Norges Bank Mini-Course: Sims’s “Paper Money”

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An Important Paper

- Real-world economic developments require fresh perspectives on price-level determination
  - conventional monetarist/new-Keynesian theories silent
    - makes strong, implicit assumptions about fiscal behavior
  - need to understand how monetary & fiscal policies are intertwined
- Key developments
  1. central bank balance sheets have exploded & become riskier
  2. government debt in advanced economies grown rapidly
  3. central banks now pay interest on reserves, expanding government interest-bearing debt
The Fed’s Balance Sheet

- Traditional Security Holdings
- Long Term Treasury Purchases
- Lending to Financial Institutions
- Fed Agency Debt Mortgage-Backed Securities Purchases
- Liquidity to Key Credit Markets

Graph showing changes in balance sheet over time.
The ECB’s Balance Sheet
The Bank of England’s Balance Sheet

Chart 5: Bank of England assets

(a) Loan to the Asset Purchase Facility (APF) to finance the purchase of assets.
Growth in Government Debt

Central Government Debt as Percent of GDP

Japan
Euro Area
UK
USA
Themes of the Paper

1. To be effective, MP actions must induce specific fiscal responses
2. Fiat currency requires fiscal backing
3. Central bank balance sheets matter because they connection MP & FP
4. Nominal & real debt are very different
   4.1 nominal debt provides a “fiscal cushion”
   4.2 nominal debt is important to lender of last resort
Consider a high-debt, high-inflation economy with unresponsive FP

- will have high interest rates
- interest expense large fraction of budget
- if inflation rises more, usual MP reaction is to raise interest rate (Taylor principle)
- but CB will realize higher interest rates will flow to higher debt service
- only result is still higher nominal debt growth
- private agents will see higher nominal debt as higher wealth
- inflation will rise more
- CB may just decide *not* to raise interest rate
- by not raising rate, CB is dampening inflation effects
Monetary Actions, Fiscal Responses

- Some CBs—Fed and ECB—now pay interest on reserves
- This implies that the explosion in CB balance sheets need not be inflationary
  - CB can raise interest on reserves to combat inflation
  - banks have little incentive to expand lending if perfectly safe reserve deposits pay interest close to loan rates
- But reserve deposits & Treasury securities close substitutes
  - higher rate on reserves $\Rightarrow$ higher rate on Treasuries
  - if debt-GDP 100%, raising rates from 2% to 6% $\Rightarrow$
  - interest expense rises from 10% to 30% of expenditures
  - if FP passive, surpluses would rise to back this MP action
  - current politics raises doubts that taxes will increase
Fiat Currency Requires Fiscal Backing

- Have known since Obstfeld-Rogoff (1983) that speculative hyperinflations cannot be ruled out
  - if money is not essential, then there is always a continuum of eq with $P_t \to \infty$ and $M_t/P_t \to 0$
  - economy approaches the barter eqm
- If taxes used to pay interest on government liabilities or contract supply of fiat currency, can resolve indeterminacy
- Fiscal backing can be...
  - explicit: taxes place a lower bound on value of nominal debt
  - implicit: commitment to recapitalize CB if CB’s balance sheet deteriorates
- EMU’s setup sidestepped this issue of fiscal backing for the ECB
  - dealt with ex-post after sovereign crises
When does the CB balance sheet not matter?

- when it can “print money”: use reserve deposits to maintain positive net worth
  - depositors assured payment, so can avoid runs
- when CB is truly part of the government
  - recapitalization always assured through taxation

These conditions fail when CB aims to control $P$ & $MP-FP$ separately chosen

- open-market sale: sell assets to reduce high-powered money
- if CB net worth negative, CB may not be able to aggressively contract
  - people will see the action requires selling more assets than CB has
- if CB contracts by raising rates on reserves, but wants to avoid increasing reserves
  - CB needs to sell assets to finance interest on reserves
  - again, may be unsustainable
CB Balance Sheets & Policy Interactions

- CB balance sheet really is irrelevant if fiscal backing assured
- CB can, to some extent, provide its own “fiscal backing”
  - can use seigniorage revenues to maintain unique price level & restore its balance sheet
- Higher seigniorage raises inflation, conflicting with mission of controlling $P$
- CB may also avoid certain actions for fear of negative net worth
  - lender-of-last-resort poses risks of losses
  - if fiscal backing not assured, CB may avoid this
- Fed now pays interest on reserves & holds a lot of long-term debt
  - high interest on reserves & low interest on long debt can create negative seigniorage even with higher inflation
CB Balance Sheets & Policy Interactions

- In both U.S. and Euro Area, fiscal backing of CBs not certain
- EMU has a “capital key” that specifies how countries will provide capital to ECB
  - if ECB needs a lot of capital & its popularity is low in some countries . . .
  - will capital infusion be assured?
  - ECB might avoid taking some actions because of this concern
- Fed most likely to have problems when it raises short rates
  - higher short rates generate capital losses on long bonds & raise interest payments on reserves
  - tighter MP would reduce growth, raise fiscal deficits & raise interest expense on debt
  - will Congress recapitalize in the current environment?
Nominal & Real Debt Are Different

- Both types of debt must satisfy eqm condition
  - for real debt, $b$:
    \[ b_{t-1} = \sum_{j=0}^{\infty} \beta^j E_t s_{t+j} \]  
  - for nominal debt, $B$:
    \[ Q_t B_{t-1} = P_t \sum_{j=0}^{\infty} \beta^j E_t s_{t+j} \]

- higher $b_{t-1}$ requires higher $s_{t+j}$
- higher $B_{t-1}$ requires mix of
  - higher $s_{t+j}$
  - higher $P_t$
  - lower $Q_t$

- Outright default far less likely for nominal than for real debt
Nominal & Real Debt Are Different

- In deterministic steady state, investors insist on higher nominal return to compensate for inflation’s effects on real value of debt holdings
  - real returns on debt will be same whether debt is real or nominal
  - if nominal interest rates fall after date of issue of long debt with fixed coupon...
  - market value of debt rises
  - bond holders get unanticipated higher return
  - if inflation higher than expected, real value of debt falls
- These mechanisms can cushion impacts of unexpected changes in FP
- Sims calculates these surprises to be on the order of ±6% of value of debt
  - treating government debt as real is a poor approximation
Surprise gains & losses to holders of U.S. government bonds, as share of GDP. Source: Sims (2013)
By joining Euro Area, countries have made their debt real
- by signing onto there being no fiscal cushion, Euro countries must accept
  - occasional defaults
  - countries in EU receivership
  - unless countries can assure requisite fiscal backing

Optimal FP exercises often take debt as real
- need to address what role fiscal cushion has in optimal policy
### Nominal & Real Debt Are Different

<table>
<thead>
<tr>
<th></th>
<th>Debt-GDP (%)</th>
<th>Interest Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>127.3</td>
<td>5.49</td>
</tr>
<tr>
<td>Spain</td>
<td>77.4</td>
<td>5.85</td>
</tr>
<tr>
<td>Japan</td>
<td>200.0</td>
<td>0.84</td>
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<tr>
<td>U.K.</td>
<td>87.4</td>
<td>1.90</td>
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<tr>
<td>U.S.</td>
<td>72.5</td>
<td>1.80</td>
</tr>
<tr>
<td>Euro Area</td>
<td>90.0</td>
<td>2.22</td>
</tr>
</tbody>
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Data approximately reflect condition in late 2012. Debt is for central government and interest rate is average yield on bonds.

- Italy & Spain: real debt
- Japan, U.K. & U.S.: nominal debt
- Default far less likely with nominal debt
Nominal debt is important for lender-of-last-resort function

In financial panic, counterparty risk widespread & credit markets freeze up

A CB—backed by a treasury that can run primary surpluses & able to issue nominal debt—is an ideal lender-of-last-resort

- CB can create reserves, so it need never default
- if CB takes capital losses, fiscal authority can back it

Doesn’t work like this in EMU

- ECB not assigned lender-of-last-resort role
- but national CBs cannot create reserves
- and member governments issue only real debt

In practice, ECB has stepped in as lender-of-last-resort

- and been highly criticized for doing so
Model 1

- Samuelson’s pure consumption loans model with storage
- Will show that
  - without tax backing for money or debt, $P$ indeterminate
  - there is one stable $P$ where allocation is efficient
  - there are a continuum of initial $P_1$’s
    - all inefficient with value of money and bonds $\to 0$
  - if govt runs primary surplus, agents see future taxes as reducing their wealth
    - will save until $P$ low enough to make value of debt equal PV(taxes)
    - this eliminates non-uniqueness, no matter how small the surplus
    - for small value of surplus, allocation arbitrarily close to efficient one
Model 1: Environment

- Infinite sequence of time periods
  - in each period, same number of 2-period-lived agents are born
  - endowed with 1 unit of consumption good (grain)
  - grain can be stored but decays at rate $\theta$
  - govt debt denominated in dollars
  - at initial date, $t = 1$, initial old have $B_0$
  - they redeem debt with govt, receiving new 1-period debt in amount $B_1 = R_0 B_0$
  - new govt paper worthless to old, so they try to sell it to initial young for grain
  - $P_t$ is rate at which grain trades for newly issued debt
  - process repeats for $t = 1, 2, \ldots, \infty$
Model 1: Environment

- Generation born at $t$ maximizes $U(C_{1,t}, C_{2,t+1})$ subject to

\[ C_{1,t} + S_t + \frac{B_t}{P_t} = 1 \quad (3) \]

\[ C_{2,t+1} = \frac{R_t B_t}{P_{t+1}} + \theta S_t \quad (4) \]

\[ S_t \geq 0, \quad B_t \geq 0 \quad (5) \]

- Govt rolls over debt, so market clearing is

\[ B_{t+1} = R_t B_t \]

- Govt sets arbitrary value for $R_t$ each period
Model 1: Deriving Equilibrium

- **FOCs for generation $t$ agent with perfect foresight**

  \[
  \partial C_1 : \quad D_1 U(C_{1,t}, C_{2,t+1}) = \lambda_t \quad (6) \\
  \partial C_2 : \quad D_2 U(C_{1,t}, C_{2,t+1}) = \mu_{t+1} \quad (7) \\
  \partial B_t : \quad \frac{\lambda_t}{P_t} = \frac{R_t \mu_{t+1}}{P_{t+1}}, \text{ if } B_t > 0 \quad (8) \\
  \partial S_t : \quad \lambda_t = \theta \mu_{t+1}, \text{ if } S_t > 0 \quad (9)
  \]

- **(8) & (9) imply**

  \[
  \frac{R_t P_t}{P_{t+1}} = \theta \quad (10)
  \]

- **Assume $R_t \equiv R$ & log preferences**

  \[
  U(C_{1,t}, C_{2,t+1}) = \log(C_{1,t}) + \log(C_{2,t+1})
  \]
Model 1: Deriving Equilibrium

- Solve for Lagrange multipliers

\[
\frac{R_t P_t}{P_{t+1}} = \frac{C_{2,t+1}}{C_{1,t}}, \text{ if } B_t > 0
\]  
(11)

\[
\theta = \frac{C_{2,t+1}}{C_{1,t}}, \text{ if } S_t > 0
\]  
(12)

- Let total savings be \( W_t = S_t + B_t/P_t \)

- With log preferences, if rate of return to savings is \( \rho_t \), then

\[
\rho_t = \frac{C_{2,t+1}}{C_{1,t}}
\]

- which implies from budget constraints

\[
C_{1,t} + W_t = 1 = C_{1,t} + \frac{C_{2,t+1}}{\rho_t} = 2C_{1,t}
\]  
(13)

- saving is always half the endowment: 0.5
Model 1: Deriving Equilibrium

- There is an eqm with $S_t \equiv 0$ & $P_t < \infty$
  - if $W_t = .5, S_t = 0$, then $C_{1,t} = B_t/P_t = .5$
  - since savings all used to buy debt from old,
    $C_{2,t} = .5 = .5R_tP_t/P_{t+1} \Rightarrow \rho_t \equiv 1$
  - so $P_{t+1} = RP_t$ all $t \geq 1$
  - price level grows at gross nominal interest rate, but
    real value of newly issued and maturing debt is constant at $B_t/P_t = B_{t+1}/P_{t+1} = .5$
  - equilibrium requires a particular initial price level, $P_1$

\[
\frac{B_1}{P_1} = \frac{RB_0}{P_1} = \frac{1}{2}
\]

\[\Rightarrow P_1 = 2B_1 = 2RB_0\]
Model 1: Deriving Equilibrium

- Are also equilibria with $S_t > 0$
  - by (11) & (12)
    \[
    \rho \equiv \theta = \frac{RP_t}{P_{t+1}}
    \]
  - price level grows at the *higher* rate $R/\theta$
  - nominal debt still grows at rate $R$, so real debt shrinks according to
    \[
    \frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}}
    \]
  - start economy with any $B_1/P_1 < .5$
  - $S_t = .5 - B_1/P_1$
  - $S_t \to .5$ because $B_t/P_t \to 0$ as $t \to \infty$,
- *Every* initial $P_1 > 2RB_0$ (including $P_1 = \infty$) is a perfect foresight equilibrium
  - price level is indeterminate
Model 1: Deriving Equilibrium

- Equilibria with $S_t > 0$ are inefficient
  - economy’s resource constraint is

\[ C_{1,t} + C_{2,t} + S_t = 1 + \theta S_{t-1} \]

  - when $S_t > 0$, $S$ is either increasing or (when $P_t = \infty$) constant
    - since $C_{1,t} = .5$, $C_{2,t} < .5$ when $S_t > 0$
    - recall that when $S_t = 0$, $C_{1,t} = C_{2,t} = .5$

- Nothing in the model makes these inefficient equilibria unlikely to occur

- Notice also that if we replace “$B''$ with “$M''$ and set $R = 1$, we get Samuelson’s model of “money,” so this indeterminacy is not about “debt” versus “money”
Model 1: Deriving Equilibrium

- What if we introduce fiscal backing for debt?
- Impose a lump-sum tax, \( \tau \), on young each period
  - govt budget constraint is now
    \[
    \frac{B_t}{P_t} = \frac{RB_{t-1}}{P_t} - \tau
    \] (14)
- Assume interior solution: bonds & storage positive
  - they must share gross rate of return \( \theta \) & (14) becomes
    \[
    \frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}} - \tau
    \] (15)
- This stable difference equation has solution
  \[
  \frac{B_t}{P_t} = \sum_{s=0}^{t-1} -\tau \theta^s + \theta^t \frac{B_0}{P_0}
  \] (16)
- Since \( \theta < 1 \), RHS \( \rightarrow -\tau/(1 - \theta) \) as \( t \rightarrow \infty \)
  - individual perceives wealth in form of govt debt cannot cover tax obligations
  - increases savings, reducing \( P_t \), pushing value of debt toward zero-storage value
Model 1: Deriving Equilibrium

- If $S_t = 0$, return on debt can be positive
  - agents see tax as reducing wealth & first-period consumption
  - agent’s budget constraint in first period is
    \[ C_{1,t} + \frac{B_t}{P_t} + \tau = 1 \tag{17} \]
  - FOCs still deliver $RP_t/P_{t+1} = \rho_t = C_{2,t+1}/C_{1,t}$
  - so $C_{2,t+1} = RB_t/P_{t+1} = \rho_t B_t/P_t$ and (17) is
    \[ C_{1,t} + \tau + \frac{C_{2,t+1}}{\rho_t} = 1 = C_{1,t} + C_{1,t} + \tau \tag{18} \]
  - so first-period consumption is
    \[ C_{1,t} = \frac{1 - \tau}{2} \]
- $S_t = 0 \Rightarrow C_{1,t} + C_{2,t} = 1$, so
  \[ C_{2,t} = \frac{1 + \tau}{2} \]
Model 1: Deriving Equilibrium

- With these allocations, utility is
  \[ \log(1 - \tau) + \log(1 + \tau) + 2 \log(1/2) < 2 \log(1/2) \]

  - with \( \tau = 0 \), equilibrium not unique
  - small \( \tau > 0 \) makes equilibrium unique
  - utility approaches the \( \tau = 0 \) level

  - Even a little bit of fiscal backing goes a long way
Model 1: Deriving Equilibrium

- Debt valuation equation holds in these equilibria
- Gross interest rate is $\rho_t = C_{2,t+1}/C_{1,t} \equiv (1 + \tau)/(1 - \tau)$ & gbc yields
  \[ \frac{B_t}{P_t} = \rho_{t-1} \frac{B_{t-1}}{P_{t-1}} - \tau \]
- Since $B/P = C_2/\rho$ is constant in the eqm
  \[ \frac{B_t}{P_t} = \frac{\tau}{\rho - 1} \quad (19) \]
  - as $\tau \to 0$, $B/P \not\to 0$ as (19) suggests
  - substitute eqm value for $\rho = (1 + \tau)/(1 - \tau)$ and (19) becomes
    \[ \frac{B_t}{P_t} = \frac{1 - \tau}{2} \quad (20) \]
  - because $\rho \to 1$ as $\tau \to 0$, $B/P \to 1/2$ even though (19) continues to hold
Model 1: Deriving Equilibrium

How is the initial price level determined?

\[
\frac{B_0}{P_0} = \frac{1 - \tau}{2} = \frac{R_{t-1}B_{t-1}}{P_t} - \tau
\]  

(21)

- \(R_{t-1}\) & \(B_{t-1}\) given
- can solve (21) for \(P_0 > 0\) so long as \(R_{t-1}B_{t-1} > 0\)
- future values of \(P_t\) come from policy choices for \(R_t\)
  (higher \(R\) produces higher inflation)

What if \(R_{t-1}B_{t-1} = 0\)?

- given the constraint \(B_0 > 0\), FP at \(t = 0\) cannot simply set \(\tau\) to its constant value
- old at time 0 cannot finance their consumption
- plausible to suppose govt imposes \(\tau\) on young, issues new debt bought by young, and uses proceeds to subsidize time-0 old
Model 2

- A Taylor rule requires fiscal backing to deliver a unique price level.
- In this model, fiscal backing is always in play:
  - can be negligibly small
  - equilibrium arbitrarily close to model without backing
- As in previous model, existence of fiscal backing is important for stable prices:
  - if the backing is credible, size of backing can be small
- EMU makes it unclear where fiscal backing will come from—or if it will come:
  - such institutions are destabilizing
  - because in normal times, large backing interventions do not occur, it is easy to overlook importance of the backing.
Model 2: Setup

- Continuous-time extension of Leeper (1991)
- Monetary policy rule
  \[ \dot{r} = \gamma (\theta \dot{p} - (r - \rho)) \]  

  - MP adjusts nominal interest rate, \( r \), with delay to inflation, \( \dot{p} \)
  - larger \( \gamma \) means longer delay
  - Taylor principle: \( \theta > 1 \)

- Fisher equation
  \[ r = \rho + \hat{\dot{p}} \]  

  - \( \rho \) is constant real interest rate
  - \( \hat{\dot{p}} \) is right time derivative of expected path of log of price level
  - on perfect foresight path, \( \hat{\dot{p}} = \dot{p} \) at all dates after initial date; \( p \) can move discontinuously at the initial date
Model 2: Solution

- Combine (22) & (23) to get second-order d.q.

\[ \ddot{p} = \gamma(\theta - 1)\dot{p} \quad (24) \]

- holds after the initial date \( t = 0 \) on any perfect foresight path
  - \( \theta > 1 \Rightarrow \) solutions of form \( \dot{p}_t = \dot{p}_0 e^{\gamma(\theta-1)t} \)

- If rule out such explosive paths, there is a single stable solution: \( \dot{p} \equiv 0 \)
  - on that path \( r = \rho \Rightarrow \dot{r} = 0 \)

- MP rule, (22), holds in actual derivatives, which implies
  - \( r - \gamma\theta p \) is differentiable even at \( t = 0 \)
  - both \( r \) and \( p \) can jump discontinuously at \( t = 0 \)
  - jumps must satisfy \( \Delta r = \gamma\theta \Delta p \)
  - delivers a determinate price level
Continuing the analysis

- Let $r_0^-$, $p_0^-$ indicate left limits of variables at time 0 (pre-jump values)
- Then we have

$$\Delta r_0 = \rho - r_0^- = \gamma \theta (p_0 - p_0^-) = \gamma \theta \Delta p_0 \tag{25}$$

- Can solve this for unique $p_0$ as function of $(\rho, r_0^-, p_0^-)$
- If initial $p$ were below the unique $p_0$, $r$ & $\hat{p}$ would also be lower
  - Inflation tends to $-\infty$ at exponential rate
  - Rule (22) cannot be maintained, since it requires $r < 0$
Model 2: Solution

- If initial $p$ were above the unique $p_0$
  - inflation tends to $+\infty$ at exponential rate
  - opportunity cost, $r$, of holding non-interest-bearing money becomes arbitrarily high
  - if money is essential, these explosive paths may be viable equilibria
  - if money not essential, there may be upper bound on $r$ where $M/P = 0$ & paths in which $p = \infty$ may also be viable equilibria
- In any case, can postulate a shift in policy at some point
  - policy *must* shift to avoid $r < 0$
  - policy could shift at high inflation rates
  - aim is to leave the stationary $\dot{p} = 0$ equilibrium viable
- Cochrane finds these hypothetical policy shifts implausible
  - they would not be observed in equilibrium
Model 2: Solution

- Can instead eliminate bad price paths with observable policies
- Add to model real debt, $b$, and primary surplus, $\tau$
  - government budget constraint
    \[ \dot{b} = (r - \dot{p})b - \tau \] (26)
  - and passive FP rule
    \[ \tau = -\phi_0 + \phi_1 b \] (27)
- Along perfect foresight path were $r = \rho + \dot{p}$
  \[ \dot{b} = (\rho - \phi_1)b + \phi_0 \] (28)
- With $\rho - \phi_1 < 0$, (28) is stable and converges to $\phi_0 / (\phi_1 - \rho)$ for all equilibrium time paths of $p$
  - evidently, (28) cannot play a role in determining $p$ and cannot eliminate the indeterminacy
Model 2: Solution

- Recall: problem is eliminating exploding price paths
- Alter fiscal policy rule to be

\[ \tau = -\phi_0 + \phi_1 b + \phi_2 \dot{p} \]  \hspace{1cm} (29)

and (28) becomes

\[ \dot{b} = (\rho - \phi_1) b + \phi_0 - \phi_2 \dot{p} \]  \hspace{1cm} (30)

- Now have 3-equation recursive system, (22), (23) & (30)
  - \( b \) appears in last equation
  - any paths that make \( \dot{p} \) explode are still mathematical solutions
Model 2: Solution

- What happens to $b$ as $\dot{p} \to \infty$?
  - $\dot{b}$ eventually negative & more negative over time
  - $b$ becomes 0 in finite time
  - agents cannot borrow from govt; they see their tax obligations exceeding their wealth in form of govt debt
  - they reduce their planned consumption & try to save to pay future taxes
  - this reduction in demand reduces price level
  - with perfect foresight, $p$ jumps immediately to $p_0$ at $t = 0$

- Can also rule out unstable paths with accelerating deflation
  - $b$ would rise without bound
  - surpluses shrink and eventually become negative
  - agents will see their wealth rising without bound with no offsetting tax obligations
  - they increase spending and raise $p$
Model 2: Solution

- Solutions do not depend on size of $\phi_2$
  - just has to be positive
- Equilibrium will always be $\dot{p} = 0$
  - may be hard to detect in data
  - presence of $\phi_2$ has no effect on first two equations of system
  - but it eliminates unstable solutions as equilibria
- This example emphasizes that fiscal backing must be of the right kind
  - the passive rule, (27), constitutes “backing,” but doesn’t resolve indeterminacy
  - need backing to kick in under the right circumstances
Model 3: Optimal Policy

- Should debt be used as a “fiscal cushion”?

1. Barro: not optimal to use distorting taxes to rapidly pay off debt
   - deadweight losses from large initial tax distortions exceed present value of lower future deadweight losses from lower debt
   - debt & total revenue optimally follow martingale processes: $E_t b_{t+1} = b_t, E_t T_{t+1} = T_t$
   - note: taxes rise with increases in debt

2. Lucas & Stokey: if govt can issue contingent liabilities, it is optimal for taxes to be set without reference to current level of debt

3. Chari, Christiano & Kehoe: MP, through inflation, can create optimal contingencies in return on debt

4. Sims: keep taxes constant and use only surprise inflation & deflation to finance random govt spending
Model 3: Optimal Policy

- All of that work operates under flexible prices
  - surprise inflation & deflation is costless
  1. Schmitt-Grohé & Uribe: in new Keynesian model with 1-period nominal debt, optimal policy close to Barro’s constant taxes
    - do not want to use surprise inflation & deflation to revalue debt
  2. Sims: with long-term debt, can also revalue debt through nominal interest rates
    - much smaller changes in inflation
    - interest-rate fluctuations not thought to be very costly
    - price fluctuations can generate inefficient output & employment

- This model revisits Barro but with endogenous price determination and short & long debt
  - concludes there is a substantial role for use of nominal debt as a fiscal cushion
  - limiting tax fluctuations may be optimal if debt is long
Model 3: Setup

- **Posit loss function**

\[
-\frac{1}{2}E \left[ \sum_{t=0}^{\infty} \beta^t \left( \tau_t^2 + \theta \left( \frac{P_{t-1}}{P_t} - 1 \right)^2 \right) \right]
\]  
(31)

- **deadweight loss from taxation proportional to** \( \tau^2 \) (revenues)
- **swings in inflation welfare reducing**
- **let** \( \nu_t = P_{t-1}/P_t \)

- **Constant real interest rate,** \( \rho \), **and two constraints**
  - **Fisher equation**
  \[
  R_tE_t\nu_{t+1} = \rho
  \]
  (32)
  - **government budget constraint**
  \[
  b_t = R_{t-1}\nu_t b_{t-1} - \tau_t + g_t
  \]
  (33)
  - **\( b \) is real debt; \( g_t \) is exogenous govt spending**
Model 3: Solution

- Government maximizes (31) subject to (32) & (33) by choosing $R, \nu, b, \tau$
  - let $\mu$ be multiplier for (32) & $\lambda$ be multiplier for (33)
  - FOCs are

\[
\begin{align*}
\partial \tau & : \quad \tau_t = \lambda_t \quad \text{(34)} \\
\partial b & : \quad \lambda_t = \beta R_t E_t \left[ \nu_{t+1} \lambda_{t+1} \right] \quad \text{(35)} \\
\partial R & : \quad \mu_t E_t \nu_{t+1} = \beta E_t \left[ \nu_{t+1} \lambda_{t+1} \right] b_t \quad \text{(36)} \\
\partial \nu & : \quad \theta (\nu_t - 1) = -\lambda_t R_{t-1} b_{t-1} + \mu_{t-1} R_{t-1} \rho \quad \text{(37)}
\end{align*}
\]

- Examine simple case: $\beta \rho = 1$
  - use (32), (35) & (36)

\[
\begin{align*}
\lambda_t &= \beta R_t E_t \left[ \nu_{t+1} \lambda_{t+1} \right], \quad \mu_t = (\beta / \rho) R_t b_t E_t \left[ \nu_{t+1} \lambda_{t+1} \right] \\
\Rightarrow \rho \mu_t &= \lambda_t b_t
\end{align*}
\]
Model 3: Solution

- Use (38) with (34) in (33)

\[ \theta(\nu_t - 1) = (\tau_{t-1} - \tau_t)R_{t-1}b_{t-1} \]  

(39)

- If no loss from inflation fluctuations, \( \theta = 0 \), optimal policy is \( \tau_t = \tau_{t-1} \) if \( R_{t-1}b_{t-1} > 0 \)

- Constant \( \tau \) & random \( g_t \) make (33) unstable
  - Feasibility (\( b > 0 \)) & transversality (\( b_t \to 0 \) while \( \tau \) constant cannot be optimal) imply \( b \) must not explode

- Solve (33) forward

\[ b_t = \frac{\tau}{\rho - 1} - E_t \sum_{i=1}^{\infty} \rho^{-i} g_{t+i} \]  

(40)

- If \( g \) is i.i.d., \( b \) constant

- Constant \( b \) maintained by fluctuations in \( \nu \) that offset fluctuations in \( g \)

- Conventional fiscal theory
Model 3: Solution

- Now suppose want to keep price *level* constant ($\theta = \infty$)
  - now only real govt debt exists
- Drop FOC for $\nu$ & set $\nu_t \equiv 1$
  - have that $R_t = \rho$ so (35) implies
    \[
    \tau_t = E_t \tau_{t+1} \tag{41}
    \]
  - govt budget constraint unstable; forward solution is
    \[
    b_t = \frac{\tau_t}{\rho - 1} - E_t \sum_{i=1}^{\infty} \rho^{-i} g_{t+i} \tag{42}
    \]
  - using (42) at $t + 1$, can see that with *i.i.d.* $g$
    \[
    b_t = E_t b_{t+1} \tag{43}
    \]
- Obtain Barro’s random walk for taxes & debt
Model 3: Solution

- Most interesting case is when $0 < \theta < \infty$
- Add a consol: pays $\$1$ coupon every period forever
  - public holds $A_t$ consols that sell at price $Q_t$
  - $1/Q_t$ is long-term nominal rate
- No-arbitrage yields
  \[
  E_t \left[ \frac{Q_{t+1} \nu_{t+1} + 1}{Q_t} \right] = \rho
  \] (44)
- Government budget constraint is
  \[
  b_t \equiv \frac{A_t Q_t}{P_t} = \left( \frac{Q_t \nu_t + 1}{Q_{t-1}} \right) b_{t-1} - \tau_t + g_t
  \] (45)
- Rely on numerical computations: calibrate as
  \[
  Eg_t = \bar{g} = 1, \rho = \beta^{-1} = 1.1, \tau = 2, \tau - g = 1
  \Rightarrow b = (\tau - g)/(\rho - 1) = 10
  \]
- Consider one-time increase in $g_t$ by 1 unit, then return to $\bar{g}$
Imagine in a steady state and at $t \Delta g_t = 1$

Real debt

- $\theta = \infty$: $\Delta g_t = 1 \Rightarrow \Delta b = 10/11$ & $\Delta \tau = 1/11$ 
  *permanently*
- first column in figure

Nominal debt

- $\theta = 0$: full adjustment to higher $g$ through inflation
- $\Delta b = \Delta \tau = \Delta Q = 0$ and from (45)

\[
0 = \frac{b_{t-1}}{Q_{t-1}}(Q_t \Delta v_t + v_t \Delta Q_t) + 1
\]

\[
\Rightarrow \Delta v_t = -d\pi_t = -1/b = -0.10
\]

because inflation is costless, all adjustment occurs at $t$ (even with long debt)
- last column in figure
Model 3: Solution

- Make inflation fluctuations costly: $\theta = 10$
- With one-period debt, optimal policy after $g$ shock is
  - 43% flows into increase in $b$ & 4.3% increase in $\tau$ permanently
  - 4.8% into one-time increase in inflation
  - nominal interest rate unchanged
  - second column of figure
- With consol debt, optimal policy after $g$ shock is
  - 6.9% flows into increase in $b$ & 0.69% increase in $\tau$ permanently
  - most of shock absorbed by small permanent increases in $1/Q$ & inflation
    - interest rate rises 0.84 percentage points
    - inflation rate rise by 0.76 percentage points
    - note: no change in real interest rate ($1/Q$ an approximation)
  - third column of figure
Effect of a Unit $g$ Shock

Barro 1yr consol flexprice

Figure 2. Responses to unit $g$ shock rates depends on the size of the real debt, and the real debt is locally non-stationary in this solution. However, the results with long and short debt contrast so sharply that this example does provide a reason for caution in interpreting the results of analyses like those of Schmitt-Grohé and Uribe (2001) and Siu (2004). These papers conclude that in normal (non-war) times, it is optimal to make very little use of surprise inflation in cushioning fiscal shocks, but both papers assume that all debt is one-period debt. It is likely that their conclusions are sensitive to this assumption.
Model 3: Wrap Up

- Not claiming that long debt & small response of taxes to fiscal shocks is optimal
- But in consol case, both taxes & inflation vary less, so it may yield higher welfare
  - magnitudes of these responses depend on size of real debt, which is locally non-stationary in these solutions
- Results do raise doubts about previous optimality results that say there is little room for surprise inflation in cushioning fiscal shocks
- Other caveats
  - model considers only responses to $g$ shocks
  - welfare function *ad hoc*
  - no complete model of economy specified
  - results can be sensitive to calibration (especially of $\theta$)