Lecture 4. Generalizing Policy Interactions (B)

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THE MESSAGES

- Difficult to obtain general analytical results with both monetary and fiscal switching
- Will examine some special cases and then turn to numerical results
- Allowing recurring regime change in both MP & FP can dramatically change nature of equilibria we study
- Raises the possibility for FP to play a role in our interpretations of business cycles
MONETARY AND FISCAL POLICY INTERACTIONS

- Standard reasoning about macro policy
  - active monetary policy necessary for stability
  - Taylor principle delivers good economic performance in many models
  - high and variable inflation due to indeterminacy
  - active monetary/passive fiscal policies insulate economy from demand shocks (e.g., fiscal)

- Reasoning rests on convenient assumptions
  - passive fiscal behavior
  - fixed policy regimes
  - local $\implies$ global
Regime Change

- Regime change: realizations of params in policy rule
  \[ R_t = \alpha_0(S_t) + \alpha_\pi(S_t)\pi_t + \alpha_x(S_t)x_t + \sigma(S_t)\varepsilon_t \]
  
  $S_t$ evolves stochastically by a known process

- Many researchers have estimated policy rules to find parameters changed over time
  - Taylor, Clarida-Galí-Gertler, Auerbach, Lubik- Schorfheide, Sala, Favero-Monacelli

- Fixed-regime theory: problematic interpretation
  - ex-ante agents put probability 0 on change
  - ex-post agents put probability 1 on new regime
  - Cooley-LeRoy-Raymon: this is logically inconsistent
What We Do

- Bring together empirical and theoretical work
- Estimate Markov-switching rules for U.S. monetary and fiscal policies
- Embed estimated joint policy process in DSGE model with rigidities
WHAT WE FIND

- Policies fluctuate between active & passive
  - some active/active; some passive/passive
- Fit is good; connects to narrative accounts
- Post-war U.S. data can be modeled as a single, locally unique equilibrium
- Fiscal theory of price level always operative
  - taxes matter even with active MP/passive FP
- Fiscal theory mechanism quantitatively important
  - $1 transitory tax cut $\Rightarrow$ PV output rises $\approx$ $1$
- Common practice: break samples into distinct regimes and embed rules in fixed-regime DSGE can produce misleading inferences
Canzoneri, Cumby, Diba: Ricardian equilibria more general than non-Ricardian

- if responses of taxes to liabilities is positive infinitely often—however small and infrequent—then eqm exhibits Ricardian equiv
- because fiscal response does not stabilize debt, these are potentially equilibria with unbounded debt-output ratios

Our example satisfies CCD’s assumptions, but deliver a unique eqm in set with bounded debt-output ratios

- this eqm is non-Ricardian
- important conclusions hinge on unboundedness ass’n of CCD
**The Model**

- MIUF, constant endowment, log prefs, constant \( g \)
- Fisher equation
  \[
  \frac{1}{R_t} = \beta E_t \frac{1}{\pi_{t+1}}
  \]
- Money demand
  \[
  m_t = \left[ \frac{R_t - 1}{R_t} \right]^{-1} c
  \]
- Monetary policy
  \[
  R_t = \exp(\alpha_0 + \alpha(S_t) \hat{\pi}_t + \theta_t)
  \]
- Tax policy
  \[
  \tau_t = \gamma_0 + \gamma(S_t)(b_{t-1} + m_{t-1}) + \psi_t
  \]

\((\theta_t, \psi_t)\) exogenous policy shocks; \( \hat{\pi} = \ln \pi \)
The Model

- $S_t$ an $N$-state Markov chain with transition probs
  \[ P[S_t = j|S_{t-1} = i] = p_{ij} \]
- Define expectation error (and use Fisher equation)
  \[ \eta_{t+1} \equiv \frac{1/\pi_{t+1}}{E_t[1/\pi_{t+1}]} = \beta \frac{R_t}{\pi_{t+1}} \]
- Then the inflation process is given by
  \[ \hat{\pi}_{t+1} = \alpha(S_t) \hat{\pi}_t + \alpha_0 + \theta_t - \hat{\eta}_{t+1} + \ln \beta \]
- Let $l_t = b_t + m_t$, real govt liabilities
- Use tax rule & money demand in govt budget constraint

\[
l_t = \left[ \frac{R_{t-1}}{\pi_t} - \gamma(S_t) \right] l_{t-1} - \frac{R_{t-1}}{\pi_t} c + D - \psi_t \]

\[ D = g - \gamma_0 \]
Solution

• Assume that
  
  I. \( E_t[\gamma_{t+1}] = \gamma \)
  
  II. \( \gamma \) satisfies \(|1/\beta - \gamma| > 1\)
  
  III. inflation process is stable in expectation (i.e., there exists a \( 0 < \xi < \infty \) such that \( |E_t\pi_{t+k}| < \xi \) for all \( k \))
  
• (I)-(II): on average FP active; (III): on average MP passive

• Iterate on \( l \) equation and take \( E_{t-1} \) and law of iterated expectations

\[
E_{t-1}[l_{t+k}] = (1/\beta - \gamma)^{k+1} \left[ l_{t-1} - c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right) \right] \\
+ c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right)
\]

Stability requires that \( l_{t-1} = c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right) \), which is positive if \( D/c < 1/\beta \).
The value of $\eta_t$ is obtained from the budget constraint after substituting in the value of $l$:

$$
\eta_t = \beta \frac{(1 + \gamma(S_t)) (\frac{1}{\beta} - \frac{D}{c}) - (\frac{D}{c}) (\frac{1}{\beta} - \gamma - 1)}{1 + \gamma - \frac{D}{c}} \\
+ \frac{\beta}{c} \left( \frac{1/\beta - \gamma - 1}{1 + \gamma - \frac{D}{c}} \right) \psi_t
$$

The unique eqm mapping from $\psi_t$ and $\gamma(S_t)$ to forecast error in inflation

$\eta$ and $\pi_t$ process yields unique solution for inflation
**Concrete Example**

- Two regimes, \( N = 2 \), and policy parameters take on the values

\[
\alpha(S_t) = \begin{cases} 
\alpha(1) & \text{for } S_t = 1 \\
\alpha(2) & \text{for } S_t = 2 
\end{cases} \quad \gamma(S_t) = \begin{cases} 
\gamma(1) & \text{for } S_t = 1 \\
\gamma(2) & \text{for } S_t = 2 
\end{cases}
\]

- Suppose \( \alpha(1) \) and \( \alpha(2) \) are sufficiently small such that the inflation process is stable in expectation

\[
E[\gamma_{t+j} | S_t = 1, \Omega_t] = \gamma(1)p_{11} + \gamma(2)p_{12}
\]

\[
= E[\gamma_{t+j} | S_t = 2, \Omega_t] = \gamma(1)p_{21} + \gamma(2)p_{22} \equiv \gamma
\]

- If either \( \gamma(1) \) or \( \gamma(2) \) is positive, then the model satisfies CCD’s premise that taxes adjust to debt infinitely often
- But negative tax shocks generate wealth effects that raise inflation
- The only eqm with bounded debt is one in which Ricardian equiv breaks down
**Policy Rule Estimates**

- Hidden Markov chain, as in Hamilton and Kim-Nelson
- Off-the-shelf policy rules; no dynamics
- Independent switching of M & F regimes

\[ r_t = \alpha_0(S^M_t) + \alpha_\pi(S^M_t)\pi_t + \alpha_x(S^M_t)x_t + \sigma_R(S^M_t)\varepsilon_t^r \]

4 states, \( \alpha \)'s have 2 sets of values, \( P^M \) transition matrix

\[ \tau_t = \gamma_0(S^F_t) + \gamma_b(S^F_t)b_{t-1} + \gamma_x(S^F_t)x_t + \gamma_g(S^F_t)g_t + \sigma_\tau(S^F_t)\varepsilon_t^\tau \]

2 states, \( P^F \) transition matrix

- \( S_t = (S^M_t, S^F_t) \). Joint distribution \( P = P^M \otimes P^F \), 8 states
Policy Rule Estimates

• U.S. data, 1948:2-2004:1

• $r$: 3-month Treasury bill
• $\pi$: log difference of GDP deflator
• $x$: log output gap using CBO potential
• $\tau$: federal receipts net transfers as share of GDP
• $b$: market value of federal debt held by public as share of GDP
• $g$: federal government consumption plus investment expenditures as a share of GDP
POLICY RULE ESTIMATES

• Four checks on plausibility of estimates
  1. Are the estimates reasonable on *a priori* grounds?
  2. Do the estimates fit the data?
  3. Do the estimates accord with narrative and other evidence on active/passive periods?
  4. Does the estimated policy process make sense in a standard DSGE model?

• Yes!
## Monetary Policy Estimates

<table>
<thead>
<tr>
<th>State</th>
<th>$S_t^M = 1$</th>
<th>$S_t^M = 2$</th>
<th>$S_t^M = 3$</th>
<th>$S_t^M = 4$</th>
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<tbody>
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<td>$\alpha_\pi$</td>
<td>1.3079</td>
<td>1.3079</td>
<td>.5220</td>
<td>.5220</td>
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<tr>
<td></td>
<td>(.0527)</td>
<td>(.0527)</td>
<td>(.0175)</td>
<td>(.0175)</td>
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<tr>
<td>$\alpha_y$</td>
<td>.0232</td>
<td>.0232</td>
<td>.0462</td>
<td>.0462</td>
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<td></td>
<td>(.0116)</td>
<td>(.0116)</td>
<td>(.0043)</td>
<td>(.0043)</td>
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<td>$\sigma_r^2$</td>
<td>1.266e-5</td>
<td>9.184e-7</td>
<td>2.713e-5</td>
<td>5.434e-7</td>
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<tr>
<td></td>
<td>(8.670e-6)</td>
<td>(1.960e-6)</td>
<td>(5.423e-6)</td>
<td>(1.512e-6)</td>
</tr>
</tbody>
</table>

**Table 1:** Log likelihood value = $-1014.737$
## Tax Policy Estimates

<table>
<thead>
<tr>
<th>State</th>
<th>$S_t^F = 1$</th>
<th>$S_t^F = 2$</th>
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</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>.0497</td>
<td>.0385</td>
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<td></td>
<td>(.0021)</td>
<td>(.0032)</td>
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<td>$\gamma_b$</td>
<td>.0136</td>
<td>-.0094</td>
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<td></td>
<td>(.0012)</td>
<td>(.0013)</td>
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<tr>
<td>$\gamma_y$</td>
<td>.4596</td>
<td>.2754</td>
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<tr>
<td></td>
<td>(.0326)</td>
<td>(.0330)</td>
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<td>$\gamma_g$</td>
<td>.2671</td>
<td>.6563</td>
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<tr>
<td></td>
<td>(.0174)</td>
<td>(.0230)</td>
</tr>
<tr>
<td>$\sigma^2_\tau$</td>
<td>4.049e-5</td>
<td>5.752e-5</td>
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<tr>
<td></td>
<td>(6.909e-6)</td>
<td>(8.472e-6)</td>
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</tbody>
</table>

**Table 2:** Log likelihood value $= -765.279$
INTEREST RATE: ACTUAL & PREDICTED
Taxes: Actual & Predicted

The graph shows the comparison between actual and predicted values of taxes from 1950 to 2000. The y-axis represents the tax value, ranging from 0.02 to 0.16. The x-axis represents the years from 1950 to 2000. The graph includes three lines: the actual values, smoothed values, and filtered values. The smoothed and filtered values are indicated by dashed lines.
MONETARY REGIME PROBABILITIES

Monetary Regime Probabilities

Active, High $\sigma$

Active, Low $\sigma$

Passive, High $\sigma$

Passive, Low $\sigma$
Fiscal Regime Probabilities

Fiscal Regime Probabilities

Passive

Active

0 0.5 1
0 0.5 1
0 0.5 1
0 0.5 1
0 0.5 1
0 0.5 1
JOINT POLICY REGIME PROBABILITIES
A Model with Nominal Rigidities

- Conventional: monopolistic competition, Calvo pricing, elastic labor, lump-sum taxes, nominal debt

- Households

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} + \delta \frac{(M_{t+i}/P_{t+i})^{1-\kappa}}{1-\kappa} \right]
\]

\[
C_t = \left[ \int_0^1 c_{j,t}^{\theta-1} \frac{dj}{\theta} \right]^{\frac{\theta}{\theta-1}}, \theta > 1
\]

\[
C_t + \frac{M_t}{P_t} + E_t \left( Q_{t,t+1} \frac{B_t}{P_t} \right) + \tau_t \leq \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \Pi_t
\]

\[
E_t[Q_{t,t+1}]^{-1} = 1 + r_t
\]
A Model with Nominal Rigidities

- Firms

\[ E_t \sum_{i=0}^{\infty} \varphi^i q_{t+i} \left[ \left( \frac{p_t^*}{P_{t+i}} \right)^{1-\theta} - \Psi_{t+i} \left( \frac{p_t^*}{P_{t+i}} \right)^{-\theta} \right] Y_{t+i} \left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{K_{1t}}{K_{2t}} \]

\[ K_{1t} = (Y_t - G_t)^{-\sigma} \Psi_t Y_t + \varphi \beta E_t K_{1t+1} \left( \frac{P_{t+1}}{P_t} \right)^{\theta} \]

\[ K_{2t} = (Y_t - G_t)^{-\sigma} Y_t + \varphi \beta E_t K_{2t+1} \left( \frac{P_{t+1}}{P_t} \right)^{\theta-1} \]

\[ \pi_t^{\theta-1} = \frac{1}{\varphi} - \frac{1-\varphi}{\varphi} \left( \mu \frac{K_{1t}}{K_{2t}} \right)^{1-\theta} \]

- Relative price dispersion

\[ \Delta_t = (1 - \varphi) \left( \frac{p_t^*}{P_t} \right)^{-\theta} + \varphi \pi_t^{\theta} \Delta_{t-1} \]
A Model with Nominal Rigidities

- Policy follows estimated rules and satisfies

\[ G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t} + E_t \left( Q_{t,t+1} \frac{B_t}{P_t} \right) - \frac{B_{t-1}}{P_t} \]

- Two information assumptions:
  - standard: \( \Omega_t = \{ \varepsilon_{t-j}^r, \varepsilon_{t-j}^\tau, S_{t-j}^M, S_{t-j}^F, j \geq 0 \} \)
  - foreknowledge: \( \Omega_t^* = \Omega_t \cup \{ \varepsilon_{t+1}^r \} \)

- Focus on stationary equilibria
  - \( b/y \to \infty \) feasible with lump-sum taxes
  - U.S. \( b/y \) appears stationary

- Use monotone map method to solve non-linear model
  - finds functions mapping state to decisions
  - state: \( \Theta_t = \{ b_{t-1}, w_{t-1}, \Delta_{t-1}, \varepsilon_t^r, \varepsilon_t^\tau, S_t \} \)
The Fiscal Theory Mechanism

- The ubiquitous equilibrium condition
  \[ \frac{M_{t-1} + B_{t-1}}{P_t} = \sum_{T=t}^{\infty} E_t \left[ q_{t,T} \left( \tau_T - G_T + \frac{r_T}{1+r_T} \frac{M_T}{P_T} \right) \right] \]

- Three sources of financing: net-of-interest surpluses; seigniorage; revaluations induced by jumps in \( P_t \)

- Cut \( \tau_t \) with exogenous \( \tau - G \) and pegged \( r \)
  - at initial prices, feel wealthier
  - increase demand for current goods
  - raises output relative to potential
  - money stock expands passively
  - must also raise inflation & lower real rates

- With positive probability of active FP, the mechanism is always operating
Characteristics of Equilibrium

- Numerical analysis of uniqueness and stationarity
- Numerical checks
  - randomly perturb decision rules at points in state space: converge back?
  - how monotone map behaves when properties known
    - indeterminacy (non-convergence)
    - non-existence (converges but solutions explode)
- zero expected present value of debt?
- histograms
Quantifying the Fiscal Theory

- Three regimes are stationary
  - AM/PF, PM/PF, PM/AF
  - AM/AF exhibits slowly growing debt
- A surprise tax cut of 2% of GDP, conditional on each stationary regime
  1. condition on remaining in prevailing regime
  2. average across future regimes
- Compute tax multipliers
  - condition on initial regime
NON-LINEAR IMPULSE RESPONSES

- Draw from regime after initial shock
**Tax Multipliers**

- Defined as

\[
PV_n(\Delta y) / \Delta \tau_0 = \frac{1}{\Delta \tau_0} \sum_{s=0}^{n} q_{0,s} (y_s - \bar{y})
\]

\(n = 5, 10, 20, \infty\)

- Size depends on conditioning regime
  - always non-trivial
  - potentially large (\(> 1\))

- Similar impacts from unanticipated and anticipated changes

- With draws from future regimes
  - size depends on initial regime
  - range can be very wide
### Output Multipliers

| Init Regime | 5 quarters $\frac{PV(\Delta y)}{\Delta \tau}$ after 10 quarters $\frac{PV(\Delta y)}{\Delta \tau}$ after 25 quarters $\frac{PV(\Delta y)}{\Delta \tau}$ |
|-------------|------------------------------|------------------------------|------------------------------|
| AM/PF       | $[-.126, -.400]$             | $[-.213, -.754]$             | $[-.430, -.922]$             |
| PM/PF       | $[-.215, -.401]$             | $[-.271, -.623]$             | $[-.414, -.764]$             |
| PM/AF       | $[-.365, -.568]$             | $[-.537, -.928]$             | $[-.993, -1.363]$            |

**Table 3:** 80th percentile bands based on 10,000 draws
**Price Level Effects**

<table>
<thead>
<tr>
<th>Regime</th>
<th>5 quarters</th>
<th>10 quarters</th>
<th>25 quarters</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AM/\text{PF}$</td>
<td>0.324</td>
<td>0.641</td>
<td>1.513</td>
<td>6.704</td>
</tr>
<tr>
<td>$PM/\text{PF}$</td>
<td>0.770</td>
<td>1.077</td>
<td>1.232</td>
<td>1.237</td>
</tr>
<tr>
<td>$PM/\text{AF}$</td>
<td>0.949</td>
<td>1.369</td>
<td>1.620</td>
<td>1.633</td>
</tr>
</tbody>
</table>

**Table 4:** Cumulative effect on price level of an *i.i.d.* unanticipated tax cut of 2 percent of output, conditional on regime.
Fiscal Theory Robust

- Percentage of time in AM/PF regime
**Some Empirical Implications**

- Observed time series produced by switching DSGE
- Correctly identified VAR, but fixed regime
- Policy rules and pattern matrix:

\[
\begin{align*}
    r_t &= \alpha_0 + \alpha_{\pi}\pi_t + \alpha_{x}x_t + \varepsilon^r_t \\
    \tau_t &= \gamma_0 + \gamma_{x}x_t + \gamma_{b}b_{t-1} + \varepsilon^\tau_t
\end{align*}
\]

<p>| | | | | | |</p>
<table>
<thead>
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<tbody>
<tr>
<td></td>
<td>$x$</td>
<td>$\pi$</td>
<td>$b$</td>
<td>$MP$</td>
<td>$FP$</td>
</tr>
<tr>
<td>$x$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\otimes$</td>
<td>$\otimes$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
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<tr>
<td>$b$</td>
<td>$\times$</td>
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</tr>
<tr>
<td>$r$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

$\times$: freely estimated; $\otimes$: imposed
Some Empirical Implications

- Two assumptions about econometrician’s information
  1. full sample from single regime (draws from shocks & regime)
  2. extra-sample information to identify regime (draws only from shocks)
- Econometrician interprets results with fixed-regime DSGE
- Accurate quantitative estimates $\hat{\alpha}_\pi, \hat{\gamma}_b$

<table>
<thead>
<tr>
<th>All Regimes</th>
<th>AM/PF</th>
<th>PM/PF</th>
<th>PM/AF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\alpha}_\pi$</td>
<td>0.723</td>
<td>1.308</td>
<td>0.595</td>
</tr>
<tr>
<td>$\hat{\gamma}_b$</td>
<td>0.002</td>
<td>0.016</td>
<td>0.018</td>
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</table>

- Inaccurate qualitative inferences
Some Empirical Implications

“Fixed”: All Regimes parameters in fixed-regime DSGE
Some Empirical Implications

- “All regimes” implies PM/AF: fiscal theory equilibrium
  - correct inference about policy impacts
- Conditioning on regime gives incorrect inferences
  - AM/PF: Taylor principle & Ricardian
  - PM/PF: Indeterminacy & sunspots
- Most accuracy from full sample and averaging across regimes
  - quantitative predictions close
  - qualitative inferences correct
Wrap Up

- Fiscal theory can break down Ricardian equivalence
  - may be quantitatively important in U.S.
  - likely still more important in other countries
- If fiscal theory important, need to modify models
- Misleading to study MP (or FP) in isolation
  - models must be consistent with evidence on both MP & FP
- Need a serious integration of MP & FP
  - tax distortions
  - other sources of non-neutrality
  - GBC met non-trivially
Wrap Up

- Empirical complications
  - identification: disentangling monetary and fiscal impacts
  - unobserved fiscal state: foreknowledge of fiscal policy
- Understanding source of regime change
  - optimal policy response?
- Holy Grail
  - joint estimation of policy and private parameters in DSGE with switching
  - some work with just MP switching (Zha et al.) and with everything switching (Svensson-Williams)
  - no work with MP & FP switching