This exercise asks you to solve some simple models of Markov switching. These are examples that I believe you can solve “by hand,” though it is possible that you may feel you need to use a computer. In any case, they can be solved exactly, without approximation.

1. Consider the equation

\[ \alpha E_t x_{t+1} = x_t - \beta z(s_t) \]  

where \( 0 < \alpha < 1, \beta > 0 \), and \( z(s_t) \) is a random variable whose realizations depend on the state, \( s_t \), which follows a Markov chain,

\[ z(s_t) = \begin{cases} 
z_1 & \text{if } s_t = 1 \\
2 & \text{if } s_t = 2 
\end{cases} \]

where \( s_t \) is governed by the transition probabilities

\[ P = \begin{bmatrix} p_{11} & p_{12} \\
p_{21} & p_{22} \end{bmatrix} \]

with \( p_{ij} = P[s_{t} = j \mid s_{t-1} = i] \) and \( \sum_{j=1}^{2} p_{ij} = 1 \). Solve for \( x_t \) as a function of \( z_t, s_t \).

2. Consider the equation

\[ \alpha(s_t) E_t x_{t+1} = x_t - \beta z_t \]  

where \( 0 < \alpha(s_t) < 1 \), for \( s_t = 1, 2, \beta > 0 \), and \( \alpha(s_t) \) is given by

\[ \alpha(s_t) = \begin{cases} 
\alpha_1 & \text{if } s_t = 1 \\
\alpha_2 & \text{if } s_t = 2 
\end{cases} \]

\( z_t \) is an exogenous AR(1) process given by

\[ z_t = \rho z_{t-1} + \varepsilon_t, \]

\( \varepsilon \sim i.i.d. \). As in question (1), the state \( s_t \) is governed by a Markov chain with transition matrix \( P \). Solve for \( x_t \) as a function of \( z_t, s_t \).