MAGYAR NEMZETI BANK MINI-COURSE
Lecture 4. Generalizing Policy Interactions (A)

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THE MESSAGES

- Draws heavily from “Generalizing the Taylor Principle,” with Troy Davig (AER, June 2007)
- We do see policy rules—or regimes—change
  - to study the implications of recurring changes, need to model them coherently
- Before studying monetary-fiscal interactions when policy regimes can change, need some preliminary analysis when only MP can switch
- This allows simple analytical derivations that build intuition and understanding
- Many of our inferences are monetary policy effects change in subtle ways once we allow recurring regime change
- Subsequent work will allow both monetary and fiscal regime to undergo recurring change
**Simplifying Policy**

- Monetary policy is complex
- For descriptive & prescriptive reasons, seek to simplify
- Most successful simplification due to Taylor

\[ i_t = \bar{i} + \alpha(\pi_t - \pi^*) + \gamma x_t + \varepsilon_t \]

**Taylor principle:** \( \alpha > 1 \)
- necessary & sufficient for unique bounded eqm (w/ bounded shocks)

- Unique & stable eqm necessary for good policy
  - rules out arbitrarily large fluctuations
THE TAYLOR RULE & PRINCIPLE

- Central banks can stabilize economy by adjusting nominal interest rate more than one-for-one with inflation
  - approximates Federal Reserve behavior since 1982
  - nearly optimal in workhorse class of monetary models
  - used by central banks as a benchmark
- Maintains two key assumptions
  - fiscal policy is perpetually passive
  - policy rule permanent & agents believe change impossible
- Here we relax this second assumption
  - rule evolves according to a Markov chain
  - consider two conventional monetary models
GENERALIZING THE TAYLOR RULE & PRINCIPLE

- $\alpha(s_t), \gamma(s_t)$ $s_t \sim$ Markov chain
- $s_t$: “rule,” “regime,” “state”
- $s_t$ exogenous (for now)
- Can believe actual policy rule time invariant
  - but Taylor rule is a gross simplification of reality
  - paper shows that a particular form of non-linearity can change predictions of models
In the Fisherian Model . . .

- Derive *long-run Taylor principle*
  - imposes much weaker conditions on MP for uniqueness
  - departures from short-run Taylor principle can be substantial—but brief—or modest—and prolonged
  - the more “hawkish” one regime is, the more “dovish” the other can be and still deliver uniqueness
  - “expectations formation effects”—beliefs about possible future regimes affect current eqm, increasing volatility even in a regime that satisfies TP
In the New-Keynesian Model . . .

- Derive long-run Taylor principle: dramatically expands region of determinacy
- Inference that inflation of the 70’s due to failure to obey TP does not hold up when expectations embed possibility of regime change
- Occasional large departures from TP—due to worries about financial instability or economic weakness—can have quantitatively important impacts even in a regime that satisfies TP
- Misleading inferences can arise from dividing data into regime-specific periods to interpret estimates as arising from distinct fixed regimes
**Why Regime Change?**

- Evidence that monetary policy regime changed
- Institutional or policy reforms
  - adoption of inflation targeting by over 20 countries
  - Fed’s “just trust us” approach
- Logical consistency
  - if regime *has* changed, regime *can* change
  - expectations depend on prob. distn. over possible regimes
- Recurring: in US, no legislated change installed Volcker or Greenspan
  - confluence of economic/political conditions allowed US to dodge a bullet and get Bernanke (coulda’ been a FOG)
Because Taylor rule a gross simplification, deviations occur

- can be large and serially correlated
- are systematic responses to state of economy

How should we model these deviations?

- shuffled into the $\varepsilon$’s?
- time-varying feedback coefficients, $\alpha_t$ & $\gamma_t$?

$\varepsilon$’s affect conditional expectations

$\alpha_t$ & $\gamma_t$ affect expectations functions

A substantive choice
MODEL OF INFLATION DETERMINATION

• A simple Fisherian economy

\[ i_t = E_t \pi_{t+1} + r_t \]
\[ r_t = \rho r_{t-1} + \nu_t, \quad \nu \text{ bounded support} \]
\[ i_t = \alpha(s_t) \pi_t, \quad \text{Markov; } s_t = 1, 2 \]

\[ p_{ij} = P[s_t = j | s_{t-1} = i] \]

\[ \alpha(s_t) = \begin{cases} \alpha_1 & \text{for } s_t = 1 \\ \alpha_2 & \text{for } s_t = 2 \end{cases} \]

• a monetary policy regime: realization of \( \alpha(s_t) \)
• a monetary policy process: collection \((\alpha_1, \alpha_2, p_{11}, p_{22})\)
• policy is active if \( \alpha_i > 1 \); passive if \( \alpha_i < 1 \)
DETERMINACY: DEFINITION

• Seek generalization of Taylor principle
  • necessary & sufficient condition for existence of unique bounded eqm
• Why boundedness?
  • consistent w/ standard definition under fixed regime
  • corresponds to locally unique eqm
    • can analyze small perturbations
  • considering log-linearized models
    • boundedness ensures approximations are good
**Determinacy: Formalism**

Model: $\alpha(s_t)\pi_t = E_t\pi_{t+1} + r_t$

- Let $\Omega_t^{-s} = \{r_t, r_{t-1}, \ldots, s_{t-1}, s_{t-2}, \ldots\}$ and $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$
- Integrating over $s_t$, for $s_t = 1$ and $s_t = 2$

$$E_t\pi_{t+1} = E[\pi_{t+1} \mid s_t = i, \Omega_t^{-s}]$$

$$= p_{i1}E[\pi_{1t+1} \mid \Omega_t^{-s}] + p_{i2}E[\pi_{2t+1} \mid \Omega_t^{-s}]$$

where $\pi_{it} = \pi_t(s_t = i, r_t)$, the solution when $s_t = i$

- The system is

$$\begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} E_t\pi_{1t+1} \\ E_t\pi_{2t+1} \end{bmatrix} + \begin{bmatrix} r_t \\ r_t \end{bmatrix}$$

where $E_t\pi_{it+1}$ denotes $E[\pi_{it+1} \mid \Omega_t^{-s}]$
Determinacy: Formalism (con’t)

- Write system as

\[ \pi_t = ME_t \pi_{t+1} + \alpha^{-1} r_t \]

- MSV solution: \( \pi_t \) function only of \((r_t, s_t)\)
- Define \( x_t = \pi_t - \pi_t^{MSV}(r_t, s_t) \)
- Bounded soln for \( \{x_t\} \iff \) bounded soln for \( \{\pi_t\} \)
- We study: \( x_t = ME_t x_{t+1} \)
- Proof of determinacy shows that under certain conditions on the policy process, \( x_t = 0 \) is the only solution
**Prop. 1** When $\alpha_i > 0$, a unique bounded solution exists iff all the eigenvalues of $M$ lie inside the unit circle

- Sufficiency: the usual proof in linear RE models
  - intuition: boundedness requires that $\lim_{n \to \infty} M^n = 0$, so $x_t = 0$ the only solution
  - delivered by eigenvalue condition
Determinacy: Formalism (con’t)

- Necessity: Suppose $\lambda_1 \geq 1$, $\lambda_2 < 1$

  - diagonalize $M$, let $y_t = V^{-1}x_t$, then

  \[
  \begin{bmatrix}
  y_{1t} \\
  y_{2t}
  \end{bmatrix} = \begin{bmatrix}
  \lambda_1 & 0 \\
  0 & \lambda_2
  \end{bmatrix} \begin{bmatrix}
  E_t y_{1t+1} \\
  E_t y_{2t+1}
  \end{bmatrix}
  \]

  bounded solutions $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}$, so

  \[
  \begin{bmatrix}
  x_{1t} \\
  x_{2t}
  \end{bmatrix} = \begin{bmatrix}
  \gamma v_{11} \lambda_1^{-t} \\
  \gamma v_{21} \lambda_1^{-t}
  \end{bmatrix}
  \]

  - also exist bounded sunspot solutions:

    $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}$, $y_{2t+1} = 0$, $E_t\phi_{t+1} = 0$, bounded

    - multiple eq & sunspots possible w/ more stringent det defn
LONG-RUN TAYLOR PRINCIPLE

• Prop. 2 Given $\alpha_i > p_{ii}$ for $i = 1, 2$, the following statements are equivalent:

  (A) All the eigenvalues of $M$ lie inside the unit circle.
  
  (B) $\alpha_i > 1$, for some $i = 1, 2$, and the long-run Taylor principle (LRTP)

  \[(1 - \alpha_2) p_{11} + (1 - \alpha_1) p_{22} + \alpha_1 \alpha_2 > 1\]

  is satisfied.

• Premise $\alpha_i > p_{ii}$ all $i$ unfamiliar
  
  • fixed regime: MP always obeys TP
  • LRTP is hyperbola w/ asymptotes $\alpha_1 = p_{11}$ & $\alpha_2 = p_{22}$
  • restricts $\alpha$’s to economically interesting portion of hyperbola
A Range of Policies Deliver Uniqueness

\[ \alpha_1 > 1: \quad p_{11}(1 - \alpha_2) + p_{22}(1 - \alpha_1) + \alpha_1 \alpha_2 > 1 \]

- Some policy processes that deliver unique equilibria
  \[ \alpha_1 \to \infty \Rightarrow \alpha_2 > p_{22} \]
  or
  \[ p_{11} = 1 \Rightarrow \text{need } \alpha_1 > 1 \text{ and } \alpha_2 > p_{22} \]

- More active is one regime, more passive the other can be
  \[ p_{22} \to 1 \text{ OK if } \alpha_2 \approx 1 \text{ (but } < 1) \]
- Ergodic prob of passive regime can be \( \approx 1 \) (but \(< 1\))
  \[ p_{11} = p_{22} = 0 \text{ need } \alpha_2 > 1/\alpha_1 \]
- More active in one regime, less active in the other

- Figure illustrates these points
**Determinacy Region: Fisherian Model**

- $p_{11} = 0.95 ; p_{22} = 0.95$
- $p_{11} = 0.8 ; p_{22} = 0.95$
- $p_{11} = 0.95 ; p_{22} = 0$
- $p_{11} = 0 ; p_{22} = 0$
FISHERIAN MODEL: SOLUTION

- Define state as \((r_t, s_t)\) & find MSV solutions
  - posit regime-dependent rules:

\[
\pi_t = a(s_t = i)r_t
\]

\[
a(s_t) = \begin{cases} 
a_1 & \text{for } s_t = 1 \\
a_2 & \text{for } s_t = 2
\end{cases}
\]

- expectations functions:

\[
E[\pi_{t+1} | s_t = 1, r_t] = [p_{11}a_1 + (1 - p_{11})a_2] \rho r_t
\]

\[
E[\pi_{t+1} | s_t = 2, r_t] = [(1 - p_{22})a_1 + p_{22}a_2] \rho r_t
\]

- solve simple \(2 \times 2\) system to get \(a_1\) and \(a_2\)
\textbf{Solution}

- Solutions are:

\[ a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

and

\[ a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

\[ p_{12} = 1 - p_{11}, \quad p_{21} = 1 - p_{22} \quad \& \quad \text{“fixed-regime” coefficients} \]

\[ a_i^F = \frac{1}{\alpha_i - \rho p_{ii}}, \quad i = 1, 2 \]

- \( \alpha_1 > \alpha_2 \iff a_1 < a_2 \)
Expectations-Formation Effects

• Solutions are:

\[ a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

and

\[ a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

• Expectations-formation effects from regime 2 to regime 1
  • through \( p_{12} a_2^F \)
  • large if \( p_{12} \) large, \( p_{22} \) large, \( \alpha_2 \) small
Special Case

- Real interest rate serially uncorrelated \((\rho = 0)\), solution is

\[
a_1 = \frac{1}{\alpha_1}
\]

and

\[
a_2 = \frac{1}{\alpha_2}
\]

- Looks like fixed-regime solution, BUT
  - determinacy in FR: \(\alpha_i > 1\) all \(i\)
  - switching allows determinacy w/ some \(\alpha_i < 1\)
  - if \(p_{22} < \alpha_2 < 1\), regime 2 *amplifies* shocks
  - possible to fit volatile data with determinate eqm?
A New-Keynesian Model

- Bare-bones model with nominal rigidities
  - from class in wide use for monetary policy analysis
  - general insights extend to more complex models now confronting data

- With recurring regime change and rational expectations:
  - How does the Taylor principle change?
  - How do impacts of demand and supply shocks change?
- Expectations-formation effects can be large
A New-Keynesian Model

- Consumption-Euler equation and AS relations

\[ x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u^D_t \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u^S_t \]

- Disturbances: bounded, autoregressive, mutually uncorrelated

\[ u^D_t = \rho_D u^D_{t-1} + \varepsilon^D_t \]
\[ u^S_t = \rho_S u^S_{t-1} + \varepsilon^S_t \]

- A Taylor rule for \( s_t = 1, 2 \)

\[ i_t = \alpha(s_t)\pi_t + \gamma(s_t)x_t \]
NEW-KEYNESIAN MODEL: DETERMINACY

• Let \( \pi_{it} = \pi_t(s_t = i) \) & \( x_{it} = x_t(s_t = i) \), \( i = 1, 2 \)

• Define forecast errors

\[
\begin{align*}
\eta_{1t+1}^\pi &= \pi_{1t+1} - E_t\pi_{1t+1} \\
\eta_{1t+1}^x &= x_{1t+1} - E_t x_{1t+1} \\
\eta_{2t+1}^\pi &= \pi_{2t+1} - E_t\pi_{2t+1} \\
\eta_{2t+1}^x &= x_{2t+1} - E_t x_{2t+1}
\end{align*}
\]

• Model is

\[
AY_t = BY_{t-1} + A\eta_t + C u_t
\]

• Unique bounded eqm requires the 4 generalized eigenvalues of \((B, A)\) to lie inside unit circle

• Derive long-run Taylor principle
**New-Keynesian Model: Determinacy**

- Set $\gamma(s_t) = 0$
- Intertemporal margins interact w/ expected policy to affect determinacy
- Determinacy regions expand w/ parameters that reduce ability to substitute away from future policy
  - increase degree of stickiness ($\kappa$)
  - reduce intertemporal elasticity of substitution ($\sigma$)
Determinacy Regions Expand

- $p_{11} = 0.95$, $p_{22} = 0.95$
- $p_{11} = 0.8$, $p_{22} = 0.95$
- $p_{11} = 0.95$, $p_{22} = 0$
- $p_{11} = 0$, $p_{22} = 0$
DET. REGIONS & PRIVATE PARAMETERS

\[ p_{11} = 0.9 \ , \ p_{22} = 0.9 \ , \ \omega = 0.01 \]

\[ p_{11} = 0.9 \ , \ p_{22} = 0.9 \ , \ \omega = 0.99 \]

\[ p_{11} = 0.9 \ , \ p_{22} = 0.9 \ , \ \sigma = 0.01 \]

\[ p_{11} = 0.9 \ , \ p_{22} = 0.9 \ , \ \sigma = 10 \]
New-Keynesian Model: Solutions

- MSV solution is straightforward to compute
- Easiest to consider numerical examples
- For inflation, intuition from fixed regimes carries through
  - more active MP process reduces inflation volatility
- For output, switching introduces non-monotonicity
  - more active MP process can raise or lower output volatility, depending on source of shock
A Return to the 1970s?

- Studies find Fed passive 1960-79; active since 1982
- Fears of reverting to 1970s behind calls for IT
- Fiscal policy may be an impetus for switching to passive MP
- Embed estimates of Lubik-Schorfheide in switching setup
  - compute set of \((p_{11}, p_{22})\) that deliver uniqueness
- Implications
  - inference that US switched from indeterminate to determinate eqm requires current state be absorbing
  - fixed regime badly mispredicts impacts of supply & demand shocks
**Determinacy Regions: L-S Estimates**

**LS:** \( \alpha_1 = 2.19, \gamma_1 = .30, \alpha_2 = .89, \gamma_2 = .15 \)

Dark: high flexibility \((\sigma = 1.04, \kappa = 1.07)\)

Light: low flexibility \((\sigma = 2.84, \kappa = .27)\)
Financial Crises & Business Cycles

- MP shifts focus from inflation to other concerns
  - financial stability & job creation
  - shift can last few months or more than year
  - during Greenspan era: 2 market crashes, 2 foreign financial crises, 2 jobless recoveries
  - documented by Marshall and Rabanal
- Take normal times to be $\alpha_1 = 1.5$, $\gamma_1 = .25$, and persistent
  - other regime: $\gamma_2 = .5$, $\alpha_2$ and $p_{22}$ vary
  - a crude characterization of those events
- Spillovers from demand shocks can make inflation much more volatile and output much less volatile than if the active regime were permanent
# Financial Crises & Business Cycles

<table>
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<tr>
<th></th>
<th>Demand</th>
<th>Supply</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Inflation</td>
<td>Output</td>
</tr>
<tr>
<td>$p_{11} = .95$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{22} = 0$</td>
<td>1.060</td>
<td>1.011</td>
</tr>
<tr>
<td>$\alpha_2 = .25$</td>
<td>1.073</td>
<td>1.014</td>
</tr>
<tr>
<td>$\alpha_2 = 0$</td>
<td>1.268</td>
<td>.886</td>
</tr>
<tr>
<td></td>
<td>1.454</td>
<td>.807</td>
</tr>
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**Standard Deviation Active Regime Relative to Fixed Regime**

Active and fixed regimes set $\alpha_1 = \alpha = 1.5, \gamma_1 = \gamma = .25; \gamma_2 = .5$
Empirical Implications of Switching

- Commonplace for empirical work to split data into regime-dependent sub-periods
- Estimates then interpreted in fixed-regime theoretical model
- We simulate switching eqm, estimate correctly-specified (fixed-regime) identified VARs
  - assume econometrician knows when regime changed
- Estimated model

\[
\begin{align*}
  x_t &= \delta i_t + u^D_t + \text{lags} \\
  \pi_t &= \theta x_t + u^S_t + \text{lags} \\
  i_t &= \alpha \pi_t + \gamma x_t + u^P_t + \text{lags}
\end{align*}
\]
Empirical Implications of Switching

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\bar{\gamma}$</th>
<th>$\delta$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>2.182</td>
<td>0.30</td>
<td>-1.690</td>
<td>0.409</td>
</tr>
<tr>
<td>Regime 2</td>
<td>0.885</td>
<td>0.15</td>
<td>-0.750</td>
<td>1.675</td>
</tr>
<tr>
<td>Full Sample</td>
<td>1.375</td>
<td>0.225</td>
<td>-1.476</td>
<td>0.657</td>
</tr>
</tbody>
</table>

Estimates from an identified VAR using simulated data. Regime 1 is conditional on remaining in regime with $\alpha_1 = 2.19$. Regime 2 is conditional on remaining in regime with $\alpha_2 = 0.89$. Full sample is recurring changes from regime 1 to regime 2. $\alpha$ is the estimated response of monetary policy to inflation. $\bar{\gamma}$ is the policy response to output, held fixed in estimation.
\( \alpha_1 = 2.19, \gamma_1 = 0.30, \alpha_2 = 0.89, \gamma_2 = 0.15, p_{11} = 0.95, p_{22} = 0.93 \)

Dashed: fixed regime; Solid: active, switching
SUMMARY

- A broader perspective on Taylor principle and range of unique bounded equilibria it supports
- Endowing conventional models with empirically relevant MP switching processes
  - drastically alters conditions for a unique bounded eqm
  - generates important expectations-formation effects
- Developed a two-step solution method to get determinacy conditions and solutions
- Conventional models extremely sensitive to deviation from usual assumption that policy is permanent
- The possibility of regime change should be the default assumption in theoretical models
Wrap Up

• Many potential applications
  • any purely forward-looking model
  • exchange rate determination: switch between fixed & floating
  • term structure: policy switching
  • technology: switch between high- and low-growth periods
  • terms of trade: persistent & transitory changes

• Need to develop methods to allow analytical solutions with endogenous state variables