ACADEMIA SINICA MINI-COURSE I

“GENERALIZING THE TAYLOR PRINCIPLE” w/ T. Davig

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Monetary policy is complex
For descriptive & prescriptive reasons, seek to simplify
Most successful simplification due to Taylor

\[ i_t = \bar{i} + \alpha(\pi_t - \pi^*) + \gamma x_t + \varepsilon_t \]

Taylor principle: \( \alpha > 1 \)
- necessary & sufficient for unique bounded eqm (w/bounded shocks)

Unique & stable eqm necessary for good policy
- rules out arbitrarily large fluctuations
Central banks can stabilize economy by adjusting nominal interest rate more than one-for-one with inflation

- approximates Federal Reserve behavior since 1982
- nearly optimal in workhorse class of monetary models
- used by central banks as a benchmark

Maintains two key assumptions

- fiscal policy is perpetually passive
- policy rule permanent & agents believe change impossible

This paper relaxes second assumption

- rule evolves according to a Markov chain
- consider two conventional monetary models
GENERALIZING THE TAYLOR RULE & PRINCIPLE

- $\alpha(s_t), \gamma(s_t) \sim$ Markov chain
- $s_t$: “rule,” “regime,” “state”
- $s_t$ exogenous (for now)
- Can believe actual policy rule time invariant
  - but Taylor rule is a gross simplification of reality
  - paper shows that a particular form of non-linearity can change predictions of models
In the Fisherian Model . . .

- Derive *long-run Taylor principle*
  - imposes much weaker conditions on MP for uniqueness
  - departures from short-run Taylor principle can be substantial—but brief—or modest—and prolonged
  - the more “hawkish” one regime is, the more “dovish” the other can be and still deliver uniqueness
  - “expectations formation effects”—beliefs about possible future regimes affect current eqm, increasing volatility even in a regime that satisfies TP
In the New-Keynesian Model . . .

- Derive *long-run Taylor principle*: dramatically expands region of determinacy
- Inference that inflation of the 70's due to failure to obey TP does not hold up when expectations embed possibility of regime change
- Occasional large departures from TP—due to worries about financial instability or economic weakness—can have quantitatively important impacts even in a regime that satisfies TP
- Misleading inferences can arise from dividing data into regime-specific periods to interpret estimates as arising from distinct fixed regimes
Why Regime Change?

- Evidence that monetary policy regime changed
- Institutional or policy reforms
  - adoption of inflation targeting by over 20 countries
  - Fed’s “just trust us” approach
- Logical consistency
  - if regime has changed, regime can change
  - expectations depend on prob. distn. over possible regimes
- Recurring: in US, no legislated change installed Volcker or Greenspan
  - confluence of economic/political conditions allowed US to dodge a bullet and get Bernanke (coulda’ been a FOG)
Because Taylor rule a gross simplification, deviations occur
- can be large and serially correlated
- are systematic responses to state of economy

How should we model these deviations?
- shuffled into the $\varepsilon$’s?
- time-varying feedback coefficients, $\alpha_t$ & $\gamma_t$?

$\varepsilon$’s affect conditional expectations
$\alpha_t$ & $\gamma_t$ affect expectations functions
A substantive choice
A simple Fisherian economy

\[ i_t = E_t \pi_{t+1} + r_t \]

\[ r_t = \rho r_{t-1} + \nu_t, \quad \nu \text{ bounded support} \]

\[ i_t = \alpha(s_t) \pi_t \]

\[ s_t \text{ Markov; } s_t = 1, 2 \]

\[ p_{ij} = P[s_t = j \mid s_{t-1} = i] \]

\[ \alpha(s_t) = \begin{cases} 
\alpha_1 & \text{for } s_t = 1 \\
\alpha_2 & \text{for } s_t = 2 
\end{cases} \]

- a monetary policy regime is a realization of \( \alpha(s_t) \)
- a monetary policy process is the collection
**Determancy: Definition**

- Seek generalization of Taylor principle
  - necessary & sufficient condition for existence of unique bounded eqm
- Why boundedness?
  - consistent w/ standard definition under fixed regime
  - corresponds to locally unique eqm
    - can analyze small perturbations
  - considering log-linearized models
    - boundedness ensures approximations are good
- Stationarity—mean-square stability—is too weak a definition
  - allows explosive solutions ($\pi$ exceeds any finite bound w/ positive probability)
Determinacy: Formalism

Model: $\alpha(s_t)\pi_t = E_t\pi_{t+1} + r_t$

- Let $\Omega_t^{-s} = \{r_t, r_{t-1}, \ldots, s_{t-1}, s_{t-2}, \ldots\}$ and $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$
- Integrating over $s_t$, for $s_t = 1$ and $s_t = 2$

$$E_t\pi_{t+1} = E[\pi_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1}E[\pi_{1t+1} | \Omega_t^{-s}] + p_{i2}E[\pi_{2t+1} | \Omega_t^{-s}]$$

where $\pi_{it} = \pi_t(s_t = i, r_t)$, the solution when $s_t = i$

- The system is

$$\begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} E_t\pi_{1t+1} \\ E_t\pi_{2t+1} \end{bmatrix} + \begin{bmatrix} r_t \\ r_t \end{bmatrix}$$

where $E_t\pi_{it+1}$ denotes $E[\pi_{it+1} | \Omega_t^{-s}]$
Determinacy: Formalism (con’t)

- Write system as
  \[ \pi_t = M E_t \pi_{t+1} + \alpha^{-1} r_t \]

- MSV solution: \( \pi_t \) function only of \((r_t, s_t)\)
- Define \( x_t = \pi_t - \pi_t^{MSV}(r_t, s_t) \)
- Bounded soln for \( \{x_t\} \) \(\iff\) bounded soln for \( \{\pi_t\} \)
- We study
  \[ x_t = M E_t x_{t+1} \]

- Proof of determinacy shows that under certain conditions on the policy process, \( x_t = 0 \) is the only solution
Prop. 1 When $\alpha_i > 0$, a unique bounded solution exists iff all the eigenvalues of $M$ lie inside the unit circle.

Sufficiency: the usual proof in linear RE models

- Intuition: boundedness requires that $\lim_{n \to \infty} M^n = 0$, so $x_t = 0$ the only solution
- Delivered by eigenvalue condition
Determination: Formalism (con’t)

- Necessity: Suppose $\lambda_1 \geq 1$, $\lambda_2 < 1$
  - diagonalize $M$, let $y_t = V^{-1}x_t$, then

\[
\begin{bmatrix}
  y_{1t} \\
  y_{2t}
\end{bmatrix} =
\begin{bmatrix}
  \lambda_1 & 0 \\
  0 & \lambda_2
\end{bmatrix}
\begin{bmatrix}
  E_t y_{1t+1} \\
  E_t y_{2t+1}
\end{bmatrix}
\]

bounded solutions $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}$,

\[
\begin{bmatrix}
  x_{1t} \\
  x_{2t}
\end{bmatrix} =
\begin{bmatrix}
  \gamma v_{11} \lambda_1^{-t} \\
  \gamma v_{21} \lambda_1^{-t}
\end{bmatrix}
\]

- also exist bounded sunspot solutions:
  $y_{1t+1} = \lambda_1^{-1}y_{1t} + \phi_{t+1}$, $y_{2t+1} = 0$, $E_t \phi_{t+1} = 0$, bounded
- multiple eq & sunspots possible w/ more stringent det defn
Long-Run Taylor Principle

Prop. 2 Given $\alpha_i > p_{ii}$ for $i = 1, 2$, the following statements are equivalent:

(A) All the eigenvalues of $M$ lie inside the unit circle.
(B) $\alpha_i > 1$, for some $i = 1, 2$, and the \textit{long-run Taylor principle (LRTP)}

$$(1 - \alpha_2) p_{11} + (1 - \alpha_1) p_{22} + \alpha_1 \alpha_2 > 1$$

is satisfied.

Premise $\alpha_i > p_{ii}$ all $i$ unfamiliar
- fixed regime: MP always obeys TP
- LRTP is hyperbola w/ asymptotes $\alpha_1 = p_{11}$ \& $\alpha_2 = p_{22}$
- restricts $\alpha$'s to economically interesting portion of hyperbola
Wide Range of Policy Processes Deliver Uniqueness

\[ \alpha_1 > 1: \quad p_{11}(1 - \alpha_2) + p_{22}(1 - \alpha_1) + \alpha_1\alpha_2 > 1 \]

- Some policy processes that deliver unique equilibria
  \[ \alpha_1 \to \infty \Rightarrow \alpha_2 > p_{22} \]
  or
  \[ p_{11} = 1 \Rightarrow \text{need } \alpha_1 > 1 \text{ and } \alpha_2 > p_{22} \]

- more active is one regime, more passive the other
  can be
  \[ p_{22} \to 1 \text{ OK if } \alpha_2 \approx 1 \text{ (but } < 1) \]
- ergodic prob of passive regime can be \( \approx 1 \) (but \( < 1 \))
  \[ p_{11} = p_{22} = 0 \text{ need } \alpha_2 > 1/\alpha_1 \]
- more active in one regime, less active in the other
**Determinacy Region: Fisherian Model**

- **Case 1:** $p_{11} = 0.95; p_{22} = 0.95$
  - Determinacy region in the $(\alpha_1, \alpha_2)$ plane.

- **Case 2:** $p_{11} = 0.8; p_{22} = 0.95$
  - Determinacy region in the $(\alpha_1, \alpha_2)$ plane.

- **Case 3:** $p_{11} = 0.95; p_{22} = 0$
  - Determinacy region in the $(\alpha_1, \alpha_2)$ plane.

- **Case 4:** $p_{11} = 0; p_{22} = 0$
  - Determinacy region in the $(\alpha_1, \alpha_2)$ plane.
FISHERIAN MODEL: SOLUTION

- Define state as \((r_t, s_t)\) & find MSV solutions

- posit regime-dependent rules:

\[\pi_t = a(s_t = i) r_t\]

\[a(s_t) = \begin{cases} 
  a_1 & \text{for } s_t = 1 \\
  a_2 & \text{for } s_t = 2
\end{cases}\]

- expectations functions:

\[E[\pi_{t+1} | s_t = 1, r_t] = [p_{11} a_1 + (1 - p_{11}) a_2] \rho r_t\]

\[E[\pi_{t+1} | s_t = 2, r_t] = [(1 - p_{22}) a_1 + p_{22} a_2] \rho r_t\]

- solve simple \(2 \times 2\) system to get \(a_1\) and \(a_2\)
Solutions are:

\[ a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

and

\[ a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right) \]

\[ p_{12} = 1 - p_{11}, \quad p_{21} = 1 - p_{22} \]

& “fixed-regime” coefficients

\[ a_i^F = \frac{1}{\alpha_i - \rho p_{ii}}, \quad i = 1, 2 \]

\[ \alpha_1 > \alpha_2 \iff a_1 < a_2 \]
### Expectations-Formation Effects

- Solutions are:

\[
a_1 = a_1^F \left( \frac{1 + \rho p_{12} a_2^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right)
\]

and

\[
a_2 = a_2^F \left( \frac{1 + \rho p_{21} a_1^F}{1 - \rho^2 p_{12} a_2^F p_{21} a_1^F} \right)
\]

- Expectations-formation effects from regime 2 to regime 1
  - through \( p_{12} a_2^F \)
  - large if \( p_{12} \) large, \( p_{22} \) large, \( \alpha_2 \) small
Special Case

- Real interest rate serially uncorrelated \((\rho = 0)\), solution is

\[
a_1 = \frac{1}{\alpha_1}
\]

and

\[
a_2 = \frac{1}{\alpha_2}
\]

- Looks like fixed-regime solution
  - determinacy in FR: \(\alpha_i > 1\) all \(i\)
  - switching allows determinacy w/ some \(\alpha_i < 1\)
  - if \(p_{22} < \alpha_2 < 1\), regime 2 amplifies shocks
  - possible to fit volatile data with determinate eqm?
A New-Keynesian Model

- Bare-bones model with nominal rigidities
  - from class in wide use for monetary policy analysis
  - general insights extend to more complex models now confronting data

- With recurring regime change and rational expectations:
  - How does the Taylor principle change?
  - How do impacts of demand and supply shocks change?

- Expectations-formation effects can be quantitatively large
A New-Keynesian Model

- Consumption-Euler equation and AS relations

\[ x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u^D_t \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u^S_t \]

- Disturbances: bounded, autoregressive, mutually uncorrelated

\[ u^D_t = \rho_D u^D_{t-1} + \varepsilon^D_t \]
\[ u^S_t = \rho_S u^S_{t-1} + \varepsilon^S_t \]

- A Taylor rule for \( s_t = 1, 2 \)

\[ i_t = \alpha(s_t)\pi_t + \gamma(s_t)x_t \]
NEW-KEYNESIAN MODEL: DETERMINACY

- Let \( \pi_{it} = \pi_t(s_t = i) \) & \( x_{it} = x_t(s_t = i) \), \( i = 1, 2 \)
- Define forecast errors
  \[
  \eta_{1t+1}^\pi = \pi_{1t+1} - E_t \pi_{1t+1} \\
  \eta_{1t+1}^x = x_{1t+1} - E_t x_{1t+1} \\
  \eta_{2t+1}^\pi = \pi_{2t+1} - E_t \pi_{2t+1} \\
  \eta_{2t+1}^x = x_{2t+1} - E_t x_{2t+1}
  \]
- Model is
  \[
  AY_t = BY_{t-1} + A\eta_t + Cu_t
  \]
- Unique bounded eqm requires the 4 generalized eigenvalues of \((B, A)\) to lie inside unit circle
- Derive long-run Taylor principle
Set $\gamma(s_t) = 0$

- Intertemporal margins interact with expected policy to affect determinacy
- Determinacy regions expand with parameters that reduce ability to substitute away from future policy
  - Increase degree of stickiness
  - Reduce intertemporal elasticity of substitution
INTRODUCTION
FISHERIAN MODEL
NEW-KEYNESIAN MODEL
APPLICATIONS
EMPIRICAL IMPLICATIONS
WRAP UP

DETERMINACY REGIONS EXPAND

\( p_{11} = 0.95, p_{22} = 0.95 \)

\( p_{11} = 0.8, p_{22} = 0.95 \)

\( p_{11} = 0.95, p_{22} = 0 \)

\( p_{11} = 0, p_{22} = 0 \)

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**Det. Regions & Private Parameters**

\[ p_{11} = 0.9, \ p_{22} = 0.9, \ \omega = 0.01 \]

\[ p_{11} = 0.9, \ p_{22} = 0.9, \ \omega = 0.99 \]

\[ p_{11} = 0.9, \ p_{22} = 0.9, \ \sigma = 0.01 \]

\[ p_{11} = 0.9, \ p_{22} = 0.9, \ \sigma = 10 \]
MSV solution is straightforward to compute
Easiest to consider numerical examples
For inflation, intuition from fixed regimes carries through
  more active MP process reduces inflation volatility
For output, switching introduces non-monotonicity
  more active MP process can raise or lower output volatility, depending on source of shock
A Return to the 1970s?

- Studies find Fed passive 1960-79; active since 1982
- Fears of reverting to 1970s behind calls for IT
- Fiscal policy may be an impetus for switching to passive MP
- Embed estimates of Lubik-Schorfheide in switching setup
  - compute set of \((p_{11}, p_{22})\) that deliver uniqueness
- Implications
  - inference that US switched from indeterminate to determinate eqm requires current state be absorbing
  - fixed regime badly mispredicts impacts of supply & demand shocks
Determinacy Regions: Lubik-Schorfheide Estimates

\[ \text{LS: } \alpha_1 = 2.19, \gamma_1 = 0.30, \alpha_2 = 0.89, \gamma_2 = 0.15 \]
Financial Crises & Business Cycles

- MP shifts focus from inflation to other concerns
  - financial stability & job creation
  - shift can last few months or more than year
  - during Greenspan era: 2 market crashes, 2 foreign financial crises, 2 jobless recoveries
  - documented by Marshall and Rabanal
- Take normal times to be $\alpha_1 = 1.5$, $\gamma_1 = 0.25$, and persistent
  - other regime: $\gamma_2 = 0.5$, $\alpha_2$ and $p_{22}$ vary
  - a crude characterization of those events
- Spillovers from demand shocks can make inflation much more volatile and output much less volatile than if the active regime were permanent
## Financial Crises & Business Cycles

<table>
<thead>
<tr>
<th>$p_{11} = .95$</th>
<th><strong>Demand</strong></th>
<th><strong>Supply</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inflation</td>
<td>Output</td>
</tr>
<tr>
<td>$p_{22} = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_2 = .25$</td>
<td>1.060</td>
<td>1.011</td>
</tr>
<tr>
<td>$\alpha_2 = 0$</td>
<td>1.073</td>
<td>1.014</td>
</tr>
<tr>
<td>$p_{22} = .75$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_2 = .25$</td>
<td>1.268</td>
<td>.886</td>
</tr>
<tr>
<td>$\alpha_2 = 0$</td>
<td>1.454</td>
<td>.807</td>
</tr>
</tbody>
</table>

### Standard Deviation Active Regime Relative to Fixed Regime

Active and fixed regimes set $\alpha_1 = \alpha = 1.5, \gamma_1 = \gamma = .25; \gamma_2 = .5$
Empirical Implications of Switching

- Commonplace for empirical work to split data into regime-dependent sub-periods
- Estimates then interpreted in fixed-regime theoretical model
- We simulate switching eqm, estimate correctly-specified (fixed-regime) identified VARs
  - assume econometrician knows when regime changed
- Estimated model

\[
\begin{align*}
    x_t &= \delta i_t + u_t^D + \text{lags} \\
    \pi_t &= \theta x_t + u_t^S + \text{lags} \\
    i_t &= \alpha \pi_t + \gamma x_t + w_t^{MP} + \text{lags}
\end{align*}
\]
Empirical Implications of Switching

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\bar{\gamma}$</th>
<th>$\delta$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>2.182</td>
<td>0.30</td>
<td>-1.690</td>
<td>0.409</td>
</tr>
<tr>
<td>Regime 2</td>
<td>0.885</td>
<td>0.15</td>
<td>-0.750</td>
<td>1.675</td>
</tr>
<tr>
<td>Full Sample</td>
<td>1.375</td>
<td>0.225</td>
<td>-1.476</td>
<td>0.657</td>
</tr>
</tbody>
</table>

Estimates from an identified VAR using simulated data.
Regime 1 is conditional on remaining in regime with $\alpha_1 = 2.19$.
Regime 2 is conditional on remaining in regime with $\alpha_2 = 0.89$.
Full sample is recurring changes from regime 1 to regime 2.
$\alpha$ is the estimated response of monetary policy to inflation.
$\bar{\gamma}$ is the policy response to output, held fixed in estimation.
Demand & Supply Shocks: Lubik-Schorfheide Parameters

\[ \alpha_1 = 2.19, \gamma_1 = 0.30, \alpha_2 = 0.89, \gamma_2 = 0.15, p_{11} = 0.95, p_{22} = 0.93 \]
SUMMARY

- A broader perspective on Taylor principle and range of unique bounded equilibria it supports
- Endowing conventional models with empirically relevant MP switching processes
  - drastically alters conditions for a unique bounded eqm
  - generates important expectations-formation effects
- Paper develops a two-step solution method to get determinacy conditions and solutions
- Conventional models extremely sensitive to deviation from usual assumption that policy is permanent
- The possibility of regime change should be the default assumption in theoretical models
WHERE WE’RE HEADED

- Expand the class of models w/ switching that can be solved analytically
  - endogenous state variables
    - neoclassical growth model
    - models of fiscal policy w/ debt
  - treat state as unobserved
    - agents face inference problem
    - increase persistence
    - connects w/ econometric literature
- Turn to estimation
  - does regime change fit data?
  - does it help explain specific time series facts?
  - revisit Lubik-Schorfheide w/ switching