are satisfied, with definitions given in Eqs. (1) and (4) and analogous foreign-variable definitions; (2) domestic and foreign government policies are $(\tau^h, i, b^g_h, b^{*g}_h, m^g) = \Pi^h(P, P^*)$ and $(\tau^f, i^*, b^g_f, b^{*g}_f, m^{*g}) = \Pi^f(P, P^*)$, respectively; and (3) markets for goods, domestic-currency bonds, foreign-currency bonds, domestic money and foreign money all clear.

Given purchasing power parity, interest rate parity, and Walras’ Law, we need to consider only three markets explicitly. As before, we consider goods and money markets. In the open economy, Proposition 1 is modified to become

**Proposition 2.** In the open economy, market equilibrium requires intertemporal government budget balance for the aggregated world government, but not for individual country governments.

To prove this, consider consumption demand and supply. Supply is constant and equal to $\tilde{y}^h + \tilde{y}^f - \tilde{g}^h - \tilde{g}^f$. Therefore, in equilibrium, consumption demand must also be constant. By purchasing power parity and interest rate parity, each country’s representative agent faces the same real interest rate. Therefore $r_t = r^*_t = \theta$, such that $\beta_t = e^{-\theta t}$. Using Eq. (20) and its foreign counterpart, consumption demands for representative domestic and foreign agents can be expressed, respectively, as

$$
c^h_t = \theta \left[ (b^h_t + b^{*h}_t + m^h_t) + \int_{s=t}^{\infty} (\tilde{y}^h - \tau^h_s - i_s m^h_s) e^{-\theta(s-t)} \, ds \right],
$$

$$
c^f_t = \theta \left[ (b^f_t + b^{*f}_t + m^f_t) + \int_{s=t}^{\infty} (\tilde{y}^f - \tau^f_s - i_s m^{*f}_s) e^{-\theta(s-t)} \, ds \right],
$$

where $\theta$ is a parameter to be determined.
World consumption demand is given by aggregating across agents and countries. Using the assumption that agents in each country have unit mass, and imposing bond and money market equilibrium, world consumption demand becomes

\[ c^h_t + c^f_t = \theta \left[ (b^g_{ht} + \theta^*_g + \theta^*_f + b^*_g + m^g_t + m^*_g) \right. \]
\[ + \int_{s=t}^{\infty} \left( \hat{y}_t^v + \hat{y}_f^v - \tau^h_t - \tau^f_s - i_t m^g_s - i_t^* m^*_g \right) e^{-\theta(s-t)} \, ds \right]. \quad (28) \]

Using the sum of domestic and foreign government budget constraints, given by (22) and its foreign counterpart, to substitute for the present value of seigniorage revenue in Eq. (28), yields world consumption demand as

\[ c^h_t + c^f_t = \theta \left[ \int_{s=t}^{\infty} (\hat{y}_t^v + \hat{y}_f^v - \theta^*_g - \theta^*_f + m^g_t + m^*_g) e^{-\theta(s-t)} \, ds \right. \]
\[ + \lim_{T \to \infty} (b^g_{ht} + b^*_g + b^g_{ft} + b^*_f + m^g_t + m^*_g) e^{-\theta(T-t)} \right]. \quad (29) \]

Performing the integration and equating consumption demand with goods available for consumption reveals that a zero limit term is necessary for equilibrium. Therefore, the world intertemporal government budget constraint, given by the sum of Eqs. (23) and (25), must be satisfied in equilibrium according to

\[ \int_{t=0}^{\infty} (\hat{g}_t^v + \hat{g}_t^v - \tau^h_t - \tau^f_t - i_t m^g_t - i_t^* m^*_g) e^{-\theta t} \, dt 
\[ + \left( b^g_{ht0} + b^*_g + m^g_{t0} + b^g_{ft0} + b^*_f + m^*_g \right) \]
\[ = \lim_{T \to \infty} (b^g_{ht} + b^*_g + b^g_{ft} + b^*_f + m^g_t + m^*_g) e^{-\theta T} = 0. \quad (30) \]

Eq. (30) is the aggregated world government intertemporal budget constraint. We have proved that this must hold in equilibrium. However, market equilibrium does not require that each government’s intertemporal budget constraint hold.

The description of equilibrium is complete with the requirement that each country’s currency is willingly held. Substituting the real quantity of each currency into Eq. (19) and its foreign counterpart yields

\[ u'(m^h_t) = \left[ \theta + \hat{P}_t \right] u'(c^h_t), \quad (31) \]
\[ u'(m^*_t) = \left[ \theta + \hat{P}^*_t \right] u'(c^*_t), \quad (32) \]

where \( c^h_t \) and \( c^*_t \) are given by Eqs. (26) and (27), respectively.

Now, consider the nature of equilibrium when both countries follow Ricardian policy. Ricardian policy implies that Eq. (30) holds for any
price path, \((P, P^*)\). Therefore, there are only two differential equations, given by (31) and (32), to determine both initial price levels and their rates of change, with the paths for domestic and foreign policy variables determined by \(\Pi^h(P, P^*)\) and \(\Pi^f(P, P^*)\), respectively. As in the closed-economy, with nothing to determine initial conditions, there is potentially an equilibrium corresponding to each pair of \(P_0, P^*_0\). Indeterminacy is two dimensional.

If policy is non-Ricardian, then Eq. (30) becomes an independent equilibrium condition. Now, there are three equations to determine the two price levels and their rates of change, with the paths for policy variables determined by \(\Pi^h(P, P^*)\) and \(\Pi^f(P, P^*)\). Non-Ricardian policy reduces the dimension of the indeterminacy by one, but does not eliminate it. This is the result emphasized by Dupor (2000) and Loyo (1998).

Note an additional characteristic of equilibrium under non-Ricardian policy. In all but one of the equilibria, one of the two governments must be willing to collect present-value taxes in excess of present-value spending, implying an intertemporal government budget surplus. An alternative expression of the same result is that the country with the intertemporal government budget surplus must be willing to run an intertemporal current account surplus, implying that its agents fail to exhaust the present value of their country’s resources. To illustrate this, note that the current account is the country’s accumulation of net foreign assets. The domestic intertemporal current account can, therefore, be expressed using the domestic government and private agent’s intertemporal budget constraints. Using the fact that optimality requires that private agents satisfy Eq. (20) and combining this with Eq. (22) yields

\[
\int_0^\infty (c^h_t + g^h - y^h) e^{-\theta t} \, dt - h^h_t - b^*_h + b^*_g + b^*_h + b^*_g = \lim_{T \to \infty} (b^g_{hT} + b^*_g + m^g_t) e^{-\theta T}. \tag{33}
\]

Eq. (33) states that when the domestic country spends more than the present value of its income combined with net foreign assets (enjoys an intertemporal current account deficit), it must have an intertemporal government budget deficit, that is, a positive limit term. It follows from Eq. (30), which imposes intertemporal balance for the aggregated world government, that the foreign country must spend less than its resources in equilibrium, experiencing both an intertemporal current account surplus and a government budget surplus. Since a policy that yields an intertemporal government surplus leads a country’s residents to fail to exhaust their country’s resources, such a policy cannot maximize their welfare. Therefore, a policy, whereby a government rules out paths leading to an intertemporal surplus, hereafter called a “no-surplus”
policy, must yield higher welfare for the country’s residents than a general non-Ricardian policy.

In the next section of the paper, we show that a “no-surplus” policy on the part of both governments yields an open-economy fiscal theory of the price level completely analogous to that of the closed-economy.

4. A determinate price and exchange rate under fiscal theory

4.1. No-surplus policy

A “no-surplus” policy rule is defined as follows:

**Definition 5.** A “no-surplus” policy rule on the part of the domestic government requires

\[
\lim_{T \to \infty} \left( b_{hT}^g + b_{hT}^g + m_T^g \right) e^{-0T}
\]

\[
= \int_{t=0}^{\infty} (\bar{g}^h - \bar{r}^h - i_t m_t^g) e^{-0t} dt + b_{h0}^g + b_{h0}^g + m_0^g \geq 0
\]

for all \( P, P^* \).

The definition of a “no-surplus” policy rule for the foreign government is completely analogous.

Since a “no-surplus” policy is welfare improving for a country’s own residents, compared with general policy, we assume in the following that both governments follow this policy. Below, we show that a “no-surplus” policy yields an open-economy fiscal theory of the price level which is an exact analogue to the closed-economy fiscal theory.

The “no-surplus” policy space can further be divided into Ricardian and non-Ricardian policy. The definition of Ricardian policy is unchanged. Therefore, a “no-surplus” Ricardian policy is simply a Ricardian policy as follows

**Definition 6.** For the domestic country, a “no-surplus” policy rule is Ricardian if

\[
\lim_{T \to \infty} \left( b_{hT}^g + b_{hT}^g + m_T^g \right) e^{-0T}
\]

\[
= \int_{t=0}^{\infty} (\bar{g}^h - \bar{r}^h - i_t m_t^g) e^{-0t} dt + b_{h0}^g + b_{h0}^g + m_0^g = 0
\]

for all \( P, P^* \). Otherwise the policy is non-Ricardian.

---

9 Be careful not to confuse a “no-surplus” policy as defined here with a policy whereby a government never runs a flow surplus. Flow surpluses are allowed, but intertemporal surpluses are ruled out.
The definition of foreign Ricardian policy is completely analogous. Note that non-Ricardian “no-surplus” policy requires the weak inequality in Definition 5 for at least some \( P, P^* \), while the reverse inequality is ruled out for all \( P, P^* \).

Consider the implications of “no-surplus” non-Ricardian policy. This implies that each government is willing to accept an intertemporal deficit, but not an intertemporal surplus. Equilibrium conditions include world goods market equilibrium, given by Eq. (30), together with the “no-surplus” inequalities for each country, given in Definition 5 and its foreign counterpart, yielding

\[
\lim_{T \to \infty} \left( b_{ht} + b^*_h + b^*_{lt} + m_t + m^*_{gt} \right) e^{-\theta T} = 0, \tag{34}
\]

\[
\lim_{T \to \infty} \left( b_{ht} + b^*_h + m_t \right) e^{-\theta T} \geq 0, \tag{35}
\]

\[
\lim_{T \to \infty} \left( b^*_{ht} + b^*_{lt} + m^*_{gt} \right) e^{-\theta T} \geq 0. \tag{36}
\]

Together, these three equations imply that the “no-surplus” inequalities, given by Eqs. (35) and (36), must be satisfied with equality in equilibrium. A “no-surplus” non-Ricardian policy, therefore, implies that each government’s intertemporal budget constraint, given by Eqs. (23) and (25), respectively, must be satisfied in equilibrium, exactly as in closed-economy fiscal theory. This yields four independent equations, given by (31), (32), (23) and (25), to determine the initial values for the two price levels and their rates of change. Open-economy indeterminacy vanishes. By purchasing power parity, the exchange rate is determined by relative prices.

A “no-surplus” policy has implications about how a government would react to price vectors, that are potentially off-equilibrium. Ricardian behavior satisfies the “no-surplus” criterion and implies that any change in the price vector must be accompanied by a change in government policy to assure intertemporal government budget balance. Therefore, all price paths are equilibrium paths. Alternatively, a “no-surplus” non-Ricardian policy implies that a change in the price vector must elicit a change in the path of government policy variables only if the new price vector at the old policy settings violates its “no-surplus” inequality to yield an intertemporal surplus. Since any price paths, satisfying goods market equilibrium, must yield either (1) an intertemporal deficit for one country and a corresponding surplus for the other, or (2) balance for both; and since a country does not choose policy paths, associated with price paths, which yield an intertemporal surplus, the only equilibrium price path consistent with a “no-surplus” non-Ricardian policy is that yielding intertemporal balance for both countries.

The next section presents an example of equilibrium under a particular “no-surplus” policy.
4.2. A simple example of “no-surplus” policy

Let \( \Pi^h(P, P^*) \) be given by

\[
\begin{align*}
    b^*_{ht} &= 0, \\
    i_t &= \theta, \\
    b^g_{ht} &= \theta b^g_{ht} + \bar{g}^h - \tau^h_t, \\
    \tau^h_t &= \begin{cases} \\
        \bar{z}^h & \text{if } b^g_{h0} \geq \bar{b}^g_h, \\
        \bar{z}^h_0 < \bar{z}^h & \text{if } b^g_{h0} < \bar{b}^g_h,
    \end{cases}
\end{align*}
\]

where

\[
\bar{b}^g_h = \frac{\bar{z}^h - \bar{g}^h}{\theta}, \quad \bar{z}_0 = \theta b^g_{h0} + \bar{g}^h
\]

and nominal money is whatever is necessary to satisfy Eq. (31) at \( i = \theta \). Foreign government policy \( \Pi^f(P^*, P) \) is defined analogously. Note that each government has a preferred tax level denoted by a tilde. If this tax level yields either intertemporal government budget balance or an intertemporal deficit for a given price path, then the government stays with this tax level. However, if this tax level yields an intertemporal surplus for a given price path, the government lowers taxes to yield intertemporal budget balance at the initial level of real government debt.

With this specification of policy, the world government intertemporal budget constraint, given by Eq. (30) and necessary for equilibrium, simplifies to

\[
\int_{t=0}^{\infty} (\bar{g}^h + g^f - \bar{t}^h_t - \tau^f_t) e^{-\theta t} \, dt + b^g_{h0} + b^*_{10} = \lim_{T \to \infty} (b^g_{ht} + b^*_{1t}) e^{-\theta T} = 0.
\]

(37)

Now, consider a particular path for prices \( (\hat{P}, \hat{P}^*) \) for which \( b^g_{h0} < \bar{b}^g_h \) and for which \( b^*_g > \bar{b}^*_g \) such that Eq. (37) holds at \( \bar{t}^h_t = \bar{z}^h \) and \( \bar{t}^f_t = \bar{z}^f \) for all \( t \geq 0 \). If governments were to choose these tax levels along this price path, then markets would clear. However, this candidate price path does not yield an equilibrium because the domestic government does not choose \( \bar{z}^h \) as its tax level along this candidate price path. This is because along this price path, the real value of domestic government bonds is low enough to generate a domestic government intertemporal budget surplus. Therefore, the domestic government lowers taxes to yield intertemporal budget balance \( (\tau^h_t = \bar{t}_0 < \bar{z}^h, \text{ for all } t \geq 0) \). However, since the foreign government enjoys a deficit along this price path, it maintains taxes at \( \bar{t}^f \), yielding a foreign intertemporal deficit. Therefore, along the candidate price path, the world government budget constraint is in intertemporal deficit; that is, Eq. (37) fails. The candidate price path is,
therefore, not an equilibrium path. The equilibrium price path under this policy rule must set $b_{g0}^g = \bar{b}_{g0}^g$ and $b_{10}^{*g} = \bar{b}_{10}^{*g}$, thereby yielding intertemporal budget balance for each country’s government.\(^{10}\)

5. Conclusion

This paper demonstrates that by carefully tailoring policy rules in an independent manner, governments can eliminate price and exchange rate indeterminacy. Specifically, if all governments refuse to engage in policy which would lead to an intertemporal government budget surplus, then the open-economy fiscal theory of the price level becomes the exact analogue of the closed-economy fiscal theory. The price level in each country is determined to assure intertemporal government budget balance. Additionally, a “no-surplus” non-Ricardian is preferable to a general non-Ricardian policy since the general non-Ricardian policy can lead to intertemporal surpluses whereby a country’s residents fail to exhaust their country’s resources.

References


\(^{10}\)To complete the description of equilibrium, note that satisfaction of Eqs. (31) and (32) under the specified policy rule requires that prices be constant at their initial levels, where the initial levels are determined to balance each country’s intertemporal government budget.