Lecture 3. Policy Interactions with Tax Distortions

Eric M. Leeper
Indiana University
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THE MESSAGES

- Will study two models with distorting taxes
- First draws on Gordon-Leeper (2005,2006): growth model w/ transactions demand for money
- Second draws on Leeper-Yun (2006): provides micro foundations for FTPL
- Once models completely solved out, can understand price-level determination more deeply
- Emphasizes the role of asset substitution, which is absent from simple models
- Gets us away from the fiscal theory story about wealth effects
- Useful models to keep in your head: “roll your own policies”
- Characterize eqm as function of general sequences of policy variables
**FIRST MODEL**

- Growth model w/ capital, money, nominal government debt
  - arbitrages among assets determine their relative demands
  - returns to real balance holdings and after-tax returns to capital determine the relative values of real and nominal assets
  - expected macro policies determine expected returns on real and nominal assets
  - so price level depends on interactions among current and expected future MP & FP
- Quantity theory and fiscal theory emerge as special cases
- QT & FT employ common money demand

\[
\frac{M^d}{P} = h(i, y)
\]

- how can this be?
THE MODEL

• We exploit the analytic convenience that comes with log prefs, C-D technology, complete depreciation of capital
  • none of the general points depend on these simplifying assumptions

• Aggregate resource constraint

\[ c_t + k_t + g_t = f(k_{t-1}) \]

• Goods producing firm rents \( k \) at rental rate \( r \) and pays taxes levied against sales of goods to solve

\[ \max_{k_{t-1}} D_{Gt} = (1 - \tau_t)f(k_{t-1}) - r_t k_{t-1} \]

• Transactions services producing firm hires labor \( l \) at wage rate \( w \) to solve

\[ \max_{l_t} D_{Tt} = P_{Tt} T(l_t) - w_t l_t \]
THE MODEL

- Household owns firms and pays taxes on capital income
- HH has income

\[ I_t = r_t k_{t-1} + D_{Gt} + w_t l_t + D_{Tt} + z_t \]

where \( z_t \geq 0 \) is lump-sum transfers from the government

- HH’s expenditures on \( c \) & \( k \) must be financed with real money balances, \( M_{t-1}/P_t \), or with transactions services, \( T_t \), to satisfy the constraint

\[ \frac{M_{t-1}}{P_t} + T_t(c_t + k_t) \geq c_t + k_t \]

\( T_t \) gives fraction of expenditures financed w/ transactions services
The Model

- HH’s problem

$$\max_{\{c_t, l_t, T_t, M_t, B_t, k_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - l_t), \quad 0 < \beta < 1$$

where $1 - l_t$ is leisure, subject to the finance constraint, the budget constraint

$$c_t + k_t + \frac{M_t + B_t}{P_t} + P_{T_tT_t} \leq I_t + \frac{M_{t-1} + R_{t-1}B_{t-1}}{P_t}$$

and $0 \leq l_t \leq 1$

- Future government policy is the sole source of uncertainty; the operator $E$ denotes equilibrium expectations of private agents over future policy
The Model

- The government finances expenditures on goods, $g_t$, and transfer payments, $z_t$, by levying taxes, issuing new debt, and creating new money to satisfy:

\[ \tau_t f(k_{t-1}) + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t + R_{t-1}B_{t-1}}{P_t} = g_t + z_t \]

- Assume the following functional forms:

\[ f(k_{t-1}) = k_{t-1}^\sigma, \quad 0 < \sigma < 1 \]

\[ T(l_t) = 1 - (1 - l_t)^\alpha, \quad \alpha > 1 \]

\[ U(c_t, 1 - l_t) = \log(c_t) + \gamma \log(1 - l_t), \quad \gamma > 0 \]
SOLVING THE MODEL

- State at $t$ depends on assets and expectations of macro policies

- denote state by
  
  $z_t = (k_{t-1}, M_{t-1}, (1 + i_{t-1})B_{t-1}, \{E_t \rho_j, E_t \tau_j, E_t s_{j}^{g}\}_{j=t}^\infty)$

- $\rho_t = M_t/M_{t-1}$, $s_{t}^{g} = g_t/f(k_{t-1})$
- emphasizes that a complete specification of policy must allow agents to form expectations over infinite future of policies
Solving the Model

- First-order conditions
  - firms' \[ 1 + r_t = \sigma(1 - \tau_t)k_{t-1}^{\sigma - 1}, \quad w_t = \alpha(1 - l_t)^{\alpha - 1}P_{Tt} \]

- household's
  \[ \varphi_t + \lambda_t = \frac{1}{c_t} + \lambda_t T_t^d \]
  \[ \frac{\gamma}{1 - l_t} = w_t \varphi_t \]
  \[ \varphi_t P_{Tt} = \lambda_t(c_t + k_t) \]
  \[ \frac{\varphi_t}{P_t} = \beta E_t \left[ \frac{\varphi_{t+1} + \lambda_{t+1}}{P_{t+1}} \right] \]
  \[ \frac{\varphi_t}{P_t} = \beta(1 + i_t)E_t \left[ \frac{\varphi_{t+1}}{P_{t+1}} \right] \]
  \[ \varphi_t + \lambda_t = \lambda_t T_t^d + \beta E_t(1 + r_{t+1})\varphi_{t+1} \]
**EQUILIBRIUM**

- Characterize eqm in terms of policy expectations functions \((\mu_t, \eta_t)\), government claims to goods, \(s_t^g\), and assets, \((k_{t-1}, M_{t-1}, (1 + i_{t-1})B_{t-1})\)

- Solution maps policy expectations into portfolio choices
  - of course, policy expectations are restricted to policy paths that are consistent with eqm

- \(\eta\) and \(\mu\) capture portfolio balance effects of policies
  - \(\eta\) measures direct tax distortion on investment & extent to which gov’t expends are financed by taxing output
  - \(\mu\) reflects expected inflation & the expected return on nominal assets

- Assume (similar to “Monetary Doctrines”)

\[
\begin{align*}
\rho_{t+j} &= \rho_F, \forall j > 0 \\
\tau_{t+j} &= \tau_F, \forall j > 0 \\
s_{t+j}^g &= s_F^g, \forall j > 0
\end{align*}
\]
EQUILIBRIUM

• Two dynamical equations to solve: real asset & nominal assets

• $k$ Euler equation in terms of $s_t = k_t / (c_t + k_t)$ yields

\[
\frac{1}{1 - s_t} = \sigma \beta E_t \left[ \frac{1 - \tau_{t+1}}{1 - s_{t+1}^g} \left( \frac{1}{1 - s_{t+1}} \right) \right] + E_t \left[ 1 - \sigma \beta \frac{\gamma}{\alpha} \frac{1 - \tau_{t+1}}{1 - s_{t+1}^g} \right]
\]

whose solution is

\[
\frac{1}{1 - s_t} = \eta_t
\]

where

\[
\eta_t \equiv E_t \sum_{i=0}^{\infty} (\sigma \beta)^i d_i \left[ 1 - \sigma \beta \frac{\gamma}{\alpha} \frac{1 - \tau_{t+i+1}}{1 - s_{t+i+1}^g} \right]
\]

\[
d_i = \prod_{j=0}^{i-1} \left( \frac{1 - \tau_{t+j+1}}{1 - s_{t+i+1}^g} \right), \quad d_0 = 1
\]
EQUILIBRIUM

• Euler equation for $M$ yields d.q. in velocity, $1 - T_t$

$$(1 - T_t) \left[ \frac{1}{1 - s_t} - \frac{\gamma}{\alpha} \right] = \beta \frac{1}{\rho_t} E_t \left\{ (1 - T_{t+1}) \left[ \frac{1}{1 - s_{t+1}} - \frac{\gamma}{\alpha} \right] + \frac{\gamma}{\alpha} \right\}$$

whose solution is

$$(1 - T_t) \left[ \frac{1}{1 - s_t} - \frac{\gamma}{\alpha} \right] = \frac{\mu_t}{\rho_t}$$

where

$$\mu_t \equiv \beta \frac{\gamma}{\alpha} E_t \sum_{i=0}^{\infty} \beta^i d_i^\mu, \quad d_i^\mu \equiv \prod_{j=0}^{i-1} \frac{1}{\rho_t+j+1}, \quad d_0^\mu = 1$$
**EQUILIBRIUM**

- Imposing the stationary policy assumptions yields the policy expectations functions

\[
\eta_t(\tau_F, s_F^g) = \frac{1 - \sigma \beta \gamma}{1 - \sigma \beta \alpha} \left( \frac{1 - \tau_F}{1 - s_F^g} \right)
\]

\[
\mu_t(\rho_F) = \frac{\beta \gamma}{1 - \beta / \rho_F}
\]

- Eqm capital stock is

\[
k_t = \left( 1 - \frac{1}{\eta_t} \right) (1 - s_t^g) f(k_{t-1})
\]

- Eqm real money balances are

\[
\frac{M_t}{P_t} = \left( \frac{\mu_t}{\eta_t - \gamma / \alpha} \right) (1 - s_t^g) f(k_{t-1})
\]
Price-Level Determination

- Can think of price level being determined “through eqm real balances”

\[
\frac{M_t}{P_t} = \Delta_t (1 - s_t^g) f(k_{t-1})
\]

where

\[
\Delta_t = \frac{\mu_t}{\eta_t - \gamma/\alpha}
\]

with

\[
\Delta_t((-), (+), (-)) = \frac{\beta \gamma}{\alpha} \left[ \frac{1 - \sigma \beta \left( \frac{1 - \tau_F}{1 - s_F^g} \right)}{1 - \gamma/\alpha} \right]
\]

- \(1/\Delta_t\) is velocity; it gives the value of nominal assets
- \(\Delta_t\) depends on expected MP & FP
THE ROLE OF POLICY EXPECTATIONS

- $\mu$ and $\eta$ capture 3 distinct influences of expectations on $P$

  1. $\mu$: the marginal value of real money balances; higher expected money growth lowers $\mu$ and induces substitution away from money, raising $P$

  2. $\eta$: direct tax distortion that alters return on investment; higher expected taxes reduce return on investment and induces substitution away from $k$ into $c$ and into $M$ (Tobin effect), raising money demand and lowering $P$

  3. $\eta$ summarizes composition of expected fiscal financing; higher $\eta$ reflects increase in expected nominal liability creation & reduction in relative role of real taxation

To see (3), note terms $(1 - \tau)/(1 - s^g)$ in $\eta$ and write gbc as

$$\frac{1 - \tau_t}{1 - s^g_t} = 1 + \frac{(M_t - M_{t-1} + B_t - (1 + i_{t-1})B_{t-1})/P_t}{(1 - s^g_t)f(k_{t-1})}$$
JOINTLY CONSISTENT (EQUILIBRIUM) POLICIES

- Dynamic interactions among policies
  - current policies constrain future policy options
  - expected fiscal financing constrains current policies
  - expected policies affect $P_t$ & real value of gov’t liabilities

- How do jointly consistent combinations of current & future policies affect $P$?
  1. Which policies are consistent with eqm given current expectations $(\mu & \eta)$?
  2. How do current policy changes affect the set of future policies that are consistent with eqm?
JOINTLY CONSISTENT POLICIES

1. Which policies are consistent with eqm given current expectations \((\mu \& \eta)\)?

2. How do current policy changes affect the set of future policies that are consistent with eqm?

- Eqm government b.c. at \(t\)

\[
\left[ \frac{\rho_t - 1}{\rho_t} + \left( \frac{B}{M} \right)_t - \frac{1 + \delta_{t-1}}{\rho_t} \left( \frac{B}{M} \right)_{t-1} \right] \Delta_t = \frac{s^g_t - \tau_t}{1 - s^g_t}
\]

where \((B/M)_s \equiv B_s/M_s\) and \(\Delta_t\) summarizes given expected policies

- Eqm government b.c. in future

\[
\Delta_t = \left( \frac{s^g_F - \tau_F}{1 - s^g_F} \right) \left[ \frac{1}{(B/M)_F - \frac{1}{\beta} (B/M)_t + \left( \frac{\rho_F - 1}{\rho_F} \right)} \right]
\]
\[ \frac{M_t}{P_t} = \beta \frac{\gamma}{\alpha} \left( \frac{1 + i_t}{i_t} \right) \frac{1}{\eta_t - \gamma/\alpha} (1 - s_t^g) f(k_{t-1}) \]

- In general, both MP and FP affect \( P \)
- When is \( P \) determined by MP alone?
- Under policy assumptions that dichotomize real & nominal sides
- Balanced net-of-interest surplus: \( \tau_t = s_t^g \) all \( t \)
- Now \( \eta_t = (1 - \sigma \beta \gamma / \alpha) / (1 - \sigma \beta) \) and \( M^d \) is
  \[ \frac{M_t}{P_t} = h(i_t, c_t + k_t) \]

- \( P \) independent of FP but \textit{not of debt}
  - money growth must finance interest obligations
  - higher \( B \) \( \Rightarrow \) higher debt service \( \Rightarrow \) higher \( P \) & \( \pi \)
- In general, cannot rid \( M/P \) of \( \eta \)
Open-market sale of $B_t$, holding $M_t + B_t$ fixed

Fix $(s^g_t, s^g_F)$ and $\tau_t$

B/c $B_t \uparrow$, some future policy must adjust—either $\tau_F$ or $\rho_F$

1. Suppose $\tau_F \uparrow$: $\eta_t \downarrow$, $k_t \downarrow$, $P_t \downarrow$ (but future $P \uparrow$)
2. Suppose $\rho_F \uparrow$: $\mu_t \downarrow$, tend to make $P_t \uparrow$ (but future $P \uparrow$)

But $M_t \downarrow$, so ultimate effect on $P_t$ can go either way, depending on $B/M$

Monetary policy is constrained by the government’s fiscal obligations

- works through seigniorage
Cannonical FTPL

- Bond-financed tax cut: $\tau_t \downarrow, B_t \uparrow$
- Fix $(\rho_F, \tau_F, {s_g}_F)$ and $s_t^g$
- B/c $B_t$ rises, if $M_t$ unchanged, $(B/M)_t$ rises and some future policy must adjust
- By ass’n no future policy can adjust
- Only eqm policy is for $M_t$ to rise in proportion to the $B_t$ increase so that $(B/M)_t$ unchanged
- Required increase in $M_t$ is exactly enough so increase in future seigniorage (b/c the level of money supplied is now higher) suffices to service higher debt
- The fixed policies peg $i_t$ and $M_t/P_t$, so $P_t \uparrow$
- Monetary policy is constrained by the government’s fiscal obligations
  - works through nominal asset revaluation
Pure Fiscal Effects

- FP can affect $P$ independently of MP
- Consider a debt-financed tax cut to which future taxes adjust
- Fix $(\rho_t, \rho_F, s^g_t, s^g_F)$
- Lower $\tau_t$ & higher $(B/M)_t$ $\Rightarrow$ higher $\tau_F$
- Lower return on capital induces substitution away from real assets toward nominal assets
- With $M_t$ fixed, $P_t$ falls
- This Tobin effect gives debt a natural role in determining $P$
- Quite non-Keynesian: current fiscal expansion reduces nominal demand and price level
- Note that even though money growth is unchanged, because $M/P$ rises, seigniorage revenues rise
SECOND MODEL

- Seek to provide micro foundations for the FTPL
- Elastic labor supply; fixed capital stock
- Proportional tax levied against labor income has both “supply” and “demand” effects
- FTPL typically focuses only on “demand” effects
- Complete contingent claims, fiat currency, nominal government debt
- CRS production in labor
- Derive effects of tax policies on balance sheets of HHs
THE MODEL

• Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)] \]

• HH budget constraint

\[ c_t + m_t + E_t \left[ Q_{t,t+1} \frac{B_{t,t+1}}{P_t} \right] \leq (1 - \tau_t)(w_t h_t + \Phi_t) + \frac{B_{t-1,t} + M_{t-1}}{P_t} \]

\( Q_{t,t+1} \) is stochastic discount factor (nominal value at \( t \) of $1 at \( t + 1 \); \( \Phi_t \) is real dividends

\( E_t \left[ Q_{t,t+1} \frac{B_{t,t+1}}{P_t} \right] \) is real value at \( t \) of nominal contingent claims

\[ 1 + i_t = \frac{1}{E_t[Q_{t,t+1}]} \]

\[ Q_{t,t+1} = q_{t,t+1} \frac{P_t}{P_{t+1}} \]
THE MODEL

• Rewrite the HH’s flow b.c. as

\[ c_t + \frac{i_t}{1 + i_t} m_t + E_t[q_{t+1} a_{t+1}] \leq (1 - \tau_t)(w_t h_t + \Phi_t) + a_t \]

\[ a_t = \frac{B_{t-1} + M_{t-1}}{P_t}, \quad \text{value of nominal assets} \]

• HH’s present-value b.c. is

\[ E_0 \sum_{t=0}^{\infty} q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t - (1 - \tau_t)(w_t h_t + \Phi_t) \right] \leq a_0 \]

with \( \lim_{t \to \infty} E_0[q_t a_t] = 0 \)
The Model

- First-order conditions

\[
\beta^t u_c(c_t, m_t) = \lambda q_t
\]

\[
\beta^t u_m(c_t, m_t) = \lambda q_t \left( \frac{i_t}{1 + i_t} \right)
\]

where \( \lambda = u_c(c_0, m_0) \)

\[
\frac{v'(1 - h_t)}{u_c(c_t, m_t)} = (1 - \tau_t) w_t
\]

- Use these in PV b.c.

\[
E_0 \sum_{t=0}^{\infty} \beta^t [u_c(c_t, m_t)c_t + u_m(c_t, m_t)m_t - (1 - \tau_t)y_t u_c(c_t, m_t)]
\]

\[
\frac{u_c(c_0, m_0)}{u_c(c_0, m_0)} = a_0
\]

- Note: LHS entirely in terms of allocations
- When allocations are unique, have a unique real value of nominal assets, \( a_0 = \frac{B_{-1,0} + M_{-1}}{P_0} \)
EQUILIBRIUM

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ u_c(c_t, m_t)c_t + u_m(c_t, m_t)m_t - (1 - \tau_t)y_t u_c(c_t, m_t) \right] = a_0
\]

\[
u_c(c_0, m_0)
\]

- Under rational expect, HH knows \(a_0\) when it optimizes
- This is an eqm balance sheet relation, where LHS is PV of HH’s assets at time 0
- Get cond in policy variables, subst \(y_t = c_t + g_t\) in relation

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_c(c_t, m_t)}{u_c(c_0, m_0)} \left[ (\tau ty_t - g_t) + \frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} m_t \right] = a_0
\]

Noting that \(q_t = \beta^t \frac{u_c(c_t, m_t)}{u_c(c_0, m_0)}\)

\[
E_0 \sum_{t=0}^{\infty} q_t \left[ (\tau ty_t - g_t) + \frac{i_t}{1 + i_t} m_t \right] = a_0
\]

An equilibrium condition!
A Fiscal Theory Equilibrium

\[ E_0 \sum_{t=0}^{\infty} q_t \left[(\tau_t y_t - g_t) + \frac{i_t}{1 + i_t} m_t\right] = a_0 \]

- Suppose: \( \tau_t y_t \) are lump-sum tax revenues; \( y \) & \( g \) exogenous, then \( q \) given; let MP peg \( i_t = \bar{i} \Rightarrow m_t = h(y_t) \) independent of MP & FP; let FP set \( \{\tau_t\} \) exogenously
- Under these ass’ns, LHS a number, call it \( PV S \), so

\[ P_0 = \frac{B_{-1,0} + M_{-1}}{PV S} \]

- At \( t = 0 \), \( B_{-1,0} + M_{-1} \) given, so this determines \( P_0 \)
- Can think of \( 1/PV S \) as price of nominal assets, which plays the same role as \( 1/\Delta \) in earlier model
POLICY EXPERIMENTS

1. Expect lower $\tau_{t+k} \Rightarrow PV S \downarrow \Rightarrow P_0 \uparrow$
2. Reduce current $\tau_0 \Rightarrow PV S \downarrow \Rightarrow P_0 \uparrow$
3. Expect lower $\bar{i}_m \Rightarrow PV S \downarrow \Rightarrow P_0 \uparrow$

- (3) seems perverse relative to standard theory
  - lower expected seigniorage iff lower $\bar{i} \Rightarrow$ lower $\pi^e$ in most monetary models $\Rightarrow$ higher expected return to $M \Rightarrow M^d \uparrow \Rightarrow P_0 \downarrow$
  - what’s going on?
  - in standard models, the ubiquitous eqm condition is present but it doesn’t restrict the nature of the eqm b/c it is assumed that taxes adjust to alter the $PV S$ for any given $P_0$
  - in FTPL, lower $\bar{i}_m \Rightarrow$ less “backing” for nominal assets, so nominal assets are worth less, meaning $1/P_0 \downarrow$

- Whether the ubiquitous eqm condition should be treated as a constraint or an eqm condition is at the heart of Buiter’s critique of the FTPL
A Price-Theoretic View of the FTPL

- Follow public finance to extend Slutsky-Hicks decomposition to include a third effect
- FT works through a type of wealth effect that arises when $\Delta P$ revalues nominal assets in HH portfolios
- Decompose impacts of tax change as
  - total effect = substitution effect + wealth effect + revaluation effect
- Let $y^F_t$ be Becker’s “full income” (dividend income + maximum labor income if HH works entire time endowment—1 unit)
  \[ y^F_t = (1 - \tau_t)w_t \cdot 1 + \Phi_t \]
- HH takes $y^F_t$ as given and from it purchases consumption, real balances, leisure
A Price-Theoretic View of the FTPL

- HH flow b.c.

\[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau_t)w_t(1 - h_t) + E_t[q_{t,t+1}a_{t+1}] \leq y^F_t + a_t \]

- HH present value b.c.

\[ E_0 \sum_{t=0}^{\infty} q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau_t)w_t(1 - h_t) \right] \leq a_0 + v_0 \]

with \( \lim_{t \to \infty} E_0[q_t a_t] = 0 \), \( \lim_{t \to \infty} E_0[q_t y^F_t] = 0 \)

- \( v_0 \) is expected PV of full income flows, \( v_0 = E_0 \sum_{t=0}^{\infty} [q_t y^F_t] \)

- HH takes both \( a_0 \) and \( v_0 \) parametrically
A Price-Theoretic View of the FTPL

- Lagrange multiplier on the PV b.c. is \( \lambda = \frac{e_0}{a_0 + v_0} \)

\[
e_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ u_c(c_t, m_t)c_t + u_m(c_t, m_t)m_t + v'(1 - h_t)(1 - h_t) \right]
\]

- \( e_0 \) is expected PV of expenditures (including leisure)
- \( \lambda \) is shadow price of wealth
  - wealth rises \((a_0 + v_0 \uparrow) \Rightarrow \lambda \downarrow\)
  - expenditures rise \((e_0 \uparrow) \Rightarrow \lambda \uparrow\)
- Demand functions

\[
c_t = c \left( \frac{q_t}{\beta^t}, \frac{i_t}{1 + i_t}, \frac{a_0 + v_0}{e_0} \right)
\]

\[
m_t = m \left( \frac{q_t}{\beta^t}, \frac{i_t}{1 + i_t}, \frac{a_0 + v_0}{e_0} \right)
\]

\[
h_t = h \left( (1 - \tau_t)w_t, \frac{q_t}{\beta^t}, \frac{i_t}{1 + i_t}, \frac{a_0 + v_0}{e_0} \right)
\]
A Price-Theoretic View of the FTPL

- Conventional wealth effect vs. revaluation effect
  - suppose \( B_{-1,0} + M_{-1} = 0 \)
  - revaluation effect is zero \( (a_0 = 0) \)
  - conventional wealth effect still operates through \( v_0 \) & \( e_0 \)
- Of course, taxes can affect \( P_0 \) even if FTPL not operative
  - suppose \( \tau_0 \uparrow \)
  - substitution effect reduces labor supply
  - wealth effect raises labor supply
  - final impact depends on relative sizes
  - but then the resulting \( \Delta P_0 \) and \( \Delta a_0 \) imposes restrictions on \( \{\tau_t\}_{t=1}^{\infty} \) necessary for eqm
SUBSTITUTION, WEALTH & REVALUATION

• Suppose \( \{\tau^*_t\}_{t=0}^{\infty} \) changes to \( \{\tau^+_t\}_{t=0}^{\infty} \)

• Problem (*)

\[
\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)]
\]

s.t. \[
E_0 \sum_{t=0}^{\infty} q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau^*_t)w_t(1 - h_t) \right] \leq a_0 + v_0
\]

yields \( \{c^*_t, m^*_t, h^*_t, a^*_t, v^*_t, e^*_t, P^*_t, q^*_t, R^*_t, \Phi^*_t\}_{t=0}^{\infty} \)

• Problem (†)

\[
\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)]
\]

s.t. \[
E_0 \sum_{t=0}^{\infty} q_t \left[ c_t + \frac{i_t}{1 + i_t} m_t + (1 - \tau^+_t)w_t(1 - h_t) \right] \leq a_0 + v_0
\]

yields \( \{c^+_t, m^+_t, h^+_t, a^+_t, v^+_t, e^+_t, P^+_t, q^+_t, R^+_t, \Phi^+_t\}_{t=0}^{\infty} \)
**Substitution Effect**

- Set lump-sum transfers, $T^s_0$, so HH can achieve same level of utility it would have obtained under the (*) tax even though it optimizes under the (†) tax

- Problem (Substitution)

\[
\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)]
\]

s.t.

\[
E_0 \sum_{t=0}^{\infty} q^\dagger_t \left[ c_t + \frac{i^\dagger_t}{1 + i^\dagger_t} m_t + (1 - \tau^\dagger_t)w^\dagger_t(1 - h_t) \right] \leq a^\dagger_0 + v^\dagger_0 + T^s_0
\]

- constraining prices to be eqm prices under (†) tax ⇒ budget line of this problem tangent to HH’s indifference surface under (†) tax
A HICKSIAN DECOMPOSITION

- Problem (No Revaluation)

$$\max_{\{c_t, m_t, h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, m_t) + v(1 - h_t)]$$

s.t. $$E_0 \sum_{t=0}^{\infty} q_t \left[ c_t + \frac{i_t^{\dagger}}{1 + i_t^{\dagger}} m_t + (1 - \tau_t^{\dagger}) w_t^{\dagger} (1 - h_t) \right] \leq a_0^{\ast} + v_0^{\dagger}$$

- HH assumes revaluation does not result from the tax change, so assets have value $$a_0 = a_0^{\ast}$$ under (\dagger) tax
A Hicksian Decomposition

- Set lump-sum transfers, $T^w_0$, so HH can achieve the same level of utility it would have obtained under the ($\dagger$) tax, with and without asset revaluation
- Problem (Revaluation)

$$\max_{\{c_t, m_t, h_t\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta^t [u(c_t, m_t) + v(1 - h_t)]$$

s.t. $$E_0 \sum_{t=0}^\infty q^\dagger_t \left[ c_t + \frac{i^\dagger_t}{1 + i^\dagger_t} m_t + (1 - \tau^\dagger_t)w^\dagger_t (1 - h_t) \right] \leq a^\dagger_0 + v^\dagger_0 + T^w_0$$

- $T^w_0$ permits Problem (No Revaluation) and Problem (Revaluation) to achieve the same level of utility
- Total Effect: Problem ($\ast$) vs. Problem ($\dagger$)
- Substitution Effect: Problem ($\ast$) vs. Problem (Substitution)
- Revaluation Effect: Problem (No Revaluation) vs. Problem (Revaluation)
- Wealth Effect = Total − Substitution − Revaluation
A HICKSIAN DECOMPOSITION

- Solutions to optimization problems are cons demands

\[ (*) : c_t^* = c \left( \frac{q_t^*}{\beta t}, \frac{i_t^*}{1 + i_t^*}, \frac{a_0^* + v_0^*}{e_0^*} \right) \]

\[ (†) : c_t^† = c \left( \frac{q_t^†}{\beta t}, \frac{i_t^†}{1 + i_t^†}, \frac{a_0^† + v_0^†}{e_0^†} \right) \]

(Substitution) : \[ c_t^† \mid u_0 = u_0^* = c \left( \frac{q_t^†}{\beta t}, \frac{i_t^†}{1 + i_t^†}, \frac{a_0^* + v_0^* + T_0^s}{e_0^† \mid u_0 = u_0^*} \right) \]

(Revaluation) : \[ c_t^† \mid u_0 = u_0^†, a_0 = a_0^* = c \left( \frac{q_t^†}{\beta t}, \frac{i_t^†}{1 + i_t^†}, \frac{a_0^* + v_0^† + T_0^w}{e_0^† \mid u_0 = u_0^*, a_0 = a_0^*} \right) \]

- \[ c_t^† \mid u_0 = u_0^* \] : planned consumption under (†) tax, with utility at \( u_0^* \), the level under (*) tax

- \[ c_t^† \mid u_0 = u_0^†, a_0 = a_0^* \] : planned consumption without revaluation under (†) tax with utility at \( u_0^† \), the level under (†) tax
A Hicksian Decomposition

- The full decomposition

\[
\log \left( \frac{c_t^*}{c_t^\dagger} \right) = \log \left( \frac{c_t^\dagger | u_0 = u_0^*}{c_t^*} \right) \\
\text{total effect}
\]

\[
+ \log \left( \frac{c_t^\dagger | u_0 = u_0^*, a_0 = a_0^*}{c_t^\dagger | u_0 = u_0^*} \right) \\
\text{substitution effect}
\]

\[
+ \log \left( \frac{c_t^\dagger | u_0 = u_0^*, a_0 = a_0^*}{c_t^\dagger | u_0 = u_0^*} \right) \\
\text{wealth effect}
\]

\[
+ \log \left( \frac{c_t^\dagger | u_0 = u_0^*, a_0 = a_0^*}{c_t^\dagger | u_0 = u_0^*} \right) \\
\text{revaluation effect}
\]
**An Example Economy**

- Assume log preferences

\[ u(c, m) + v(1 - h) = \log c + \log m + \log(1 - h) \]

Then

\[ e_0 = \frac{3}{1 - \beta} \]

\[ c_t = \left( \frac{1 - \beta}{3} \right) \left( \frac{\beta^t}{q_t} \right) (a_0 + v_0) \]

\[ m_t = \left( \frac{1 - \beta}{3} \right) \left( \frac{\beta^t}{q_t} \right) \left( \frac{1 + i_t}{i_t} \right) (a_0 + v_0) \]

\[ h_t = 1 - \left( \frac{1 - \beta}{3} \right) \left( \frac{\beta^t}{(1 - \tau_t)w_tq_t} \right) (a_0 + v_0) \]

- Then can compute all the objects in the decomposition
- Assume MP pegs \( i_t \) to satisfy: \( \beta(1 + i_t) = 1, \quad t \geq 0 \)
- See Leeper-Yun (2006) for details and case of lump-sum
AN EXAMPLE ECONOMY

- Income taxes set $\tau_t > 0$

\[
y_t = \frac{1 - \tau_t}{2 - s^g - \tau_t}
\]

\[
q_t = \beta^t \left( \frac{1 - \tau_0}{1 - \tau_t} \right) \frac{(2 - s^g - \tau_t)}{(2 - s^g - \tau_0)}
\]

- Present value full income flows is

\[
v_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(1 - \tau_0)(2 - s^g - \tau_t)}{2 - s^g - \tau_0} \right]
\]

- HH present value b.c.

\[
a_0 + v_0 = \frac{3}{1 - \beta} \frac{(1 - \tau_0)(1 - s^g)}{2 - \tau_0 - s^g}
\]

- A Laffer curve in $\tau_t y_t$ with revenues maximized at

\[
\bar{\tau} = 2 - s^g - \sqrt{(2 - s^g)(1 - s^g)}
\]
An Example Economy

• Suppose taxes constant at $\tau$
• $y$ & $c$ constant; $i$ pegged; $q_t = \beta^t$

\[ v_0 = \frac{1 - \tau}{1 - \beta}, \quad a_0 = \frac{1 - \beta}{1 - \tau} \frac{2 - s^g - \tau}{1 - 2s^g + \tau} \]

• Equilibrium price level

\[ P_0 = \frac{1 - \beta}{1 - \tau} \frac{2 - s^g - \tau}{1 - 2s^g + \tau} (B_{-1} + M_{-1}) \]

• Note from $a_0 = \frac{1 - \beta}{1 - \tau} \frac{2 - s^g - \tau}{1 - 2s^g + \tau}$, quadratic in $\tau$ \implies Laffer curve in sum of PV surpluses + seigniorage
• Laffer curves in $\tau_t y_t$ and in $a_0$ can look very different
CONVENTIONAL LAFFER CURVE

- Income Tax Rate ($\tau$)
- Units of Goods
- Direct tax revenue flow
- Tax base (real GDP)

$\tau = 58.6\%$
**Fiscal Theory Laffer Curve**

Income Tax Rate ($\tau$) vs. Units of Goods

- Direct tax revenue flow
- Tax base (real GDP)
- Primary surplus + seigniorage (present value)

$\tau = 26.8\%$

$\tau = 58.6\%$
**Two Laffer Curves**

- Why are these different?
- Tax bases differ
  - conventional: $\tau_t y_t$
  - fiscal theory: $PV \left( \tau_t y_t + \frac{i_t}{1+i_t} m_t \right)$
  - changes in conventional tax base, $y_t$, feed into $m_t$ and the seigniorage tax base
- Should we care about this?
  - presents tradeoffs
  - relevant for inflation-targeting countries to think about the fiscal consequences of MP
Wrap Up

- Fiscal theory has been accused of being “incoherent,” “inconsistent with economic theory,” and worse.
- This shows that with the right kind of price-theoretic analysis, the revaluation effect that lies at the heart of the FTPL can be understood as a natural extension in an environment with nominal assets of standard the Slutsky-Hicks decomposition.
- Critics have also accused FTPL of ignoring the government’s budget constraint.
- Here we have shown that you can get an eqm condition that determines $P_0$ without any reference to government variables.
- Introducing distorting taxes to a FTPL analysis reveals a second kind of Laffer curve that has been largely overlooked.