Norges Bank Mini-Course: Optimal Monetary Policy

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Overview

- Conventional optimal monetary policy, as in Woodford or Galí

- Sources of suboptimality in new Keynesian model
  1. monopolistic competition
     - firms produce at below the efficient level
     - firms set prices of differentiated goods, rather than taking prices as given
  2. nominal rigidities à la Calvo pricing
     - infrequent price adjustment by firms introducing price dispersion
     - relative prices will move in ways unrelated to changes in preferences or technology

- Typically...
  - consider efficient steady state (labor subsidy)
  - MP left to deal with distortions from nominal rigidities
New Keynesian Model

- Continuum of differentiated goods indexed by $i \in [0, 1]$
- Representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where $C_t = \left( \int_0^1 C_t(i)^{1 - \frac{1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}$

subject to

$$\int_0^1 P_t(i) C_t(i) \, di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

and solvency condition

$$\lim_{T \to \infty} E_t B_T \geq 0, \text{ for all } t$$
New Keynesian Model

- HHs must decide how to allocate consumption expenditures among goods
  - maximize $C_t$ for given expenditure level $\int_0^1 P_t(i)C_t(i)di$
  - yields set of demand functions

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t, \ i \in [0, 1] \quad (1)$$

where aggregate price level is

$$P_t \equiv \left[ \int_0^1 P_t(i)^{1-\epsilon} di \right]^\frac{1}{1-\epsilon}$$

- Conditional on such optimal behavior, $P_tC_t = \int_0^1 P_t(i)C_t(i)$ and budget constraint is

$$P_tC_t + Q_tB_t \leq B_{t-1} + W_tN_t + T_t$$
New Keynesian Model

- **HHs first-order conditions are**

\[-\frac{U_{Nt}}{U_{Ct}} = \frac{W_t}{P_t}\]

\[Q_t = \beta E_t \left[ \frac{U_{Ct+1}}{U_{Ct}} \frac{P_t}{P_{t+1}} \right]\]

- **With preferences**

\[U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}\]

log-linearized FOCs are

\[w_t - p_t = \sigma c_t + \varphi n_t \tag{2}\]

\[c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( i_t - E_t \pi_{t+1} - \rho \right) \tag{3}\]

where \(i_t = -\log Q_t\), \(\rho = -\log \beta\)
Each firm produces differentiated good but with identical technology

\[ Y_t(i) = A_t N_t(i)^{1-\alpha} \]  

(4)

technology shock, \( A \), common across firms

All firms face identical isoelastic demand schedule, (1), & take \( P_t, C_t \) as given

- firm may reset its price only with probability \( 1 - \theta \) in any period
- measure \( 1 - \theta \) producers adjust prices & \( \theta \) do not
- \( (1 - \theta)^{-1} \) is average duration of a price

Can show aggregate price dynamics obey

\[ \Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \]  

(5)

\( \Pi_t \equiv P_t/P_{t-1} \), \( P_t^* \) is new price set at \( t \) by firms that reoptimize
Aggregate price dynamics, (5), implies that in a steady state with zero inflation ($\Pi = 1$)

- $P_t^* = P_{t-1} = P_t$ for all $t$
- log-linearization of (5) yields

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}) \quad (6)$$
New Keynesian Model

- Reoptimizing firm at $t$ chooses $P_t^*$ to maximize current market value of profits generated while that price prevails.
  - Firm maximizes
    \[
    \sum_{k=0}^{\infty} \theta^k E_t \left( Q_{t,t+k} \left[ P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right] \right)
    \]
  subject to demand constraints
  \[
  Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k}, k = 0, 1, 2, ...
  \] (7)

where

- $Q_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)$: stochastic discount factor

- $\Psi_{t+k}(Y_{t+k|t})$: cost function

- $Y_{t+k|t}$: output at $t + k$ for firm that last reset prices at $t$
First-order condition for firm is

\[
\sum_{k=0}^{\infty} \theta^k E_t \left[ Q_{t,t+k} Y_{t+k|t} \left( P_t^* - M \psi_{t+k|t} \right) \right]
\]  

(8)

\[\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t}) \text{ is nominal marginal cost in } t + k \text{ for firm that last reset price in } t \text{ and } M \equiv \epsilon/(\epsilon - 1) \text{ is markup} \]

Note: when \(\theta = 0\) (no price rigidities), (8) yields

\[P_t^* = M \psi_{t|t} \]

\(M\) is the “desired” or “frictionless” markup

Rewrite (8): divide by \(P_{t-1}\) & let \(\Pi_{t,t+k} = P_{t+k}/P_t\)

\[
\sum_{k=0}^{\infty} \theta^k E_t \left[ Q_{t,t+k} Y_{t+k|t} \left( \frac{P_t^*}{P_{t-1}} - M M C_{t+k|t} \Pi_{t-1,t+k} \right) \right]
\]

(9)

\(M C_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}\) is real MC at \(t + k\) for firm that last set price at \(t\)
In a zero-inflation steady state

\[ \frac{P_t^*}{P_{t-1}} = 1, \Pi_{t-1,t+k} = 1, Y_{t+k|t} = Y, Q_{t,t+k} = \beta^k, MC = 1/M \]

Log-linearize (9) around \( \Pi = 1 \)

\[ p_t^* - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t [\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})] \quad (10) \]

\( \hat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc \) is log deviation of MC from steady state value \( mc = -\mu, \mu = \log M \)

rewrite (10) as

\[ p_t^* = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t [mc_{t+k|t} + p_{t+k}] \]

new price reflects markup plus present value of nominal marginal cost, using discount factor that reflects probability of resetting price
New Keynesian Model

- When $MC_{t+k|t} = MC_{t+k}$
  - write optimal price-setting rule in recursive form
    \[
    p_t^* = \beta \theta E_t p_{t+1}^* + (1 - \beta \theta) \hat{m}_c_t + (1 - \beta \theta) p_t
    \]
  - combine this with (6)
    \[
    \pi_t = \beta E_t \pi_{t+1} + \lambda \hat{m}_c_t, \quad \lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta}
    \]
- Implying equilibrium, $Y_t(i) = C_t(i)$ all $i, t$, $Y_t = C_t$
  - in equilibrium, consumption Euler equation (3) is
    \[
    y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho)
    \]
New Keynesian Model

- Can write marginal cost as

\[ mc_t = (w_t - p_t) - mpn_t \]
\[ = (w_t - p_t) - (a_t - \alpha n_t) - \log(1 - \alpha) \]
\[ = (w_t - p_t) - (1 - \alpha)^{-1}(a_t - \alpha y_t) - \log(1 - \alpha) \]

\[ mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t} \]
\[ = (w_{t+k} - p_{t+k}) - (1 - \alpha)^{-1}(a_{t+k} - \alpha y_{t+k|t}) - \log(1 - \alpha) \]

\[ mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k|t} - y_{t+k}) \]
\[ = mc_{t+k} - \frac{\alpha \epsilon}{1 - \alpha} (p^*_t - p_{t+k}) \]  

(12)
New Keynesian Model

- **Use (12) in (10)**

\[ p_t^* - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left[ \Theta \hat{m}_c_{t+k} + (p_{t+k} - p_{t-1}) \right] \]

\[ = (1 - \beta \theta) \Theta \sum_{k=0}^{\infty} (\beta \theta)^k E_t \hat{m}_c_{t+k} + \sum_{k=0}^{\infty} (\beta \theta)^k E_t \pi_{t+k} \]

\[ \Theta \equiv (1 - \alpha)/(1 - \alpha + \alpha \epsilon) \leq 1 \]

- **Write this recursively as**

\[ p_t^* - p_t = \beta \theta E_t [p_{t+1}^* - p_t] + (1 - \beta \theta) \Theta \hat{m}_c + \pi_t \]  \hspace{1cm} (13)

- **Combine (13) with (6)**

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda \hat{m}_c, \quad \lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \Theta \]  \hspace{1cm} (14)
Finally, can express $MC$ in terms of an output gap, as

$$\hat{m}c_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) (y_t - y^n_t) \quad (15)$$

$y^n_t$ is flexible-price level of output ($\theta = 0$)

- let $\tilde{y}_t \equiv y_t - y^n_t$
- use (15) in (14) to yield

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t, \quad \kappa \equiv \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \quad (16)$$

- Rewrite Euler equation in terms of output gap

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t) \quad (17)$$

- $r^n_t \equiv \rho + \sigma E_t \left[\Delta y^n_{t+1}\right]$ is “natural rate of interest”
Efficient Allocation

- Optimal allocation maximizes

\[ U(C_t, N_t) \]

where \( C_t = \left( \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \)

subject to \( C_t(i) = A_t N_t(i)^{1-\alpha} \), for all \( i \in [0, 1] \)

and \( N_t = \int_0^1 N_t(i) di \)

- Optimality conditions are

\[ C_t(i) = C_t, \text{ for all } i \quad (18) \]
\[ N_t(i) = N_t, \text{ for all } i \quad (19) \]
\[ -\frac{U_{N_t}}{U_{C_t}} = MPN_t = (1 - \alpha)A_t N_t^{-\alpha} \quad (20) \]
Efficient Allocation

- Each firm perceives demand for its good to be imperfectly elastic
  - firm has market power & prices $> \text{marginal cost}$
- Flexible prices: profit-maximizing price same across firms
  - with isoelastic demand (w/ price elasticity $\epsilon$), optimal price-setting rule is
    $$ P_t = \mathcal{M} \frac{W_t}{MPN_t}, \quad \mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1} > 1 $$
  - $W/MPN$ is marginal cost, so
    $$ - \frac{U_{Nt}}{U_{Nt}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t $$
  - optimality condition (20) is violated when there is non-trivial markup
- $MRS$ increasing in $N$ & $MPN$ non-increasing in $N$
  - markup distortion $\Rightarrow$ inefficiently low $N$ & output
Efficient Allocation

- Can eliminate this inefficiency with employment subsidy
  - let $\tau$ denote subsidization rate
  - subsidy financed with lump-sum taxes
  - under flexible prices

$$P_t = \mathcal{M} \frac{(1 - \tau) W_t}{MPN_t}$$

- so that

$$-\frac{U_{Nt}}{U_{Ct}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1 - \tau)}$$

- optimal allocation comes when
  $\mathcal{M}(1 - \tau) = 1 \Rightarrow \tau = 1/\epsilon$

- Often assume such a subsidy is in place
Calvo pricing introduces two sources of inefficiency

1. Prices not adjusted continuously $\Rightarrow$ economy’s markup with vary with shocks
   - markup will generally differ from the constant, frictionless markup $\mathcal{M}$
   - let $\mathcal{M}_t$ be economy’s average markup (ratio of average price to average $MC$)

\[
\mathcal{M}_t = \frac{P_t}{(1 - \tau)\left(\frac{W_t}{MPN_t}\right)} = \frac{\mathcal{M}P_t}{W_t/MPN_t}
\]

(assuming subsidy exactly offsets monopolistic competition distortion)

then

\[
-\frac{U_{Nt}}{U_{Ct}} = \frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t}
\]

violates condition (20) whenever $\mathcal{M} \neq \mathcal{M}_t$
Calvo pricing introduces two sources of inefficiency

2. Relative price distortions
   - lack of synchronization in price adjustments ⇒ relative prices move because of nominal rigidities
   - relative price movements not driven by shocks to preferences of technology
   - generally, \( P_t(i) \neq P_t(j) \) for any \((i, j)\) pair whose prices are not adjusted in same period
   - this price dispersion ⇒ \( C_t(i) \neq C_t(j) \) & \( N_t(i) \neq N_t(j) \)
   - violates optimality conditions (18) & (19)
Efficient Allocation

- Assume...
  - optimal subsidy in place
  - no inherited relative price distortions: \( P_{-1}(i) = P_{-1} \) all \( i \in [0, 1] \)
- Efficient allocation obtained by a policy that stabilizes MC at a level consistent with firms’ desired markup, given the prices in place
  - if policy expected to last forever, no firm has incentive to adjust price
  - \( P_t^* = P_{t-1} \), so \( P_t = P_{t-1} \) for \( t \geq 0 \)
  - aggregate price level fully stabilized & there are no relative price distortions
  - also, \( M_t = M \) all \( t \), so employment & output match their flexible-price counterparts
Efficient Allocation

- Optimal policy requires
  
  \[ \hat{y}_t = 0, \] close the output gap
  \[ \pi_t = 0, \] zero inflation
  \[ i_t = r^n_t, \] target natural rate

- Optimal policy does not stabilize output
  
  \[ y_t = y^n_t \] all \( t \), so output mimics an RBC world

- Price stability emerges as a feature of optimal policy
  
  - policymaker puts no weight on inflation \textit{per se}
  - stable prices are a way to replicate the efficient flexible-price allocation
  - with Calvo pricing, do this by making firms happy with their existing prices
  - constraints on price adjustment effectively non-binding
Extending the Analysis

> Note that the previous analysis made scant mention of fiscal policy
  > only as a lump-sum tax to finance the employment subsidy
  > no mention at all about what else fiscal policy does
> This is typical of the optimal monetary policy literature
> Now turn to Benigno-Woodford, “Optimal Inflation Targeting Under Alternative Fiscal Regimes”
  > uses same new Keynesian framework
  > adds distorting tax on labor
  > explicitly considers optimal MP under different fiscal behavior
The Model

- Representative HH is a yeoman farmer with preferences

\[ U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj \right] \quad (21) \]

with \( C_t \equiv \left[ \int_0^1 c_t(i) \frac{\theta}{\theta-1} di \right]^{\theta-1}_{\theta} \) and \( \theta > 1 \)

- \( H_t(j) \) is quantity of labor supplied of type \( j \)
- goods of type \( j \) are produced using labor of type \( j \)
- Specialize preferences to

\[ \tilde{u}(C_t; \xi_t) \equiv \frac{C_t^{1-\frac{1}{\tilde{\sigma}}} - \frac{1}{\tilde{\sigma}}}{1 + \frac{1}{\tilde{\sigma}}} \], \quad \tilde{v}(H_t; \xi_t) \equiv \frac{\lambda}{1 + \nu} H_t^{1+\nu} \bar{H}_t^{-\nu} \]

- \( \tilde{\sigma}, \nu > 0 \) & \( \{ \bar{C}_t, \bar{H}_t \} \) bounded exogenous processes
- notation: \( \xi_t \) is vector of all exogenous shocks
The Model

- Common technology for all goods
  \[ y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{\frac{1}{\phi}}, \quad \phi > 1 \]

- Aggregate resource constraint
  \[ Y_t = C_t + G_t \]

- Can invert production function and use ARC to write utility as function of expected production plan, \( \{y_t(i)\} \)
The Model

- Produce constrained by Calvo pricing
  - \( 0 \leq \alpha < 1 \) is fraction of prices that remain unchanged each period
  - suppliers choose new price \( p_t(i) \) to maximize

\[
E_t \left[ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi \left( p_t(i), p_j^j, P_T; Y_T, \tau_T, \xi_T \right) \right]
\]  

- \( Q_{t,T} \) is stochastic discount factor to discount random nominal income in \( T \) to determine nominal value of a claim to that income in \( t \)
- \( \alpha^{T-t} \) is probability that price chosen in \( t \) will continue to hold in \( T \)
- in equilibrium

\[
Q_{t,T} = \beta^{T-t} \frac{\tilde{u}_c(C_T; \xi_T) P_t}{\tilde{u}_c(C_t; \xi_t) P_T}
\]
The Model

- Profit function is

$$\Pi(p, p^j, P; Y, \tau, \xi) \equiv (1 - \tau)pY(p/P)^{-\theta}$$

$$- \mu_r^w \frac{\tilde{h} \left[ f^{-1} \left( \frac{Y(p/P)^{-\theta}}{A} \right); \xi \right]}{\tilde{u}(Y - G; \xi)} p \cdot f^{-1} \left( \frac{Y(p/P)^{-\theta}}{A} \right)$$

- this is after-tax profits of supplier with price $p$ in industry with common price $p^j$
- aggregate price level is $P$, aggregate demand is $Y$, sales revenues tax rate is $\tau$
- $\mu_r^w$ is exogenous markup factor for real wage for labor of type $j$ (produces cost-push shocks that affects pricing behavior, but not efficient allocation of resources)
The Model

- Central bank controls the risk-free short-term nominal interest rate, \( i_t \)

\[
1 + i_t = \left[ E_t Q_{t,t+1} \right]^{-1} \tag{24}
\]

- Govt issues only one-period riskless nominal bonds, \( B_t \)

- Govt budget constraint is

\[
B_t = (1 + i_{t-1})B_{t-1} - P_t s_t \tag{25}
\]

\[
s_t \equiv \tau_t Y_t - G_t - Z_t \tag{26}
\]

\( Z_t \) is lump-sum transfers

- In equilibrium, must satisfy solvency condition

\[
b_{t-1} \frac{P_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T} s_T \tag{27}
\]

\[
R_{t,T} \equiv Q_{t,T} P_T / P_t \quad \text{is discount factor for real income,}
\]

\[
b_{t-1} \equiv B_{t-1} / P_{t-1}
\]
An important contribution of Benigno-Woodford (2004) is to show how to setup the optimal policy problem as a linear-quadratic problem

- quadratic loss function & linear constraints

They derive the *micro-founded* loss function: maximizing utility is locally equivalent to minimizing

\[
E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} q_y \left( \hat{Y}_t - \hat{Y}_t^* \right)^2 + \frac{1}{2} q_\pi \pi_t^2 \right]
\]

(28)

- if steady-state distortions are not too extreme, \(q_y, q_\pi > 0\) & loss function is convex
- note that \(q_y, q_\pi\) are functions of “deep” parameters
- \(q_y, q_\pi\) and other coefficients defined in B-W (2004)
The LQ Problem

Constraints for LQ problem are

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (\hat{Y}_t + \psi \hat{\tau}_t + c'_\xi \xi_t), \quad \kappa, \psi > 0 \quad (29) \]

introduce the welfare-relevant output gap,

\[ y_t \equiv \hat{Y}_t - \hat{Y}_t^*, \text{ so (29) is} \]

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (y_t + \psi \hat{\tau}_t + u_t) \quad (30) \]

\( u \) a composite cost-push terms from shocks other than taxes

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa [y_t + \psi (\hat{\tau}_t - \hat{\tau}_t^*)] \quad (31) \]

\( u_t \equiv \hat{Y}_t^* + c'_\xi \text{ and } \hat{\tau}_t^* \equiv -\psi^{-1} u_t \)
The LQ Problem

- Log-linearize solvency condition (27)

\[
\hat{b}_{t-1} - \pi_t - \sigma^{-1} y_t = -f_t + (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} [b_y y_T + b_\tau (\hat{\tau}_T - \hat{\tau}_T^*)]
\]

(32)

where

\[
f_t \equiv \sigma^{-1} (g_t - \hat{Y}_t^*) - (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} [b_y \hat{Y}_T^* + b_\tau \hat{\tau}_T^* + b'_\xi \xi_T]
\]

- note that \(f_t\) depends only on current & expected exogenous disturbances
The LQ Problem

\[ \hat{b}_{t-1} - \pi_t - \sigma^{-1}y_t = -f_t + (1 - \beta)E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y y_T + b_\tau (\hat{\tau}_T - \hat{\tau}_T^*)] \] (32)

- Interpreting this solvency condition
  - \( \hat{\tau}_t - \hat{\tau}_t^* \): tax gap—departures of tax rate from level consistent with complete stabilization of inflation & output gap
    - if \( \hat{\tau}_t = \hat{\tau}_t^* \), then \( y_t = 0 \Rightarrow \pi_t = 0 \)
  - \( \hat{b}_{t-1} + f_t \): measures extent to which intertemporal solvency prevents complete achievement of stabilization goals in (28)
    - from (32), if \( \hat{\tau}_t = \hat{\tau}_t^* \) and \( y_t = 0 \Rightarrow \pi_t = 0 \), then \( \hat{b}_{t-1} + f_t = 0 \)
  - have used (23) to solve out stochastic discount factor
    - effect of MP will operate through \( R_{t,T} \) in (27), which is connected via \( 1 + i_t = [E_t Q_{t,T+1}]^{-1} \) to MP
The LQ Problem

\[ \hat{b}_{t-1} - \pi_t - \sigma^{-1} y_t = -f_t + (1 - \beta) E_t \sum_{T=t}^{\infty} \beta^{T-t} [b y_T + b_T (\hat{\tau}_T - \hat{\tau}_T^*)] \] (32)

- Interpreting this solvency condition
  - (32) treats transfers, \( Z_t \), as exogenous, but to study passive FP will need to separate them out of \( f_t \)
    - when FP passive, (32) is never binding & return to previous analysis
  - if \( \hat{b}_{t-1} + f_t \neq 0 \), then cannot achieve complete stabilization because solvency requirements prevent it
    - \( f_t \) depends on exogenous shocks, so whenever \( \hat{b}_{t-1} + f_t \neq 0 \), some adjustment in endogenous variables is required to ensure solvency
    - FP is passive when those adjustments occur through lump-sum taxes/transfers
    - FP is active when those adjustments occur through prices & allocations
Conventional Analysis

- Conventional optimal monetary policy ignores solvency constraint
  - maintains assumption of passive fiscal policy
  - all fiscal adjustments are lump sum
- Assume \( \{G_t, \tau_t\} \) exogenous; \( \{Z_t\} \) endogenous & passive
  - passive transfers \( \Rightarrow \) solvency constraint never binds
  - \( \tau_t \) exogenous \( \Rightarrow \) treat it as a cost-push shock in (31)
- Choose \( \{\pi_t, y_t\} \) to minimize (28) subject to
  \[
  \pi_t = \beta E_t \pi_{t+1} + \kappa(y_t + u_t) \tag{33}
  \]
- If \( u_t \equiv 0 \), optimal policy is \( \pi_t = y_t \equiv 0 \)
  - if distortions from market power & taxes are small and stable, then low, stable inflation is optimal regardless of how real activity is affected through shocks to \( Y^*_t \)
Conventional Analysis

- We focus on optimal policy with commitment
- When only constraint on policy is $E_t \pi_{t+1}$ in (31)
  - expected inflation only aspect of expectations that affects $\pi_t$ & $y_t$
  - argues for a commitment to particular inflation rate in future
  - commit to achieve a particular inflation rate in the future, which might be different from what would otherwise be chosen later to achieve stabilization goals in future (time consistency)
  - this is the underpinning for CB communication about outlook for inflation: aim to anchor inflation expectations
Conventional Analysis

► Policy problem yields Lagrangian

\[\mathcal{L}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \left[ \frac{1}{2} (q_y y_t^2 + q_\pi \pi_t^2) + \varphi_t (\pi_t - \beta \pi_{t+1} - \kappa (y_t + u_t)) \right]\]

► FOCs

\[q_y y_t - \kappa \varphi_t = 0\]
\[q_\pi \pi_t + \varphi_t - \varphi_{t-1} = 0\]

► These imply

\[\varphi_t = \frac{q_y}{\kappa} y_t\]  \hspace{1cm} (34)

\[q_\pi \pi_t = \varphi_{t-1} - \varphi_t = \frac{q_y}{\kappa} (y_{t-1} - y_t)\]  \hspace{1cm} (35)

► (34) & (35) hold for \( t \geq 0 \)

► use (34), (35) & (33) to solve for \( \{\pi_t, y_t, \varphi_t\} \) given \( \{u_t\} \)
Conventional Analysis

- FOC (35) implies (where $p_t \equiv \log P_t$)

\[ p_t - p_{t-1} = -\frac{q_y}{\kappa q_\pi} (y_t - y_{t-1}) \]

or

\[ p_t + \frac{q_y}{\kappa q_\pi} y_t = p_{t-1} + \frac{q_y}{\kappa q_\pi} y_{t-1} \]

- optimal policy keeps $\tilde{p}_t$ constant

\[ \tilde{p}_t \equiv p_t + \frac{q_y}{\kappa q_\pi} y_t \tag{36} \]

- (36) depicts the optimal tradeoff between price increase & output decrease

- Write the targeting rule as

\[ \pi_t + \frac{q_y}{\kappa q_\pi} \Delta y_t = 0 \tag{37} \]

- This is conventional optimal MP with lump-sum passive FP
Conventional Analysis

- Expression (36) implies
  - policy should not be so tight as to make $\tilde{p}_t$ rise nor too loose to make $\tilde{p}_t$ fall
  - commitment to the rule in the future
    - implies invariance of expected long-run price level & output gap
    - determines optimal rate of return of $\pi$ & $y$ to those long-run levels
  - figure 1 depicts optimal responses to positive cost-push shock
  - gradual return to long-run levels
  - targeting rule (37) also says acceptable rate of inflation at any time varies, depending on rate of change of output gap
Conventional Analysis

Figure 1. Impulse Responses to a Transitory Cost-Push Shock under Discretionary Policy and an Optimal Commitment.

*Inflation*

*Output*

*Price level*

Source: Woodford (2003, chap. 7, fig. 7.3).
Note on Calibration

- B-W cite Woodford (2003) for calibration of parameters

\[ \beta = 0.99, \quad \kappa = 0.024, \quad \frac{q_y}{q_\pi} = 0.048 \]

- \( q_y, \frac{q_y}{q_\pi} \) come from calibrations of deep parameters
- in terms of the loss function, the calibration implies inflation stabilization get more than 20 times the weight as output-gap stabilization
- targeting rule becomes: \( \pi_t = -2\Delta y_t \)

- Interesting to ask
  - how do changes in model structure affect these relative weights?
  - to what extent are inflation-targeting results driven by the weights?
  - is there empirical support for these weights?
A note on obtaining Benigno-Woodford’s FOCs

Write solvency condition recursively

- at $t$ and $t+1$, this is

$$\hat{b}_{t-1} - \pi_t - \sigma^{-1}y_t = -f_t + (1 - \beta)E_t \sum_{T=t}^{\infty} \beta^{T-t} [b_T y_T + b_T (\hat{T} - \hat{T}^*)]$$

$$\hat{b}_t - \pi_{t+1} - \sigma^{-1}y_{t+1} = -f_{t+1} + (1 - \beta)E_{t+1} \sum_{T=t+1}^{\infty} \beta^{T-(t+1)} [b_T y_T + b_T (\hat{T} - \hat{T}^*)]$$

- combine to yield recursive form

$$b_{t-1} - \pi_t - \sigma^{-1} + f_t$$

$$= (1 - \beta) [b_T y_t + b_T (\hat{T} - \hat{T}^*)] + \beta [b_t - E_t \pi_{t+1} - \sigma^{-1}E_t y_{t+1} + E_t f_{t+1}]$$  \hspace{1cm} (38)

- use (38) as constraint on policy problem
Optimal Tax Smoothing: Flexible Prices

- Assume only distorting taxes available to ensure solvency
  - \( \{Z_t\} \) now an exogenous process
  - now intertemporal solvency condition binds on \( \pi \) & \( y \)
- Planner chooses \( \{\pi_t, y_t, \hat{\tau}_t\} \) to minimize (28) subject to (31) & (38)
- Flexible prices \( q_\pi = \kappa = 0 \), so minimize

\[
\frac{1}{2} q_y E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} y_t^2
\]

subject to (38) and

\[
y_t + \psi (\hat{\tau}_t - \hat{\tau}_t^*) = 0
\]

- Substitute (40) into (39), so objective is entirely tax smoothing

\[
E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\hat{\tau}_t - \hat{\tau}_t^*)^2
\]
Optimal Tax Smoothing: Flexible Prices

- Optimal solution makes $y_t \equiv 0 \Rightarrow \hat{\tau}_t = \hat{\tau}_t^*$
- Monetary policy adjusts $\pi_t$ to ensure solvency holds

$$\pi_t = \hat{b}_{t-1} + f_t$$

$$\Rightarrow \pi_t - E_{t-1} \pi_t = f_t - E_{t-1} f_t = 0$$

1. Tax rate should fluctuate only to extent that $\hat{\tau}_t^*$ fluctuates: a supply-side policy to offset inefficient supply shocks
2. Unexpected changes in fiscal stress, $f_t$, should be offset by inflation to ensure solvency
3. Yields volatile inflation that is sensitive to fiscal shocks
Optimal Tax Smoothing: Flexible Prices

- Nominal debt is playing a critical role
- If govt issues only riskless real debt, then variations in $f_t$ require changes in tax rate
  - suppose govt issues $b_{t-1} \equiv \hat{b}_{t-1} - \pi_t$
  - this is predetermined at beginning of $t$
  - rewrite solvency constraint in terms of $b_{t-1}$

\[
\mathcal{L}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} q_y y_t^2 + + \varphi_{1t} [y_t + \psi (\hat{\tau}_t - \hat{\tau}_t^*)] \\
+ \varphi_{2t} \left[ b_{t-1} - \sigma^{-1} y_t + f_t - (1 - \beta)[b_y y_t + b_{\tau} (\hat{\tau}_t - \hat{\tau}_t^*)] \right. \\
- \beta (b_t - \sigma^{-1} y_{t+1} + f_{t+1}) \left. \right\} + \sigma \varphi_{2,t_0-1} y_{t_0}
\]

- $\sigma \varphi_{2,t_0-1}$ is multiplier for constraint $y_{t_0} = \bar{y}_{t_0}$
- this delivers the timeless perspective solution of Woodford
Optimal Tax Smoothing: Flexible Prices

- FOCs for setup with real debt

\[
q_yy_t = -\varphi_{1t} + [(1 - \beta)b_y + \sigma^{-1}]\varphi_{2t} - \sigma^{-1}\varphi_{2,t-1}
\]

\[
\psi \varphi_{1t} = (1 - \beta)b_T \varphi_{2t}
\]

\[
\varphi_{2t} = E_t \varphi_{2,t+1}
\]

- these hold for \( t \geq t_0 \), given initial conditions, \( b_{t_0}, y_{t_0} \)
- solution is

\[
\varphi_{2t} = \frac{m_b}{m_b + n_b} \varphi_{2,t-1} - \frac{1}{m_b + n_b} [f_t + b_{t-1}]
\]

\[
b_t = -E_t f_{t+1} - n_b \varphi_{2t}
\]

- delivers \( \{b_t, \varphi_{2t}\} \) given process for \( \{f_t\} \) and initial conditions, \( b_{t_0-1}, \varphi_{2,t_0-1} \)
Optimal Tax Smoothing: Flexible Prices

- Solutions for output and tax rates are

\[ y_t = m\varphi_{2t} + n\varphi_{2,t-1} \]
\[ \hat{\tau}_t = \hat{\tau}_t^* - \psi^{-1} y_t \]

- Evolution of inflation is indeterminate

- With real government debt...
  - Inflation not affected by \( f_t \) (fiscal stress)
  - Output & tax rates are affected by \( f_t \)
  - \( \{\varphi_{2t}, y_t, \hat{\tau}_t\} \) have unit roots
  - Only in special cases are \( y_t, \hat{\tau}_t \) random walks
  - Optimal policy smooths \( \varphi_{2t} \)—value, in units of marginal utility, of additional government revenue at \( t \)
  - Optimal policy makes \( \varphi_{2t} \) a random walk
Optimal Policy: Sticky Prices

- Form the Lagrangian

\[
\mathcal{L}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} (q_y y_t^2 + q_\pi \pi_t^2) + \varphi_1 \left[ -\kappa^{-1} \pi_t + y_t + \psi (\hat{\pi}_t - \hat{\pi}_t^*) + \kappa^{-1} \beta \pi_{t+1} \right] + \varphi_2 \left[ b_{t-1} - \pi_t - \sigma^{-1} y_t + f_t - (1 - \beta) [b_y y_t + b_\pi (\hat{\pi}_t - \hat{\pi}_t^*)] - \beta (b_t - \pi_{t+1} - \sigma^{-1} y_{t+1} + f_{t+1}) \right] \right\} + \left[ \kappa^{-1} \varphi_{1,t_0-1} + \varphi_{2,t_0-1} \right] \pi_{t_0} + \sigma^{-1} \varphi_{2,t_0} y_{t_0}
\]

- last line adds constraints that \( \pi_{t_0} = \bar{\pi}_{t_0} \) & \( y_{t_0} = \bar{y}_{t_0} \)
- ensures timeless perspective
Optimal Policy: Sticky Prices

- FOCs are

\[ q_\pi \pi_t = \kappa^{-1}(\varphi_{1t} - \varphi_{1,t-1}) + (\varphi_{2t} - \varphi_{2,t-1}) \]  
\[ q_y y_t = -\varphi_{1t} + [\sigma^{-1} + (1 - \beta) b_y] \varphi_{2t} - \sigma^{-1} \varphi_{2,t-1} \]  
\[ \psi \varphi_{1t} = (1 - \beta) b_\tau \varphi_{2t} \]  
\[ \varphi_{2t} = E_t \varphi_{2,t+1} \]  

(42)–(45), (31) & (38) hold for \( t \geq t_0 \) and solve for \( \{\pi_t, y_t, \hat{\tau}_t, \hat{b}_t, \varphi_{1t}, \varphi_{2t}\} \)

- Also have at \( t_0 \)

\[ \varphi_{2,t_0-1} = \kappa^{-1} \varphi_{1,t_0-1} \]  
\[ \sigma^{-1} \varphi_{2,t_0} = 0 \]

- Output & inflation dynamics are

\[ y_t = \frac{1}{q_y} \left[ (1 - \beta) b_\tau (1 - \psi^{-1}) + \sigma^{-1} \right] \varphi_{2t} - \sigma^{-1} q_y^{-1} \varphi_{2,t-1} \]  
\[ \pi_t = \frac{1}{q_\pi} \left[ 1 + \frac{(1 - \beta) b_\tau}{\kappa \psi} \right] (\varphi_{2t} - \varphi_{2,t-1}) \]  

(48)  
(49)
What do these FOCs mean?

- (42) ⇒ there are forecastable changes in $\pi_t$ only if there are forecastable changes in $\phi_{1t}$ or $\phi_{2t}$
- (45) ⇒ there are no forecastable changes in $\phi_{2t}$
- (44) ⇒ $\phi_{1t}$ & $\phi_{2t}$ covary perfectly ∴ there are no forecastable changes in $\phi_{1t}$
- taken together ⇒ no forecastable changes in $\pi_t$

Benigno-Woodford calibration

\[
\beta = 0.99, \quad \sigma^{-1} = 0.157, \quad \kappa = 0.0236, \quad \psi = 0.397, \quad b_\tau = 8.33, \quad \frac{q_y}{q_\pi} = 0.0024(?)
\]

Shock lump-sum transfers, $z_t$, by 1% of steady-state GDP & vary persistence, $\rho$
Optimal Policy: Sticky Prices

- Optimal responses to a persistent increase in transfers (“pure fiscal shock”)
  - permanent increase in debt
  - planned permanent increase in tax rate
  - adjustments more gradual the more persistent the shock
  - some (small) unanticipated inflation
    - larger the more persistent the shock
  - inflation rises only in the period of the shock
  - just a little bit of stickiness makes inflation rise small
  - see figures 2–5

- Summary
  - transitory rise in $\pi_t$ robust to type of shock (due to unforecastable nature of $\Delta \pi_t$)
  - not strict price stability, but expected $\pi$ does not vary, regardless of shock
  - no longer optimal to target constant $\tilde{p}_t$
    - price level exhibits base drift
Sticky Prices

Figure 2: Alternative Fiscal Shocks

![Graph showing alternative fiscal shocks]

Source: Authors' computations.

Figure 3: Impulse Response of the Public Debt to a Pure Fiscal Shock, for Alternative Degrees of Persistence

![Graph showing impulse response]

Source: Authors' computations.
As shown in figure 5, the inflationary impact of a fiscal shock under the optimal policy regime is quite small. In the case of a purely transitory (one-quarter) increase in the size of transfer programs by an amount equal to one percent of GDP, optimal policy allows an increase in the inflation rate that quarter of only two basis points (at an annualized rate). Moreover, the increase in inflation is . The log price level is thus allowed to increase that quarter by only half a basis point.
A Non-Binding Solvency Constraint

Three ways to make solvency constraint not bind

1. Allow two distinct—independent—tax rates
   - \( \psi \hat{\tau}_t \) in (31) becomes \( \psi_t \hat{\tau}_{1t} + \psi_2 \hat{\tau}_{2t} \) & \( b_t \hat{\tau}_t \) in (38) becomes \( b_1 \hat{\tau}_{1t} + b_2 \hat{\tau}_{2t} \)
   - elasticities will generally be different
   - can use one tax rate to shift (31) & other to shift (32)
   - increase tax revenues without affecting (31)
   - optimal policy: strict IT \( \pi_t \equiv 0 \)

2. Arbitrary kinds of state-contingent government debt
   - make \( \hat{b}_{t-1} \) vary with state at \( t \)
   - can have \( \hat{b}_{t-1} + f_t \) not vary, regardless of shocks
   - optimal policy: strict IT \( \pi_t \equiv 0 \)

3. Extend maturity of government debt
   - \( \hat{b}_{t-1} \) varies at \( t \) with market value of debt & different maturity bond prices respond distinctly to shocks
   - effectively makes government debt state-contingent
   - optimal policy: strict IT \( \pi_t \equiv 0 \)
Assume \{G_t, Z_t, \tau_t\} all exogenous processes

Now both the solvency constraint, (38), & the aggregate supply curve, (31), constrain possible \{\pi_t, y_t\} paths

- no endogenous fiscal instruments to shift these relations

Full price stability is generally not feasible

- if \pi_t \equiv 0, (31) \Rightarrow y_t = -\psi(\hat{\tau}_t - \hat{\tau}_t^*)
- but then solvency constraint is not satisfied

 Monetary policy now chooses among alternative paths for \{\pi_t\}—not just \pi_t

FOCs (42), (43) & (45) continue to apply

- but (44) need not hold \Rightarrow \varphi_{1t} \& \varphi_{2t} may not covary perfectly

As before, consider a “pure fiscal shock”: serially correlated increase in lump-sum transfers, Z_t

- figures 6 & 7: exogenous vs. endogenous tax rates
Figure 6: Impulse Response of the Public Debt under Optimal Monetary Policy and Two Assumptions about Tax Policy

The figure shows the impulse response of the real public debt to a pure fiscal shock under optimal monetary policy, both under the assumption that tax policy also responds optimally (solid line; same as in figure 3) and under the assumption that the path of the tax rate does not respond at all (dashed line).
Figure 7: Impulse Response of the Inflation Rate under Optimal Monetary Policy and Two Assumptions about Tax Policy

a. The figure shows the impulse response of the real public debt to a pure fiscal shock under optimal monetary policy, both under the assumption that tax policy also responds optimally (solid line; same as in figure 3) and under the assumption that the path of the tax rate does not respond at all (dashed line).
Exogenous Fiscal Instruments

- Optimal to allow fiscal shock to raise inflation more
  - inflation rises about 8 times more than
  - inflation’s rise is persistent (not optimal to make $\Delta \pi_t$ unforecastable)
  - since taxes exogenous, long-run level of debt cannot rise
  - need big unexpected inflation to reduce real debt burden
  - real debt falls even though transfers rise
    - $EPV(s)$ is lower initially, but then rises back to long-run value
    - as $EPV(s)$ begins to rise, real debt rises back to long-run value
  - inflation’s rise is smaller than in flexible-price case
    - with sticky prices, real economic activity increases
    - higher incomes raise tax revenues (even with fixed tax rate)
Exogenous Fiscal Instruments

- Benefit of an appropriately managed IT regime
  - expected inflation is anchored even when transitory inflation erodes value of debt
  - if $E_t \pi_t + 1$ does not rise much, then aggregate supply tradeoff, (31), yields a relatively large increase in real output for given increase in $\pi_t$

$$
\pi_t = \beta E_t \pi_{t+1} + \kappa [y_t + \psi (\hat{\tau}_t - \hat{\tau}^*_t)]
$$

(31)

- if $E_t \pi_{t+1}$ were to rise as much as $\pi_t$, get small—if any—increase in output
  - this would raise revenues by less
  - requires a larger increase in surprise inflation to devalue debt
Illustrate tradeoff: consider alternative responses to increase in $f_t$.

Suppose MP allows path of inflation to obey

$$E_t \pi_{t+j} - E_{t-1} \pi_{t+j} = \tilde{\pi}_t \lambda^j$$

for all $j \geq 0$, some initial inflation response, $\tilde{\pi}_t$, some persistent factor, $0 \leq \lambda \leq 1$.

- Assume expected tax gap unchanged:
  $$E_t (\hat{\tau}_{t+j} - \hat{\tau}^*_t) = 0$$

- For any $\lambda$ there is a unique $\tilde{\pi}_t$ so this is a possible equilibrium under suitable MP.

- Can ask how $\tilde{\pi}_t$ & path of inflation vary with $\lambda$. 

Exogenous Fiscal Instruments
Exogenous Fiscal Instruments

- Solve (31) for $y_t$ and compute

$$E_t y_{t+j} - E_{t-1} y_{t+j} = \frac{1}{\kappa} [E_t (\pi_{t+j} - \beta \pi_{t+j+1}) - E_{t-1} (\pi_{t+j} - \beta \pi_{t+j+1})]$$

$$= \frac{1 - \beta \lambda}{\kappa} \lambda^j \tilde{\pi}_t, \quad \text{for each } j$$

- Substitute this and conjectured inflation response into solvency constraint, (38), requires that

$$\tilde{\pi}_t = \frac{\tilde{f}_t}{1 + \sigma^{-1} (1 - \beta \lambda) + (1 - \beta) b_y / \kappa} \quad (50)$$

- the more persistent the shock is expected to affect inflation, larger $\lambda$, the higher $\tilde{\pi}_t$
- if policy makes the shock’s effect on inflation more persistent, will get higher initial—and all subsequent—inflation rates
- Still need CB to communicate that it will bring inflation back to target, even in the face of fiscal stress
Optimal Targeting Rule

- Seek a targeting rule for MP that is a commitment about behavior in the face of fiscal constraints
  - need to anchor inflation expectations
  - need behavior to be time-consistent so it’s credible
  - target criterion should explain how much inflation is allowed in short run, given the shocks
  - guarantee no significant fluctuations in inflation will be forecasted beyond a few quarters

- Follow previous method: use FOCs
  - (42), (43) & (45) must hold for optimal MP
  - target criterion should follow from these
  - these conditions must hold for any disturbances, so will be robust
Optimal Targeting Rule

- Derive a robustly optimal target criterion equivalent to demanding that $\{\varphi_{1t}, \varphi_{2t}\}$ satisfy (42), (43) & (45)
  - A commitment to a target for the output-gap-adjusted price level, $\tilde{p}_t$
  - CB should use policy instrument to ensure that each period $p_t$ satisfies

$$
\tilde{p}_t = p_{t-1}^* + (1 + \eta)(p_t^* - p_{t-1}^*) \quad (51)
$$

where $\eta \equiv \frac{\sigma^{-1}}{(1 - \beta) \tilde{b}_t + \kappa} > 0$

$p_t^*$ is CB’s estimate at $t$ of the long-run (output-gap-adjusted) price level consistent with solvency

- $p_t^*$ is determined by the implication of (51) that

$$
E_t \tilde{p}_T = p_t^*, \quad \text{for all } T \geq t + 1
$$
Optimal Targeting Rule

- Value for $p_t^*$ implies
  - value for $\tilde{p}_t$
  - complete expected path $\{E_t\tilde{p}_T\}$
- CB uses its model of the economy to derive forecast paths for endogenous variables given current estimate of $p_t^*$
  - right estimate of $p_t^*$ leads forecast paths to be consistent with solvency
- To implement this, CB must forecast fiscal variables, as well as prices & real activity
  - this is a type of fiscal dominance
  - CB can be understood to be operating as best it can given the fiscal constraints
  - optimal MP, then, must include reaction to fiscal developments