MAGYAR NEMZETI BANK MINI-COURSE

LECTURE 5. LUCAS CRITIQUE & MODEST POLICY INTERVENTIONS

Eric M. Leeper

Indiana University

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THE MESSAGES

• Draws on Leeper-Zha (*JME*, 2003)
• There are two typical reactions to empirical work on policy
  • acknowledge the Lucas critique, assert it doesn’t apply to what you’re doing, and move on
  • say “Lucas critique!” claim that the empirical work is invalid, and chant the DSGE mantra
• These responses are neither constructive nor thoughtful
• Both responses ignore a key point by Hurwicz
  • a model is *structural*—meaning invariant—only with respect to *some* class of interventions
  • so “structural” is a concept that is *relative* to some set of questions
• Taking Hurwicz seriously leads to desire to assess the quantitative importance of the LC
FRAMEWORK FOR POLICY ANALYSIS

- Compute and evaluate linear projections conditional on hypothetical paths of monetary policy
- Framework includes:
  - theory that reports when linear projections reliable even though policy switches regime
  - empirical model of U.S. monetary policy, used to probe range of interventions that do not generate large expectations-formation effects, which Lucas (1976) emphasizes
- True economy: policy regime a Markov chain; regime hidden state variable
- Private agents Bayesian updaters—infer regime from policy history
- Both anticipated and unanticipated money growth affect output
Framework for Policy Analysis

- True economy generates 3 objects of interest:
  1. nonlinear dynamics with agents updating beliefs about regime
  2. linear dynamics conditional on given regime
  3. large-sample linear dynamics that average across regimes

- Policy advisor: positive policy evaluation to inform policymakers
  - does not have access to (1)
  - (3) not interesting linear combination systematically wrong
  - armed with misspecified linear model like (2)
  - want to know the class of interventions for which linear projections are good approximations to the truth
Framework for Policy Analysis

• Advisor estimates linear model over single regime
  • conditioning on regime, report projections conditional on hypothetical policies PLUS
    • class of interventions that make current regime tenuous ⇒ linear projections unreliable
    • class of interventions consistent with current regime ⇒ linear projections good approximations

• Theory decomposes total impact of policy intervention into:
  • direct effects: usual impacts when regime fixed
    • include intra-regime shifts in expectations–do not shift decision rules
  • expectations-formation effects: arise from agents updating beliefs about regime
    • include inter-regime shifts in expectations–shift expectations-formation rules and decision rules
    • we associate EFE with behavior Lucas emphasizes
Framework for Policy Analysis

- Consider interventions that vary in magnitude and dynamic pattern
  - compute a statistic indicating if DE are improbably large relative to history
  - intervention is modest when statistic close to its mean
- We find:
  - modest policy interventions may have big DE’s without big EFE’s
  - linear model more likely to break down after small, persistent interventions than after large, fleeting ones
  - interventions that describe routine Fed choices are unlikely to change beliefs about prevailing regime
  - modest interventions matter: shift probability distributions of variables
  - modest interventions capture Fed’s appraisal/reappraisal process
CONTACTS WITH LITERATURE

• Most analyses mimic Lucas’s experiment of once-for-all policy choice
• Logical problems with once-for-all [Cooley, LeRoy, Raymon; Sargent; Sims]
  • regime change as a surprise that will never again occur is inconsistent with actual behavior—government takes actions agents thought were impossible
  • CLR: “...any entity which changes over time in a way that is not completely predictable should be modeled as a sequence of random variables.”
• Place probability distribution over all possible rules and define interventions as realizations of policy variables
  • decision rules incorporate belief that it’s always possible for policy to return to its past ways [Sargent’s Conquest]
• Sims interprets LC as pointing to a source of nonlinearity
• None of this denies potential importance of LC
  • framework isolates & quantifies beh. Lucas emphasizes
THEORETICAL FRAMEWORK

• Extend’s Cochrane’s use of Rotemberg’s costly price adjustment
  • $\alpha \in [0, 1]$ cost of adjusting prices
• Monopolistically competitive firm chooses $\{p_t\}$ to max profits cond’l on information at $t - 1$

$$-.5E \sum_{t} \beta^t [(1 - \alpha)(1 - \alpha \beta)(p_t - m_t)^2 + \alpha(p_t - p_{t-1})^2].$$

$p_t = m_t$ is the eqm when $\alpha = 0$

• Solve first-order condition for the price level to yield

$$p_t = \alpha p_{t-1} + (1 - \alpha)(1 - \alpha \beta)E_{t-1} \sum_{j=0}^{\infty} (\alpha \beta)^j m_{t+j}$$

where $m$ is nominal money stock; all variables in logs

• Add simple aggregate demand: $m_t - p_t = y_t$
Theoretical Framework

• Equilibrium output

\[ y_t = m_t - \frac{1 - \alpha}{1 - \alpha L} E_{t-1} \frac{1 - \alpha \beta}{1 - \alpha \beta L^{-1}} m_t \]

• Note that
  • \( \alpha \to 0 : y_t = m_t - E_{t-1} m_t \)
    Lucas’s unanticipated money model
  • \( \alpha \to 1 : y_t = m_t - p \)
    anticipated and unanticipated money matter
**Policy Specification**

- Monetary policy: $g_t$ is money growth between $t - 1$ and $t$

  $$m_t = g_t + m_{t-1}$$

  given $m_0 > 0$

- Letting $R_t$ be regime at $t$, the policy rule is

  $$g_t = \mu(R_t) + \rho(R_t)g_{t-1} + \sigma(R_t)\varepsilon_{Pt}, \quad \varepsilon_{Pt} \sim N(0, 1), \quad g_0 > 0$$

- Label the two policy regime $R^1$ and $R^2$

- Regime switches obey a Markov chain with transition probabilities

  $$P = \begin{bmatrix}
P[R_t = R^1 | R_{t-1} = R^1] & P[R_t = R^1 | R_{t-1} = R^2] \\
P[R_t = R^2 | R_{t-1} = R^1] & P[R_t = R^2 | R_{t-1} = R^2]
\end{bmatrix} = \begin{bmatrix}
p_{11} & 1 - p_{22} \\
1 - p_{11} & p_{22}
\end{bmatrix}$$

  and associated policy parameters

  $$(\mu(R_t), \rho(R_t), \sigma(R_t)) = \begin{cases}
  (\mu_1, \rho_1, \sigma_1^2) & \text{if } R_t = R^1 \\
  (\mu_2, \rho_2, \sigma_2^2) & \text{if } R_t = R^2
\end{cases}$$
The *policy process* is defined by above equations and values for the vector of policy parameters
\[ \Pi \equiv (\mu_1, \mu_2, \rho_1, \rho_2, \sigma_1^2, \sigma_2^2, p_{11}, p_{22}) \]

A *realization of policy* at \( t \) is the pair \((g_t, R_t)\)

Let \( \Omega_t = \{p(R_0), m_0, g_0, g_1, \ldots, g_t\} \), where \( p(R_0) \) is agents’ prior belief about regime at the initial date \( 0 \)

Agents’ decisions at \( t \) are based on information contained in \( \Omega_{t-1} \), along with \( \Pi \) and their beliefs about regime,
\[ P (R_{t-1} = R^s \mid \Omega_{t-1}) \text{, for } s = 1, 2 \]

We assume agents observe the history of money growth realizations but none of the realizations of regime
DIRECT & EXPECTATIONS FORMATION EFFECTS

- Fixed regime $\Rightarrow$ constant-coeff VAR rep
- Forecast conditional on Regime 1

\[ x_{T+K} = \sum_{s=0}^{K-1} C_s \varepsilon_{T+K-s} + E \left( x_{T+K} \mid \Omega_T, R_{t+k} = R^1, k = 1, 2, \ldots, K \right) \]

where $x_t = (p_t, y_t, m_t)'$ is a vector of variables from the model, $C_s$ is the impulse response matrix at horizon $s$, and $E \left( x_{T+K} \mid \Omega_T, R_{t+k} = R^1, k = 1, 2, \ldots, K \right)$ is the projection conditional on information in $\Omega_T$ and on policy remaining in Regime 1 over the projection period

- Intervention at $T$: $I_T = \{ \tilde{\varepsilon}_{PT+1}, \ldots \tilde{\varepsilon}_{PT+K} \}$
DEs & EFES

• Now can define

\[
\text{Direct Effects } \equiv \eta_{PT+K} = \sum_{s=0}^{K-1} C_s \tilde{\varepsilon}_{PT+K-s}
\]

\[
= E (x_{T+K} | \Omega_T^I(k), k = 1, 2, \ldots, K; R_{t+k} = R^1, k = 1, 2, \ldots, K)
\]

\[
- E (x_{T+K} | \Omega_T, R_{t+k} = R^1, k = 1, 2, \ldots, K)
\]

\(\eta\) expresses direct effects as a percentage difference from a baseline forecast of no intervention

• Direct effects arise when regime is fixed and, therefore, the model is linear

• In the linear case, direct effects are impulse responses following the contemplated intervention
**DEs & EFES**

- Intervention may trigger changes in agents’ beliefs about policy regime
- Changing beliefs about regime affect agents’ expectations of future policy and, therefore, their optimal choices
- Total effects relative to the no-intervention projection in the linear model are:

\[
\text{Total Effects } \equiv E \left( x_{T+K} \mid \Omega_T^I(k), k = 1, 2, \ldots, K \right) \\
- E \left( x_{T+K} \mid \Omega_T, R_{t+k} = R^1, k = 1, 2, \ldots, K \right)
\]

where the same intervention is conditioned on in DE & TE

- Because regime can shift, the total effects of an intervention depend on agents’ beliefs about regime at the time of the intervention

Expectations-Formation Effects \( \equiv \) Total Effects – Direct Effects
DEs & EFES

- Expectations-formation effects arise from the changes in behavior that lie at the heart of Lucas’s critique
- Natural way to judge whether the Lucas critique is important is to check if expectations-formation effects are small
- If expectations-formation effects are small, then forecasts from a model that assumes policy regime is fixed will be reasonably accurate
- If, in contrast, expectations-formation effects are large relative to direct effects, then the fixed-regime model’s predictions will be systematically wrong because the model does not capture expectations-formation effects
- In this case, the linear approximation is likely to breakdown as the nonlinearity triggered by expectations-formation effects is relatively important
- This is the situation on which Lucas focuses
Learning About Regime

- Bayesian updating about hidden regime
- prediction step:

\[
P (R_{t+h} \mid \Omega_{t+h-1}) = \sum_{R_{t+h-1}=R^1,R^2} \{ P (R_{t+h} \mid R_{t+h-1}) \ P (R_{t+h-1} \mid \Omega_{t+h-1}) \}
\]

- updating step:

\[
P (R_{t+h} \mid \Omega_{t+h}) = \frac{\varphi \left( g_{t+h} - \mu(R_{t+h}) - \rho(R_{t+h})g_{t+h-1}; \sigma^2(R_{t+h}) \right) \ P (R_{t+h} \mid \Omega_{t+h-1})}{\sum_{R_{t+h}=R^1,R^2} \left\{ \varphi \left( g_{t+h} - \mu(R_{t+h}) - \rho(R_{t+h})g_{t+h-1}; \sigma^2(R_{t+h}) \right) \ P (R_{t+h} \mid \Omega_{t+h-1}) \right\}}
\]

where \( \varphi(x; y) \) is the standard normal pdf
Simulating the Model

- Have defined a modest policy intervention in terms of the economic behavior that Lucas emphasizes
- By separating DEs & EFEs of an intervention, the theory implies a natural measure of whether a particular intervention is modest
- The theory offers a laboratory for finding examples of interventions where the Lucas critique bites
- Inferences about whether the LC bites for an intervention depend on parameters
- Focus on two different sets of parameters
  1. policy regimes are far apart so shifts in beliefs about regime can generate quantitatively important EFEs under certain conditions
  2. loosely calibrated to U.S. monetary data, so regimes much closer and EFEs tend to be small for many hypothetical interventions
Simulating the Model

- Parameters calibrated to match a monthly model
  - $\beta \Rightarrow 4\%$ real rate
  - $\alpha = .9 \Rightarrow$ costly price adjustment
  - $p_{11} \Rightarrow 30$ years (low $g$)
  - $p_{22} \Rightarrow 10$ years (high $g$)

- Two processes for $g$
  - extreme differences
  - U.S. data
A MODESTY METRIC

• For a given intervention, the distribution of direct effects may be obtained from the sequence of forecast errors computed in DE
  • \( \eta_{PT+K} \sim N(0, \sum_{s=0}^{K-1} C_s^2) \)
  • scale the statistic by the standard error of the direct effect on each variable, denoted by \( \eta^*_ {PT+K} \)
  • scale total effects similarly

Definition. An intervention is *modest* if its direct effects are small. More precisely, an intervention is modest over a specified forecast horizon, \( K \), and for variable \( i \), if

\[
| e_i \eta^*_ {PT+K} | < 2
\]

where \( e_i \) is a row vector of zeros with unity in the \( i^{th} \) column

• \( I_T \) is modest if its effects are “small” relative to typical random variation in MP (i.e., DE’s)
A Modesty Metric

- $\eta^*$ reports how unusual a conditional forecast is relative to the typical size of the direct effects, as measured in units of standard deviations of direct effects.
- With $\eta^*$ a standard normal random variable, the interval $[-2, 2]$ defines a 95 percent confidence interval.
- Large values of the statistic suggest the forecasted paths are unlikely to be due to direct effects alone, so EFEs must be important.
- When an intervention violates the MPI definition, we infer that the behavior underlying the Lucas critique is likely to be quantitatively important, making a linear approximation poor.
Regimes with Extreme Differences

- Regimes have money growth rates of 3.04% and 13.08%
- Special case of a one-period intervention:
  \[ I_T = \{1, 0, \ldots, 0\} \]
- Conventional impulse response function
  - conditions on being in and remaining in Regime 1
  - DE’s of alternative interventions are functions of this IRF
- Two kinds of interventions of same cumulative size, both 48 months
  - Extreme A: small and persistent—\[ I_T = \{2/3, 2/3, \ldots, 2/3\} \]
    std. devs.
  - Extreme B: large and fleeting—\[ I_T = \{8, 8, 8, 8, 0, \ldots, 0\} \]
    std. devs.
Regimes with Extreme Differences

- Money growth processes

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Parameters for Money Growth Processes
• Small and persistent (Figure)
  • beliefs shift away from Regime 1 ⇒ EFE’s grow
  • on $p$: DE and EFE reinforcing
  • on $y$: DE and EFE opposing
  • statistic ⇒ intervention immodest

• Large and fleeting (Figure)
  • beliefs shift quickly but briefly
  • DE’s on $y$ close to fixed-regime impulse responses
  • EFE’s small on $p$ and $y$
  • statistic ⇒ intervention immodest
  • horizon matters: linear model reliable at longer horizons
REGIMES WITH EXTREME DIFFERENCES

**Specification A**
- Money stock
- Prob[Regime 1] (right scale)

**Specification B**
- Money stock
- Prob[Regime 1] (right scale)

**Graphs**
- Direct Expectations—formation
- Money stock
- Prob[Regime 1] (right scale)

**Axes**
- Months
- Percent
- Price Level (Percent)
- Output (Percent)
Direct Effects ($\eta^*_{PT+K}$) and Expectations-Formation Effects scaled by standard errors of direct effects based on 5000 draws.

Interventions:
A: $\tilde{\varepsilon}_P = \frac{2}{3}$ in each of 48 months
B: $\tilde{\varepsilon}_P = 8.0$ for first 4 months, $\tilde{\varepsilon}_P = 0$ for next 44 months
C: $\tilde{\varepsilon}_P = \frac{1}{3}$ in each of 48 months
D: $\tilde{\varepsilon}_P = \frac{2}{3}$ in each of 48 months, but $p_{22} = .9167$ (1-year duration of Regime 2)
In Specifications A-D, $P\left(R_T = R^1\right) = .98.$
E: $\tilde{\varepsilon}_P = 0.2$ in each of 48 months, but $P\left(R_T = R^1\right) = .02.$

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**Regimes with Less Extreme Differences**

- Money growth processes

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Parameters for Money Growth Processes
Regimes with Less Extreme Differences

- Loosely calibrated to U.S. money growth: money growth rates of 6.4% and 9.4%
- Small and persistent
  - substantial DE’s: $p$ rises 19% and $y$ rises 4%
  - beliefs don’t move away from Regime 1
    - conditional likelihood more dispersed under Regime 2 ($\sigma_2 > \sigma_1$), so intervention must be larger to make Regime 2 more likely
    - money growth less persistent in Regime 2 ($\rho_2 < \rho_1$), so expect more rapid mean reversion in Regime 2 than a persistent intervention implies
  - Regime 2 less likely than Regime 1 given the intervention
- small EFE’s: statistic $\Rightarrow$ intervention immodest—reject too often
Regimes with Less Extreme Differences

- Large and fleeting
  - beliefs shift quickly but briefly
  - DE’s large: $y$ rises 10% in short run
  - EFE’s tiny because two regimes are close
  - statistic $\Rightarrow$ intervention **modest**
Direct Effects ($\eta^*_{PT+K}$) and Expectations-Formation Effects scaled by standard errors of direct effects based on 5000 draws.

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E: $\tilde{\varepsilon}_P = 0.2$ in each of 48 months, but $P (R_T = R^1) = .02$.

Impacts of Policy Interventions at 48-Month Horizon
Practical Implications

- Examine U.S. monetary policy in identified VAR
- Many economists reject VARs—identified or otherwise—as incapable of doing policy analysis
- This is one reason that DSGE modeling is so popular in central banks
- But recall Hurwicz: a model is “structural” only with respect to some class of interventions
- DSGE models are not structural with respect to arbitrary interventions
- And identified VARs may be structural with respect to some useful interventions
- This becomes a quantitative question and the Lucas critique tends to be perceived as theorem that applies globally
**Practical Implications**

- Much routine MP amounts to implementing the “existing regime”
  - in DSGE terms...applying the prevailing MP rule
- *By definition*, regime change must be relatively rare
  - otherwise, MP isn’t really following a rule
- The compelling policy question: How much structure is enough to do policy analysis?
  - Answer: It depends on the analysis being conducted
- Zha and I argue that most routine FOMC questions involve conditioning on *modest* interventions
  - EFE’s are small
  - even though DE’s are large
  - particularly true of the appraisal/reappraisal process that is central to routine MP analysis
PRACTICAL IMPLICATIONS

- Checking the modesty of the interventions being conducted in VARs ought to become standard practice
  - for example, Hamilton-Herrera have done this to examine Bernanke-Gertler-Watson’s study on oil prices & MP
  - Sveriges Riksbank does this regularly to examine the interventions they examine
- An appreciation of the class of interventions for which DSGE models are structural would be helpful
  - would combat the tendency to believe that maximizing utility ensures immunity from the Lucas critique regardless of the counterfactuals being conducted in the DSGE model
**Practical Implications**

- Example: no one believes Calvo pricing, Rotemberg pricing, habit formation, various indexation schemes, and a host of other bells & whistles are “structural”
  - we calibrate parameters of those features to historical moments
  - then we compute optimal MP, holding those parameters fixed
  - logic of this exercise: one of two possible inferences
    1. Historically MP was nearly optimal, so no big welfare gains are available
    2. If there are big welfare gains, then the move to optimal policy will create incentives for private sector to update its behavior ⇒ these feature are not structural

- Either way, we ought to think harder about which features of our models *really are* structural with respect to the interventions we contemplate