Lecture 3. Policy Interactions with Tax Distortions

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THE MESSAGES

• Will study three models with distorting taxes
• First draws on Gordon-Leeper (2005,2006): growth model w/ transactions demand for money
• Second draws on Leeper-Yun (2006): provides micro foundations for FTPL
• Once models completely solved out, can understand price-level determination more deeply
• Emphasizes the role of asset substitution, which is absent from simple models
• Gets us away from the fiscal theory story about wealth effects
• Useful models to keep in your head: “roll your own policies”
• Characterize eqm as function of general sequences of policy variables
FIRST MODEL

- Growth model w/ capital, money, nominal government debt
  - arbitrages among assets determine their relative demands
  - returns to real balance holdings and after-tax returns to capital determine the relative values of real and nominal assets
  - expected macro policies determine expected returns on real and nominal assets
  - so price level depends on interactions among current and expected future MP & FP
- Quantity theory and fiscal theory emerge as special cases
- QT & FT employ common money demand

\[
\frac{M^d}{P} = h(i, y)
\]

- how can this be?
THE MODEL

- We exploit the analytic convenience that comes with log prefs, C-D technology, complete depreciation of capital
  - none of the general points depend on these simplifying assumptions
- Aggregate resource constraint
  \[ c_t + k_t + g_t = f(k_{t-1}) \]

- Goods producing firm rents \( k \) at rental rate \( r \) and pays taxes levied against sales of goods to solve
  \[ \max_{k_{t-1}} D_{Gt} = (1 - \tau_t) f(k_{t-1}) - r_t k_{t-1} \]

- Transactions services producing firm hires labor \( l \) at wage rate \( w \) to solve
  \[ \max_{l_t} D_{Tt} = P_{Tt} T(l_t) - w_t l_t \]
THE MODEL

- Household owns firms and pays taxes on capital income
- HH has income

\[
I_t = r_t k_{t-1} + D_{Gt} + w_t l_t + D_{Tt} + z_t
\]

where \( z_t \geq 0 \) is lump-sum transfers from the government

- HH’s expenditures on \( c \) & \( k \) must be financed with real money balances, \( M_{t-1}/P_t \), or with transactions services, \( T_t \), to satisfy the constraint

\[
\frac{M_{t-1}}{P_t} + T_t (c_t + k_t) \geq c_t + k_t
\]

\( T_t \) gives fraction of expenditures financed w/ transactions services
**The Model**

- HH’s problem

\[
\max_{\{c_t, l_t, T_t, M_t, B_t, k_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t), \quad 0 < \beta < 1
\]

where \(1 - l_t\) is leisure, subject to the finance constraint, the budget constraint

\[
c_t + k_t + \frac{M_t + B_t}{P_t} + P_{TT}T_t \leq I_t + \frac{M_{t-1} + R_{t-1}B_{t-1}}{P_t}
\]

and \(0 \leq l_t \leq 1\)

- Future government policy is the sole source of uncertainty; the operator \(E\) denotes equilibrium expectations of private agents over future policy
The Model

• The government finances expenditures on goods, $g_t$, and transfer payments, $z_t$, by levying taxes, issuing new debt, and creating new money to satisfy:

$$\tau_t f(k_{t-1}) + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t + R_{t-1}B_{t-1}}{P_t} = g_t + z_t$$

• Assume the following functional forms:

$$f(k_{t-1}) = k_{t-1}^\sigma, \quad 0 < \sigma < 1$$

$$T(l_t) = 1 - (1 - l_t)^\alpha, \quad \alpha > 1$$

$$U(c_t, 1 - l_t) = \log(c_t) + \gamma \log(1 - l_t), \quad \gamma > 0$$
Solving the Model

• State at $t$ depends on assets and expectations of macro policies

• denote state by
  
  $z_t = (k_{t-1}, M_{t-1}, (1 + i_{t-1})B_{t-1}, \{E_t\rho_j, E_t\tau_j, E_t s^g_j\}_{j=t})$

  - $\rho_t = M_t/M_{t-1}$, $s^g_t = g_t/f(k_{t-1})$
  - emphasizes that a complete specification of policy must allow agents to form expectations over infinite future of policies
SOLVING THE MODEL

• First-order conditions
  • firms’
    \[ 1 + r_t = \sigma(1 - \tau_t)k_{t-1}^{\sigma-1} \]
    \[ w_t = \alpha(1 - l_t)^{\alpha-1}P_{Tt} \]

• household’s
  \[ \varphi_t + \lambda_t = \frac{1}{c_t} + \lambda_t T^d_t \]
  \[ \gamma \frac{1}{1 - l_t} = w_t \varphi_t \]
  \[ \varphi_t P_{Tt} = \lambda_t (c_t + k_t) \]
  \[ \frac{\varphi_t}{P_t} = \beta E_t \left[ \frac{\varphi_{t+1} + \lambda_{t+1}}{P_{t+1}} \right] \]
  \[ \frac{\varphi_t}{P_t} = \beta(1 + i_t)E_t \left[ \frac{\varphi_{t+1}}{P_{t+1}} \right] \]
  \[ \varphi_t + \lambda_t = \lambda_t T^d_t + \beta E_t (1 + r_{t+1})\varphi_{t+1} \]
EQUILIBRIUM

• Characterize eqm in terms of policy expectations functions $(\mu_t, \eta_t)$, government claims to goods, $s^g_t$, and assets, $(k_{t-1}, M_{t-1}, (1 + i_{t-1}) B_{t-1})$

• Solution maps policy expectations into portfolio choices
  • of course, policy expectations are restricted to policy paths that are consistent with eqm

• $\eta$ and $\mu$ capture portfolio balance effects of policies
  • $\eta$ measures direct tax distortion on investment & extent to which gov’t expends are financed by taxing output
  • $\mu$ reflects expected inflation & the expected return on nominal assets

• Assume (similar to “Monetary Doctrines”)

$$\rho_{t+j} = \rho_F, \forall j > 0$$

$$\tau_{t+j} = \tau_F, \forall j > 0$$

$$s^g_{t+j} = s^g_F, \forall j > 0$$
EQUILIBRIUM

- Two dynamical equations to solve: real asset & nominal assets
- $k$ Euler equation in terms of $s_t = k_t/(c_t + k_t)$ yields

$$\frac{1}{1 - s_t} = \sigma \beta E_t \left[ \frac{1 - \tau_{t+1}}{1 - s_{t+1}^g} \left( \frac{1}{1 - s_{t+1}} \right) \right] + E_t \left[ 1 - \sigma \beta \frac{\gamma}{\alpha} \frac{1 - \tau_{t+1}}{1 - s_{t+1}^g} \right]$$

whose solution is

$$\frac{1}{1 - s_t} = \eta_t$$

where

$$\eta_t \equiv E_t \sum_{i=0}^{\infty} (\sigma \beta)^i \eta_i^n \left[ 1 - \sigma \beta \frac{\gamma}{\alpha} \frac{1 - \tau_{t+i+1}}{1 - s_{t+i+1}^g} \right]$$

$$\eta_i^n = \prod_{j=0}^{i-1} \left( \frac{1 - \tau_{t+j+1}}{1 - s_{t+i+1}^g} \right), \quad \eta_0^n = 1$$
EQUILIBRIUM

- Euler equation for $M$ yields d.q. in velocity, $1 - T_t$

$$(1 - T_t) \left[ \frac{1}{1 - s_t} - \frac{\gamma}{\alpha} \right] = \beta \frac{1}{\rho_t} E_t \left\{ (1 - T_{t+1}) \left[ \frac{1}{1 - s_{t+1}} - \frac{\gamma}{\alpha} \right] + \frac{\gamma}{\alpha} \right\}$$

whose solution is

$$(1 - T_t) \left[ \frac{1}{1 - s_t} - \frac{\gamma}{\alpha} \right] = \frac{\mu_t}{\rho_t}$$

where

$$\mu_t \equiv \beta \frac{\gamma}{\alpha} E_t \sum_{i=0}^{\infty} \beta^i d_i^\mu, \quad d_i^\mu \equiv \prod_{j=0}^{i-1} \frac{1}{\rho_{t+j+1}}, \quad d_0^\mu = 1$$
EQUILIBRIUM

- Imposing the stationary policy assumptions yields the policy expectations functions

\[ \eta_t(\tau_F, s^g_F) = \frac{1 - \sigma \beta \frac{\gamma}{\alpha}}{1 - \sigma \beta} \left( \frac{1 - \tau_F}{1 - s^g_F} \right) \]

\[ \mu_t(\rho_F) = \frac{\beta \frac{\gamma}{\alpha}}{1 - \beta / \rho_F} \]

- Eqm capital stock is

\[ k_t = \left( 1 - \frac{1}{\eta_t} \right) (1 - s^g_t) f(k_{t-1}) \]

- Eqm real money balances are

\[ \frac{M_t}{P_t} = \left( \frac{\mu_t}{\eta_t - \frac{\gamma}{\alpha}} \right) (1 - s^g_t) f(k_{t-1}) \]
PRICE-LEVEL DETERMINATION

- Can think of price level being determined “through eqm real balances”

\[ \frac{M_t}{P_t} = \Delta_t (1 - s_t^g) f(k_{t-1}) \]

where

\[ \Delta_t = \frac{\mu_t}{\eta_t - \gamma/\alpha} \]

with

\[ \Delta_t(\rho_F, \tau_F, s_F^g) = \frac{\beta \gamma}{\alpha} \left[ 1 - \sigma \beta \left( \frac{1 - \tau_F}{1 - s_F^g} \right) \right] \]

- \( 1/\Delta_t \) is velocity; it gives the value of nominal assets
  - \( \Delta_t \) depends on expected MP & FP
THE ROLE OF POLICY EXPECTATIONS

• \( \mu \) and \( \eta \) capture 3 distinct influences of expectations on \( P \)
  1. \( \mu \): the marginal value of real money balances; higher expected money growth lowers \( \mu \) and induces substitution away from money, raising \( P \)
  2. \( \eta \): direct tax distortion that alters return on investment; higher expected taxes reduce return on investment and induces substitution away from \( k \) into \( c \) and into \( M \) (Tobin effect), raising money demand and lowering \( P \)
  3. \( \eta \) summarizes composition of expected fiscal financing; higher \( \eta \) reflects increase in expected nominal liability creation & reduction in relative role of real taxation

To see (3), note terms \( (1 - \tau)/(1 - s^g) \) in \( \eta \) and write gbc as

\[
\frac{1 - \tau_t}{1 - s^g_t} = 1 + \frac{(M_t - M_{t-1} + B_t - (1 + i_{t-1})B_{t-1})/P_t}{(1 - s^g_t)f(k_{t-1})}
\]
JOINTLY CONSISTENT (EQUILIBRIUM) POLICIES

- Dynamic interactions among policies
  - current policies constrain future policy options
  - expected fiscal financing constrains current policies
  - expected policies affect $P_t$ & real value of gov’t liabilities
- How do jointly consistent combinations of current & future policies affect $P$?
  1. Which policies are consistent with eqm given current expectations ($\mu$ & $\eta$)?
  2. How do current policy changes affect the set of future policies that are consistent with eqm?
JOINTLY CONSISTENT POLICIES

1. Which policies are consistent with eqm given current expectations \((\mu, \eta)\)?

2. How do current policy changes affect the set of future policies that are consistent with eqm?

- Eqm government b.c. at \(t\)

\[
\left[ \frac{\rho_t - 1}{\rho_t} + \left( \frac{B}{M} \right)_t - \frac{1 + \delta_{t-1}}{\rho_t} \left( \frac{B}{M} \right)_{t-1} \right] \Delta_t = \frac{s^g_t - \tau_t}{1 - s^g_t}
\]

where \((B/M)_s \equiv B_s/M_s\) and \(\Delta_t\) summarizes given expected policies

- Eqm government b.c. in future

\[
\Delta_t = \left( \frac{s^g_F - \tau_F}{1 - s^g_F} \right) \left[ \frac{1}{(B/M)_F - \frac{1}{\beta} (B/M)_t + \left( \frac{\rho_F}{\rho_F} - 1 \right)} \right]
\]
Money Demand

\[
\frac{M_t}{P_t} = \beta \frac{\gamma}{\alpha} \left( \frac{1 + i_t}{i_t} \right) \frac{1}{\eta_t - \gamma/\alpha} (1 - s^g_t) f(k_{t-1})
\]

• In general, both MP and FP affect \( P \)
• When is \( P \) determined by MP alone?
• Under policy assumptions that dichotomize real & nominal sides
• Balanced net-of-interest surplus: \( \tau_t = s^g_t \) all \( t \)
• Now \( \eta_t = (1 - \sigma \beta \gamma/\alpha)/(1 - \sigma \beta) \) and \( M^d \) is

\[
\frac{M_t}{P_t} = h(i_t, c_t + k_t)
\]

• \( P \) independent of FP but not of debt
  • money growth must finance interest obligations
  • higher \( B \) ⇒ higher debt service ⇒ higher \( P \) & \( \pi \)
• In general, cannot rid \( M/P \) of \( \eta \)
Unpleasant Monetarist Arithmetic

- Open-market sale of $B_t$, holding $M_t + B_t$ fixed
- Fix $(s_t^g, s_F^g)$ and $\tau_t$
- B/c $B_t \uparrow$, some future policy must adjust—either $\tau_F$ or $\rho_F$
  1. suppose $\tau_F \uparrow$: $\eta_t \downarrow$, $k_t \downarrow$, $P_t \downarrow$ (but future $P \uparrow$)
  2. suppose $\rho_F \uparrow$: $\mu_t \downarrow$, tend to make $P_t \uparrow$ (but future $P \uparrow$)
    But $M_t \downarrow$, so ultimate effect on $P_t$ can go either way, depending on $B/M$
- Monetary policy is constrained by the government’s fiscal obligations
  - works through seigniorage
 Canonical FTPL

- Bond-financed tax cut: \( \tau_t \downarrow, B_t \uparrow \)
- Fix \( (\rho_F, \tau_F, s^g_F) \) and \( s^g_t \)
- B/c \( B_t \) rises, if \( M_t \) unchanged, \( (B/M)_t \) rises and some future policy must adjust
- By ass’n no future policy can adjust
- Only eqm policy is for \( M_t \) to rise in proportion to the \( B_t \) increase so that \( (B/M)_t \) unchanged
- Required increase in \( M_t \) is exactly enough so increase in future seigniorage (b/c the level of money supplied is now higher) suffices to service higher debt
- The fixed policies peg \( i_t \) and \( M_t/P_t \), so \( P_t \uparrow \)
- Monetary policy is constrained by the government’s fiscal obligations
  - works through nominal asset revaluation
**PURE FISCAL EFFECTS**

- FP can affect $P$ independently of MP
- Consider a debt-financed tax cut to which future taxes adjust
- Fix $(\rho_t, \rho_F, s_t^g, s_F^g)$
- Lower $\tau_t$ & higher $(B/M)_t \Rightarrow$ higher $\tau_F$
- Lower return on capital induces substitution away from real assets toward nominal assets
- With $M_t$ fixed, $P_t$ falls
- This Tobin effect gives debt a natural role in determining $P$
- Quite non-Keynesian: current fiscal expansion reduces nominal demand and price level
- Note that even though money growth is unchanged, because $M/P$ rises, seigniorage revenues rise
**Countercyclical Fiscal Policies**

- Extend previous models in several ways
  - add human capital, $h: f(k, h), f$ CRS
  - incomplete depreciation of both $k$ and $h$
  - total investment, $x = x_k + x_h$, and consumption enters finance constraint
  - calibrate to U.S. data
- Need to compute expectations functions, $\{\eta_t, \mu_t\}$
  - assume perfect foresight
  - use data on $\{s_t^g, s_t^z, \tau_t, \rho_t\}$
  - handle “infinite sum” in a couple of ways
- Simulate time paths of investment & velocity
Countercyclical Fiscal Policies

- Basic intuition:
  - economic downturn: $g/y \uparrow$ and $T/y \downarrow$
  - debt-financed deficit
  - if agents expect higher future taxes, return on investment $\downarrow$
  - investment in the downturn declines *more* than in absence of countercyclical policy
  - capital stock *lower* than in absence of countercyclical policy
  - downturn is deeper and more prolonged that in absence of countercyclical policy
COUNTERCYCLICAL FISCAL POLICIES

• Capital accumulation

\[ k_t + h_t = \left(1 - \frac{1}{\eta_t}\right) \left(1 - \delta s_t^g\right) f(k_{t-1}, h_{t-1}) \]

• \( s_t^g \uparrow \Rightarrow \text{capital} \downarrow \)
• \( \eta_t \) plays two roles
  1. heavy dependence of direct taxation \( \Rightarrow \eta \) high
     • elasticity of capital wrt/ \( s_t^g \) is high
  2. if future taxes rise, \( \eta_t \) rises
     • further raising elasticity of capital wrt/ \( s_t^g \)

• How countercyclical policies are expected to be financed influences their effectiveness
COUNTERCYCLICAL FISCAL POLICIES

U.S. Policy Variables

Purchases as Share of Output

Transfers as Share of Output

Tax Revenues as Share of Output

Seigniorage as Share of Output
Portfolio Choices (U.S. Data)

Cyclical

\[ \frac{X_k}{C + X_k} \]


-0.15
-0.10
-0.05
0.00
0.05
0.10


-0.125
-0.100
-0.075
-0.050
-0.025
-0.000
0.025
0.050
0.075
COUNTERCYCLICAL FISCAL POLICIES

Portfolio Choices (U.S. Data)

Cyclical

Velocity of C+Xk

-0.056 -0.042 -0.028 -0.014 -0.000 0.014 0.028 0.042

Velocity of Y

-0.08 -0.06 -0.04 -0.02 0.00 0.02 0.04 0.06
COUNTERCYCLICAL FISCAL POLICIES

Model Portfolio Choices

Cyclical

$\frac{X_k}{C+X_k}$


-0.032
-0.024
-0.016
-0.008
0.000
0.008
0.016
0.024

$\frac{X}{Y}$


-0.040
-0.032
-0.024
-0.016
-0.008
0.000
0.008
0.016
0.024
0.032
COUNTERCYCLICAL FISCAL POLICIES

Model Portfolio Choices

Cyclical

Velocity of C+Xk

Velocity of Y

-0.045
-0.036
-0.027
-0.018
-0.009
0.000
0.009
0.018
0.027
Third Model

- Seek to provide micro foundations for the FTPL
- Elastic labor supply; fixed capital stock
- Proportional tax levied against labor income has both “supply” and “demand” effects
- FTPL typically focuses only on “demand” effects
- Complete contingent claims, fiat currency, nominal government debt
- CRS production in labor
- Derive effects of tax policies on balance sheets of HHs
The Model

- Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)] \]

- HH budget constraint

\[ c_t + m_t + E_t \left[ Q_{t,t+1} \frac{B_{t,t+1}}{P_t} \right] \leq (1 - \tau_t)(w_t h_t + \Phi_t) + \frac{B_{t-1,t} + M_{t-1}}{P_t} \]

- \( Q_{t,t+1} \) is stochastic discount factor (nominal value at \( t \) of $1 at \( t + 1 \); \( \Phi_t \) is real dividends
- \( E_t \left[ Q_{t,t+1} \frac{B_{t,t+1}}{P_t} \right] \) is real value at \( t \) of nominal contingent claims

\[ 1 + i_t = \frac{1}{E_t[Q_{t,t+1}]}, \quad Q_{t,t+1} = q_{t,t+1} \frac{P_t}{P_{t+1}} \]
THE MODEL

• Rewrite the HH’s flow b.c. as

\[ c_t + \frac{i_t}{1 + i_t} m_t + E_t[q_{t+1} a_{t+1}] \leq (1 - \tau_t) (w_t h_t + \Phi_t) + a_t \]

\[ a_t = \frac{B_{t-1} + M_{t-1}}{P_t} \], \text{ value of nominal assets} \]

• HH’s present-value b.c. is

\[ E_0 \sum_{t=0}^{\infty} q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t - (1 - \tau_t) (w_t h_t + \Phi_t) \right] \leq a_0 \]

with \( \lim_{t \to \infty} E_0[q_t a_t] = 0 \)
The Model

• First-order conditions

\[ \beta^t u_c(c_t, m_t) = \lambda q_t \]
\[ \beta^t u_m(c_t, m_t) = \lambda q_t \left( \frac{i_t}{1 + i_t} \right) \]

where \( \lambda = u_c(c_0, m_0) \)

\[ \frac{v'(1 - h_t)}{u_c(c_t, m_t)} = (1 - \tau_t)w_t \]

• Use these in PV b.c.

\[ E_0 \sum_{t=0}^{\infty} \beta^t [u_c(c_t, m_t)c_t + u_m(c_t, m_t)m_t - (1 - \tau_t)y_t u_c(c_t, m_t)] \]

\[ u_c(c_0, m_0) = a_0 \]

• Note: LHS entirely in terms of allocations
• When allocations are unique, have a unique real value of nominal assets, \( a_0 = \frac{B_{-1,0} + M_{-1}}{P_0} \)
EQUILIBRIUM

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u_c(c_t, m_t)c_t + u_m(c_t, m_t)m_t - (1 - \tau_t)y_t u_c(c_t, m_t) \right] / u_c(c_0, m_0) = a_0$$

- Under rational expect, HH knows $a_0$ when it optimizes
- This is an eqm balance sheet relation, where LHS is PV of HH’s assets at time 0
- Get cond in policy variables, subst $y_t = c_t + g_t$ in relation

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_c(c_t, m_t)}{u_c(c_0, m_0)} \left[ (\tau_t y_t - g_t) + \frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} m_t \right] = a_0$$

Noting that $q_t = \beta^t \frac{u_c(c_t, m_t)}{u_c(c_0, m_0)}$

$$E_0 \sum_{t=0}^{\infty} q_t \left[ (\tau_t y_t - g_t) + \frac{i_t}{1 + i_t} m_t \right] = a_0$$

An equilibrium condition!
A Fiscal Theory Equilibrium

\[ E_0 \sum_{t=0}^{\infty} q_t \left[ (\tau_t y_t - g_t) + \frac{i_t}{1+i_t} m_t \right] = a_0 \]

- Suppose: \( \tau_t y_t \) are lump-sum tax revenues; \( y \) & \( g \) exogenous, then \( q \) given; let MP peg \( i_t = \bar{i} \Rightarrow m_t = h(y_t) \) independent of MP & FP; let FP set \( \{\tau_t\} \) exogenously
- Under these ass’ns, LHS a number, call it \( PV S \), so

\[ P_0 = \frac{B_{-1,0} + M_{-1}}{PV S} \]

- At \( t = 0 \), \( B_{-1,0} + M_{-1} \) given, so this determines \( P_0 \)
- Can think of \( 1/PV S \) as price of nominal assets, which plays the same role as \( 1/\Delta \) in earlier model
**Policy Experiments**

1. Expect lower $\tau_{t+k} \Rightarrow PV S \downarrow \Rightarrow P_0 \uparrow$
2. Reduce current $\tau_0 \Rightarrow PV S \downarrow \Rightarrow P_0 \uparrow$
3. Expect lower $\frac{\bar{i}}{1+i} m \Rightarrow PV S \downarrow \Rightarrow P_0 \uparrow$

   - (3) seems perverse relative to standard theory
   - lower expected seigniorage iff lower $\bar{i} \Rightarrow$ lower $\pi^e$ in most monetary models $\Rightarrow$ higher expected return to $M \Rightarrow M^d \uparrow \Rightarrow P_0 \downarrow$
   - what’s going on?
   - in standard models, the **ubiquitous eqm condition** is present but it doesn’t restrict the nature of the eqm b/c it is assumed that taxes adjust to alter the $PV S$ for any given $P_0$
     - in FTPL, lower $\frac{\bar{i}}{1+i} m \Rightarrow$ less “backing” for nominal assets, so nominal assets are worth less, meaning $1/P_0 \downarrow$
   - Whether the **ubiquitous eqm condition** should be treated as a *constraint* or an *eqm condition* is at the heart of Buiter’s critique of the FTPL
A Price-Theoretic View of the FTPL

- Follow public finance to extend Slutsky-Hicks decomposition to include a third effect
- FT works through a type of wealth effect that arises when $\Delta P$ revalues nominal assets in HH portfolios
- Decompose impacts of tax change as
  - total effect = substitution effect + wealth effect + revaluation effect
- Let $y_t^F$ be Becker’s “full income” (dividend income + maximum labor income if HH works entire time endowment—1 unit)

$$y_t^F = (1 - \tau_t)w_t \cdot 1 + \Phi_t$$

- HH takes $y_t^F$ as given and from it purchases consumption, real balances, leisure
A Price-Theoretic View of the FTPL

- HH flow b.c.

\[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau_t)w_t(1 - h_t) + E_t[q_{t,t+1}a_{t+1}] \leq y_F^t + a_t \]

- HH present value b.c.

\[ E_0 \sum_{t=0}^{\infty} q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau_t)w_t(1 - h_t) \right] \leq a_0 + v_0 \]

with \( \lim_{t \to \infty} E_0[q_t a_t] = 0 \), \( \lim_{t \to \infty} E_0[q_t y_F^t] = 0 \)

- \( v_0 \) is expected PV of full income flows, \( v_0 = E_0 \sum_{t=0}^{\infty} [q_t y_F^t] \)

- HH takes both \( a_0 \) and \( v_0 \) parametrically
A Price-Theoretic View of the FTPL

- Lagrange multiplier on the PV b.c. is $\lambda = \frac{e_0}{a_0 + v_0}$

$$e_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ u_c(c_t, m_t)c_t + u_m(c_t, m_t)m_t + v'(1 - h_t)(1 - h_t) \right]$$

$e_0$ is expected PV of expenditures (including leisure)

- $\lambda$ is shadow price of wealth
  - wealth rises ($a_0 + v_0 \uparrow$) $\Rightarrow \lambda \downarrow$
  - expenditures rise ($e_0 \uparrow$) $\Rightarrow \lambda \uparrow$

- Demand functions

$$c_t = c \left( \frac{q_t}{\beta^t}, \frac{i_t}{1 + i_t}, \frac{a_0 + v_0}{e_0} \right)$$

$$m_t = m \left( \frac{q_t}{\beta^t}, \frac{i_t}{1 + i_t}, \frac{a_0 + v_0}{e_0} \right)$$

$$h_t = h \left( \left(1 - \tau_t\right)w_t, \frac{q_t}{\beta^t}, \frac{i_t}{1 + i_t}, \frac{a_0 + v_0}{e_0} \right)$$
A Price-Theoretic View of the FTPL

- Conventional wealth effect vs. revaluation effect
  - suppose $B_{-1,0} + M_{-1} = 0$
  - revaluation effect is zero ($a_0 = 0$)
  - conventional wealth effect still operates through $v_0$ & $e_0$
- Of course, taxes can affect $P_0$ even if FTPL not operative
  - suppose $\tau_0 \uparrow$
  - substitution effect reduces labor supply
  - wealth effect raises labor supply
  - final impact depends on relative sizes
  - but then the resulting $\Delta P_0$ and $\Delta a_0$ imposes restrictions on $\{\tau_t\}_{t=1}^{\infty}$ necessary for eqm
\textbf{SUBSTITUTION, WEALTH & REVALUATION}

- Suppose \( \{ \tau_t^* \}_{t=0}^\infty \) changes to \( \{ \tau_t^\dagger \}_{t=0}^\infty \)
- Problem (*)

\[
\max_{\{c_t, m_t, h_t\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta_t [u(c_t, m_t) + v(1 - h_t)]
\]

\[
\text{s.t. } E_0 \sum_{t=0}^\infty q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau_t^*) w_t (1 - h_t) \right] \leq a_0 + v_0
\]

yields \( \{ c_t^*, m_t^*, h_t^*, w_t^*, a_t^*, v_t^*, e_t^*, P_t^*, q_t^*, R_t^*, \Phi_t^* \}_{t=0}^\infty \)

- Problem (\( \dagger \))

\[
\max_{\{c_t, m_t, h_t\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta_t [u(c_t, m_t) + v(1 - h_t)]
\]

\[
\text{s.t. } E_0 \sum_{t=0}^\infty q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau_t^\dagger) w_t (1 - h_t) \right] \leq a_0 + v_0
\]

yields \( \{ c_t^\dagger, m_t^\dagger, h_t^\dagger, w_t^\dagger, a_t^\dagger, v_t^\dagger, e_t^\dagger, P_t^\dagger, q_t^\dagger, R_t^\dagger, \Phi_t^\dagger \}_{t=0}^\infty \)
• Set lump-sum transfers, $T_0^s$, so HH can achieve same level of utility it would have obtained under the (*) tax even though it optimizes under the (†) tax

• Problem (Substitution)

$$
\max_{\{c_t, m_t, h_t\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta^t [u(c_t, m_t) + v(1 - h_t)]
$$

subject to

$$
E_0 \sum_{t=0}^\infty q_t^\dagger \left[ c_t + \frac{i_t^\dagger}{1 + i_t^\dagger} m_t + (1 - \tau_t^\dagger) w_t^\dagger (1 - h_t) \right] \leq a_0^\dagger + v_0^\dagger + T_0^s
$$

• constraining prices to be eqm prices under (†) tax ⇒ budget line of this problem tangent to HH’s indifference surface under (†) tax
A HICKSIAN DECOMPOSITION

- Problem (No Revaluation)

\[
\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)]
\]

s.t.

\[
E_0 \sum_{t=0}^{\infty} q_t^\dagger \left[ c_t + \frac{i_t^\dagger}{1 + i_t^\dagger} m_t + (1 - \tau_t^\dagger) w_t^\dagger (1 - h_t) \right] \leq a_0^* + v_0^\dagger
\]

- HH assumes revaluation does not result from the tax change, so assets have value \(a_0 = a_0^*\) under (\(\dagger\)) tax
A Hicksian Decomposition

- Set lump-sum transfers, $T_0^w$, so HH can achieve the same level of utility it would have obtained under the ($\dagger$) tax, with and without asset revaluation
- Problem (Revaluation)

$$\max_{\{c_t, m_t, h_t\}} \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)]$$

s.t. $$\sum_{t=0}^{\infty} q_t^\dagger \left[ c_t + \frac{i_t^\dagger}{1 + i_t^\dagger} m_t + (1 - \tau_t^\dagger) w_t^\dagger (1 - h_t) \right] \leq a_0^\dagger + v_0^\dagger + T_0^w$$

- $T_0^w$ permits Problem (No Revaluation) and Problem (Revaluation) to achieve the same level of utility
- Total Effect: Problem (*) vs. Problem ($\dagger$)
- Substitution Effect: Problem (*) vs. Problem (Substitution)
- Revaluation Effect: Problem (No Revaluation) vs. Problem (Revaluation)
- Wealth Effect = Total – Substitution – Revaluation
A HICKSIAN DECOMPOSITION

- Solutions to optimization problems are cons demands

\[(\ast): c^*_t = c \left( \frac{q^*_t}{\beta^t}, \frac{i^*_t}{1 + i^*_t}, \frac{a^*_0 + v^*_0}{e^*_0} \right)\]

\[(\dagger): c^t = c \left( \frac{q^\dagger_t}{\beta^t}, \frac{i^\dagger_t}{1 + i^\dagger_t}, \frac{a^\dagger_0 + v^\dagger_0}{e^\dagger_0} \right)\]

(Substitution): \[c^\dagger_t \Big|_{u_0 = u^*_0} = c \left( \frac{q^\dagger_t}{\beta^t}, \frac{i^\dagger_t}{1 + i^\dagger_t}, \frac{a^*_0 + v^*_0 + T^s_0}{e^\dagger_0 \big|_{u_0 = u^*_0}} \right)\]

(Revaluation): \[c^\dagger_t \big|_{u_0 = u^\dagger_0, a_0 = a^*_0} = c \left( \frac{q^\dagger_t}{\beta^t}, \frac{i^\dagger_t}{1 + i^\dagger_t}, \frac{a^\dagger_0 + v^\dagger_0 + T^w_0}{e^\dagger_0 \big|_{u_0 = u^*_0, a_0 = a^*_0}} \right)\]

- \[c^\dagger_t \big|_{u_0 = u^*_0}: \text{planned consumption under (\dagger) tax, with utility at } u^*_0, \text{ the level under } (\ast) \text{ tax}\]

- \[c^\dagger_t \big|_{u_0 = u^\dagger_0, a_0 = a^*_0}: \text{planned consumption without revaluation under (\dagger) tax with utility at } u^\dagger_0, \text{ the level under (\dagger) tax}\]
A Hicksian Decomposition

- The full decomposition

$$\log \left( \frac{c_t^\dagger}{c_t^*} \right) = \log \left( \frac{c_t^\dagger|_{u_0=u_0^*}}{c_t^*} \right)$$

**total effect**

$$= \log \left( \frac{c_t^\dagger|_{u_0=u_0^*}}{c_t^*} \right)$$

**substitution effect**

$$+ \log \left( \frac{c_t^\dagger|_{u_0=u_0^*,a_0=a_0^*}}{c_t^*|_{u_0=u_0^*}} \right)$$

**wealth effect**

$$+ \log \left( \frac{c_t^\dagger}{c_t^*|_{u_0=u_0^*,a_0=a_0^*}} \right)$$

**revaluation effect**
**An Example Economy**

- Assume log preferences

\[ u(c, m) + v(1 - h) = \log c + \log m + \log(1 - h) \]

Then

\[ e_0 = \frac{3}{1 - \beta} \]

\[ c_t = \left( \frac{1 - \beta}{3} \right) \left( \frac{\beta^t}{q_t} \right) (a_0 + v_0) \]

\[ m_t = \left( \frac{1 - \beta}{3} \right) \left( \frac{\beta^t}{q_t} \right) \left( \frac{1 + i_t}{i_t} \right) (a_0 + v_0) \]

\[ h_t = 1 - \left( \frac{1 - \beta}{3} \right) \left( \frac{\beta^t}{(1 - \tau_t)w_tq_t} \right) (a_0 + v_0) \]

- Then can compute all the objects in the decomposition
- Assume MP pegs \( i_t \) to satisfy: \( \beta(1 + i_t) = 1, \quad t \geq 0 \)
- See Leeper-Yun (2006) for details and case of lump-sum
AN EXAMPLE ECONOMY

• Income taxes set $\tau_t > 0$

$$y_t = \frac{1 - \tau_t}{2 - s^g - \tau_t}$$

$$q_t = \beta^t \frac{(1 - \tau_0)(2 - s^g - \tau_t)}{(1 - \tau_t)(2 - s^g - \tau_0)}$$

• Present value full income flows is

$$v_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(1 - \tau_0)(2 - s^g - \tau_t)}{2 - s^g - \tau_0} \right]$$

• HH present value b.c.

$$a_0 + v_0 = \frac{3}{1 - \beta} \frac{(1 - \tau_0)(1 - s^g)}{2 - \tau_0 - s^g}$$

• A Laffer curve in $\tau_t y_t$ with revenues maximized at

$$\bar{\tau} = 2 - s^g - \sqrt{(2 - s^g)(1 - s^g)}$$
**An Example Economy**

- Suppose taxes constant at $\tau$
- $y$ & $c$ constant; $i$ pegged; $q_t = \beta^t$

\[ v_0 = \frac{1 - \tau}{1 - \beta}, \quad a_0 = \frac{1 - \beta}{1 - \tau} \frac{2 - s^g - \tau}{1 - 2s^g + \tau} \]

- Equilibrium price level

\[ P_0 = \frac{1 - \beta}{1 - \tau} \frac{2 - s^g - \tau}{1 - 2s^g + \tau} (B_{-1} + M_{-1}) \]

- Note from $a_0 = \frac{1 - \beta}{1 - \tau} \frac{2 - s^g - \tau}{1 - 2s^g + \tau}$, quadratic in $\tau \implies$ Laffer curve in sum of PV surpluses + seigniorage
- Laffer curves in $\tau_t y_t$ and in $a_0$ can look very different
**CONVENTIONAL LAFFER CURVE**

Income Tax Rate ($\tau$) vs. Units of Goods (direct tax revenue flow (real GDP))

- Income Tax Rate ($\tau$) = 58.6%
FISCAL THEORY LAFFER CURVE

![Graph showing the relationship between income tax rate and various fiscal indicators.]

- Income Tax Rate ($\tau$)
- Units of Goods
- Direct tax revenue flow
- Tax base (real GDP)
- Primary surplus + seigniorage (present value)

Key points:
- $\tau = 26.8\%$
- $\tau = 58.6\%$

Legend:
- Red circles: Units of Goods
- Black stars: Tax base (real GDP)
- Blue triangles: Direct tax revenue flow
**Two Laffer Curves**

- Why are these different?
- Tax bases differ
  - conventional: $\tau_t y_t$
  - fiscal theory: $PV \left( \tau_t y_t + \frac{i_t}{1+i_t} m_t \right)$
  - changes in conventional tax base, $y_t$, feed into $m_t$ and the seigniorage tax base
- Should we care about this?
  - presents tradeoffs
  - relevant for inflation-targeting countries to think about the fiscal consequences of MP
Wrap Up

• Fiscal theory has been accused of being “incoherent,” “inconsistent with economic theory,” and worse

• This shows that with the right kind of price-theoretic analysis, the revaluation effect that lies at the heart of the FTPL can be understood as a natural extension in an environment with nominal assets of standard the Slutsky-Hicks decomposition

• Critics have also accused FTPL of ignoring the government’s budget constraint

• Here we have shown that you can get an eqm condition that determines $P_0$ without any reference to government variables

• Introducing distorting taxes to a FTPL analysis reveals a second kind of Laffer curve that has been largely overlooked