ZEI Summer School 2008

Lecture 4. Generalizing Policy Interactions (B)

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THE MESSAGES

• Draws heavily from “Fluctuating Macro Policies and the Fiscal Theory” with Troy Davig (NBER Macroeconomics Annual, 2006) and “Monetary and Fiscal Policy Switching” with Hess Chung and Troy Davig (JMCB, June 2007)
• Difficult to obtain general analytical results with both monetary and fiscal switching
• Will examine some special cases and then turn to numerical results
• Allowing recurring regime change in both MP & FP can dramatically change nature of equilibria we study
• Raises the possibility for FP to play a role in our interpretations of business cycles
**Monetary and Fiscal Policy Interactions**

- Standard reasoning about macro policy
  - active monetary policy necessary for stability
  - Taylor principle delivers good economic performance in many models
  - high and variable inflation due to indeterminacy
  - active monetary/passive fiscal policies insulate economy from demand shocks (e.g., fiscal)

- Reasoning rests on convenient assumptions
  - passive fiscal behavior
  - fixed policy regimes
  - local $\Rightarrow$ global
Regime Change

- Regime change: realizations of params in policy rule

\[ R_t = \alpha_0(S_t) + \alpha_{\pi}(S_t)\pi_t + \alpha_x(S_t)x_t + \sigma(S_t)\varepsilon_t \]

- \( S_t \) evolves stochastically by a known process

- Many researchers have estimated policy rules to find parameters changed over time
  - Taylor, Clarida-Galí-Gertler, Auerbach, Lubik- Schorfheide, Sala, Favero-Monacelli

- Fixed-regime theory: problematic interpretation
  - ex-ante agents put probability 0 on change
  - ex-post agents put probability 1 on new regime
  - Cooley-LeRoy-Raymon: this is logically inconsistent
WHAT WE DO

- Bring together empirical and theoretical work
- Estimate Markov-switching rules for U.S. monetary and fiscal policies
- Embed estimated joint policy process in DSGE model with rigidities
WHAT WE FIND

• Policies fluctuate between active & passive
  • some active/active; some passive/passive

• Fit is good; connects to narrative accounts

• Post-war U.S. data can be modeled as a single, locally unique equilibrium

• Fiscal theory of price level always operative
  • taxes matter even with active MP/passive FP

• Fiscal theory mechanism quantitatively important
  • $1 transitory tax cut \implies PV output rises \approx $1

• Common practice: break samples into distinct regimes and embed rules in fixed-regime DSGE can produce misleading inferences
An Analytical Example

- Canzoneri, Cumby, Diba: Ricardian equilibria more general than non-Ricardian
  - if responses of taxes to liabilities is positive infinitely often—however small and infrequent—then eqm exhibits Ricardian equiv
  - because fiscal response does not stabilize debt, these are potentially equilibria with unbounded debt-output ratios
- Our example satisfies CCD’s assumptions, but deliver a unique eqm in set with bounded debt-output ratios
  - this eqm is non-Ricardian
  - important conclusions hinge on unboundedness ass’n of CCD
THE MODEL

- MIUF, constant endowment, log prefs, constant $g$
- Fisher equation
  \[
  \frac{1}{R_t} = \beta E_t \frac{1}{\pi_{t+1}}
  \]
- Money demand
  \[
  m_t = \left[ \frac{R_t - 1}{R_t} \right]^{-1}
  \]
- Monetary policy
  \[
  R_t = \exp(\alpha_0 + \alpha(S_t)\hat{\pi}_t + \theta_t)
  \]
- Tax policy
  \[
  \tau_t = \gamma_0 + \gamma(S_t)(b_{t-1} + m_{t-1}) + \psi_t
  \]

$(\theta_t, \psi_t)$ exogenous policy shocks; $\hat{\pi} = \ln \pi$
**The Model**

- $S_t$ an $N$-state Markov chain with transition probs $P[S_t = j | S_{t-1} = i] = p_{ij}$
- Define expectation error (and use Fisher equation)

$$
\eta_{t+1} \equiv \frac{1/\pi_{t+1}}{E_t[1/\pi_{t+1}]} = \beta \frac{R_t}{\pi_{t+1}}
$$

- Then the inflation process is given by

$$
\hat{\pi}_{t+1} = \alpha(S_t)\hat{\pi}_t + \alpha_0 + \theta_t - \hat{\eta}_{t+1} + \ln \beta
$$

- Let $l_t = b_t + m_t$, real govt liabilities
- Use tax rule & money demand in govt budget constraint

$$
l_t = \left[ \frac{R_{t-1}}{\pi_t} - \gamma(S_t) \right] l_{t-1} - \frac{R_{t-1}}{\pi_t} c + D - \psi_t
$$

$$
D = g - \gamma_0
$$
**Solution**

- Assume that
  
  1. $E_t[\gamma_{t+1}] = \gamma$
  2. $\gamma$ satisfies $|1/\beta - \gamma| > 1$
  3. Inflation process is stable in expectation (i.e., there exists a $0 < \xi < \infty$ such that $|E_t\pi_{t+k}| < \xi$ for all $k$

- (I)-(II): on average FP active; (III): on average MP passive

- Iterate on $l$ equation and take $E_{t-1}$ and law of iterated expectations

\[
E_{t-1} [l_{t+k}] = (1/\beta - \gamma)^{k+1} \left[ l_{t-1} - c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right) \right]
+ c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right)
\]

Stability requires that $l_{t-1} = c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right)$, which is positive if $D/c < 1/\beta$
Solution

- The value of $\eta_t$ is obtained from the budget constraint after substituting in the value of $l$:

$$\eta_t = \beta \frac{(1 + \gamma(S_t))(1/\beta - D/c) - (D/c)(1/\beta - \gamma - 1)}{1 + \gamma - D/c} + \frac{\beta}{c} \left( \frac{1/\beta - \gamma - 1}{1 + \gamma - D/c} \right) \psi_t$$

- The unique eqm mapping from $\psi_t$ and $\gamma(S_t)$ to forecast error in inflation

- $\eta$ and $\pi_t$ process yields unique solution for inflation
**Concrete Example**

- Two regimes, \( N = 2 \), and policy parameters take on the values
  \[
  \alpha(S_t) = \begin{cases} 
  \alpha(1) & \text{for } S_t = 1 \\
  \alpha(2) & \text{for } S_t = 2 
  \end{cases} 
  \quad \gamma(S_t) = \begin{cases} 
  \gamma(1) & \text{for } S_t = 1 \\
  \gamma(2) & \text{for } S_t = 2 
  \end{cases} 
  \]

- Suppose \( \alpha(1) \) and \( \alpha(2) \) are sufficiently small such that the inflation process is stable in expectation
  \[
  E[\gamma_{t+j} | S_t = 1, \Omega_t] = \gamma(1)p_{11} + \gamma(2)p_{12} \\
  = E[\gamma_{t+j} | S_t = 2, \Omega_t] = \gamma(1)p_{21} + \gamma(2)p_{22} \equiv \gamma 
  \]

- If either \( \gamma(1) \) or \( \gamma(2) \) is positive, then the model satisfies CCD’s premise that taxes adjust to debt infinitely often
- But negative tax shocks generate wealth effects that raise inflation
- The only eqm with bounded debt is one in which Ricardian equiv breaks down
Policy Rule Estimates

- Hidden Markov chain, as in Hamilton and Kim-Nelson
- Off-the-shelf policy rules; no dynamics
- Independent switching of M & F regimes

\[ r_t = \alpha_0(S^M_t) + \alpha_\pi(S^M_t)\pi_t + \alpha_x(S^M_t)x_t + \sigma_R(S^M_t)\varepsilon^r_t \]

4 states, \( \alpha \)'s have 2 sets of values, \( P^M \) transition matrix

\[ \tau_t = \gamma_0(S^F_t) + \gamma_b(S^F_t)b_{t-1} + \gamma_x(S^F_t)x_t + \gamma_g(S^F_t)g_t + \sigma_\tau(S^F_t)\varepsilon^\tau_t \]

2 states, \( P^F \) transition matrix

- \( S_t = (S^M_t, S^F_t) \). Joint distribution \( P = P^M \otimes P^F \), 8 states
Policy Rule Estimates

• U.S. data, 1948:2-2004:1
  • $r$: 3-month Treasury bill
  • $\pi$: log difference of GDP deflator
  • $x$: log output gap using CBO potential
  • $\tau$: federal receipts net transfers as share of GDP
  • $b$: market value of federal debt held by public as share of GDP
  • $g$: federal government consumption plus investment expenditures as a share of GDP
**Policy Rule Estimates**

- Four checks on plausibility of estimates
  1. Are the estimates reasonable on *a priori* grounds?
  2. Do the estimates fit the data?
  3. Do the estimates accord with narrative and other evidence on active/passive periods?
  4. Does the estimated policy process make sense in a standard DSGE model?

- Yes!
## Monetary Policy Estimates

<table>
<thead>
<tr>
<th>State</th>
<th>$S_t^M = 1$</th>
<th>$S_t^M = 2$</th>
<th>$S_t^M = 3$</th>
<th>$S_t^M = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_\pi$</td>
<td>1.3079</td>
<td>1.3079</td>
<td>.5220</td>
<td>.5220</td>
</tr>
<tr>
<td></td>
<td>(.0527)</td>
<td>(.0527)</td>
<td>(.0175)</td>
<td>(.0175)</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>.0232</td>
<td>.0232</td>
<td>.0462</td>
<td>.0462</td>
</tr>
<tr>
<td></td>
<td>(.0116)</td>
<td>(.0116)</td>
<td>(.0043)</td>
<td>(.0043)</td>
</tr>
<tr>
<td>$\sigma_r^2$</td>
<td>1.266e-5</td>
<td>9.184e-7</td>
<td>2.713e-5</td>
<td>5.434e-7</td>
</tr>
<tr>
<td></td>
<td>(8.670e-6)</td>
<td>(1.960e-6)</td>
<td>(5.423e-6)</td>
<td>(1.512e-6)</td>
</tr>
</tbody>
</table>

**Table 1:** Log likelihood value = $-1014.737$
# Tax Policy Estimates

<table>
<thead>
<tr>
<th>State</th>
<th>$S_t^F = 1$</th>
<th>$S_t^F = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>.0497</td>
<td>.0385</td>
</tr>
<tr>
<td></td>
<td>(.0021)</td>
<td>(.0032)</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>.0136</td>
<td>-.0094</td>
</tr>
<tr>
<td></td>
<td>(.0012)</td>
<td>(.0013)</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>.4596</td>
<td>.2754</td>
</tr>
<tr>
<td></td>
<td>(.0326)</td>
<td>(.0330)</td>
</tr>
<tr>
<td>$\gamma_g$</td>
<td>.2671</td>
<td>.6563</td>
</tr>
<tr>
<td></td>
<td>(.0174)</td>
<td>(.0230)</td>
</tr>
<tr>
<td>$\sigma^2_{\tau}$</td>
<td>4.049e-5</td>
<td>5.752e-5</td>
</tr>
<tr>
<td></td>
<td>(6.909e-6)</td>
<td>(8.472e-6)</td>
</tr>
</tbody>
</table>

**Table 2:** Log likelihood value $= -765.279$
INTEREST RATE: ACTUAL & PREDICTED
TAXES: ACTUAL & PREDICTED

0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16
τ
Smoothed
Filtered
Monetary Regime Probabilities

Active, High σ

Active, Low σ

Passive, High σ

Passive, Low σ
FISCAL REGIME PROBABILITIES

Fiscal Regime Probabilities

Passive

Active

0 0.5 1

0 0.5 1
JOINT POLICY REGIME PROBABILITIES
A Model with Nominal Rigidities

- Conventional: monopolistic competition, Calvo pricing, elastic labor, lump-sum taxes, nominal debt
- Households

\[ E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} + \delta \frac{(M_{t+i}/P_{t+i})^{1-\kappa}}{1-\kappa} \right] \]

\[ C_t = \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \theta > 1 \]

\[ C_t + \frac{M_t}{P_t} + E_t \left( Q_{t,t+1} \frac{B_t}{P_t} \right) + \tau_t \leq \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \Pi_t \]

\[ E_t[Q_{t,t+1}]^{-1} = 1 + r_t \]
A Model with Nominal Rigidities

- Firms

\[ E_t \sum_{i=0}^{\infty} \phi^i q_{t,i} + \left( \left( \frac{p^*}{P_{t+i}} \right)^{1-\theta} - \Psi_{t+i} \left( \frac{p^*}{P_{t+i}} \right)^{-\theta} \right) Y_{t+i} \frac{p^*_t}{P_t} = \left( \frac{\theta}{\theta-1} \right) \frac{K_{1t}}{K_{2t}} \]

\[ K_{1t} = (Y_t - G_t)^{-\sigma} \Psi_t Y_t + \phi \beta E_t K_{1t+1} \left( \frac{P_{t+1}}{P_t} \right)^{\theta} \]

\[ K_{2t} = (Y_t - G_t)^{-\sigma} Y_t + \phi \beta E_t K_{2t+1} \left( \frac{P_{t+1}}{P_t} \right)^{\theta-1} \]

\[ \pi_{t-1}^{\theta-1} = \frac{1}{\phi} - \frac{1-\varphi}{\varphi} \left( \mu \frac{K_{1t}}{K_{2t}} \right)^{1-\theta} \]

- Relative price dispersion

\[ \Delta_t = (1 - \varphi) \left( \frac{p^*_t}{P_t} \right)^{-\theta} + \varphi \pi_t^\theta \Delta_{t-1} \]
A Model with Nominal Rigidities

- Policy follows estimated rules and satisfies

\[ G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t} + E_t \left( Q_{t,t+1} \frac{B_t}{P_t} \right) - \frac{B_{t-1}}{P_t} \]

- Two information assumptions:
  - standard: \( \Omega_t = \{ \varepsilon^r_{t-j}, \varepsilon^\tau_{t-j}, S^M_{t-j}, S^F_{t-j}, j \geq 0 \} \)
  - foreknowledge: \( \Omega^*_t = \Omega_t \cup \{ \varepsilon^\tau_{t+1} \} \)

- Focus on stationary equilibria
  - \( b/y \rightarrow \infty \) feasible with lump-sum taxes
  - U.S. \( b/y \) appears stationary

- Use monotone map method to solve non-linear model
  - finds functions mapping state to decisions
  - state: \( \Theta_t = \{ b_{t-1}, w_{t-1}, \Delta_{t-1}, \varepsilon^r_t, \varepsilon^\tau_t, S_t \} \)
**The Fiscal Theory Mechanism**

- The **ubiquitous equilibrium condition**

\[
\frac{M_{t-1} + B_{t-1}}{P_t} = \sum_{T=t}^{\infty} E_t \left[ q_{t,T} \left( \tau_T - G_T + \frac{r_T}{1+r_T} \frac{M_T}{P_T} \right) \right]
\]

- Three sources of financing: net-of-interest surpluses; seigniorage; revaluations induced by jumps in \( P_t \)

- Cut \( \tau_t \) with exogenous \( \tau - G \) and pegged \( r \)
  - at initial prices, feel wealthier
  - increase demand for current goods
  - raises output relative to potential
  - money stock expands passively
  - must also raise inflation & lower real rates

- With positive probability of active FP, the mechanism is always operating
CHARACTERISTICS OF EQUILIBRIUM

- Numerical analysis of uniqueness and stationarity
- Numerical checks
  - randomly perturb decision rules at points in state space: converge back?
  - how monotone map behaves when properties known
    - indeterminacy (non-convergence)
    - non-existence (converges but solutions explode)
- zero expected present value of debt?
- histograms
Quantifying the Fiscal Theory

- Three regimes are stationary
  - AM/PF, PM/PF, PM/AF
  - AM/AF exhibits slowly growing debt
- A surprise tax cut of 2% of GDP, conditional on each stationary regime
  1. condition on remaining in prevailing regime
  2. average across future regimes
- Compute tax multipliers
  - condition on initial regime
NON-LINEAR IMPULSE RESPONSES

- Draw from regime after initial shock

![Images of graphs showing non-linear impulse responses](image-url)
**Tax Multipliers**

- Defined as

\[ PV_n(\Delta y)/\Delta \tau_0 = \frac{1}{\Delta \tau_0} \sum_{s=0}^{n} q_{0,s}(y_s - \bar{y}) \]

\( n = 5, 10, 20, \infty \)

- Size depends on conditioning regime
  - always non-trivial
  - potentially large (\( > 1 \))

- Similar impacts from unanticipated and anticipated changes

- With draws from future regimes
  - size depends on initial regime
  - range can be very wide
## Output Multipliers

<table>
<thead>
<tr>
<th>Init Regime</th>
<th>5 quarters</th>
<th>$\frac{PV(\Delta y)}{\Delta \tau}$ after 10 quarters</th>
<th>25 quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM/ PF</td>
<td>$[-.126, -.400]$</td>
<td>$[-.213, -.754]$</td>
<td>$[-.430, -.922]$</td>
</tr>
<tr>
<td>PM/ PF</td>
<td>$[-.215, -.401]$</td>
<td>$[-.271, -.623]$</td>
<td>$[-.414, -.764]$</td>
</tr>
<tr>
<td>PM/ AF</td>
<td>$[-.365, -.568]$</td>
<td>$[-.537, -.928]$</td>
<td>$[-.993, -1.363]$</td>
</tr>
</tbody>
</table>

**Table 3:** 80th percentile bands based on 10,000 draws
## Price Level Effects

<table>
<thead>
<tr>
<th>Regime</th>
<th>5 quarters</th>
<th>10 quarters</th>
<th>25 quarters</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM/PF</td>
<td>0.324</td>
<td>0.641</td>
<td>1.513</td>
<td>6.704</td>
</tr>
<tr>
<td>PM/PF</td>
<td>0.770</td>
<td>1.077</td>
<td>1.232</td>
<td>1.237</td>
</tr>
<tr>
<td>PM/AF</td>
<td>0.949</td>
<td>1.369</td>
<td>1.620</td>
<td>1.633</td>
</tr>
</tbody>
</table>

Table 4: Cumulative effect on price level of an i.i.d. unanticipated tax cut of 2 percent of output, conditional on regime
Fiscal Theory Robust

- Percentage of time in AM/PF regime

![Diagram showing the relationship between Tax Shock and changes in Δy (%) and Δπ (basis pts)]
Some Empirical Implications

- Observed time series produced by switching DSGE
- Correctly identified VAR, but fixed regime
- Policy rules and pattern matrix:

\[
\begin{align*}
    r_t &= \alpha_0 + \alpha_\pi \pi_t + \alpha_x x_t + \epsilon_r^r \\
    \tau_t &= \gamma_0 + \gamma_x x_t + \gamma_b b_{t-1} + \epsilon_\tau^\tau
\end{align*}
\]

\[
\begin{array}{ccccccc}
    x & \pi & b & MP & FP \\
    x & \times & \times & \times & \times & \otimes & \otimes \\
    \pi & \times & \times & \times & \times \\
    b & \times & \times & \times & \times \\
    r & \times & \times & \times & \times & \times & \times \\
    \tau & \times & \times & \times & \times & \times & \times \\
\end{array}
\]

\times: freely estimated; \otimes: imposed
Some Empirical Implications

- Two assumptions about econometrician’s information
  1. full sample from single regime (draws from shocks & regime)
  2. extra-sample information to identify regime (draws only from shocks)
- Econometrician interprets results with fixed-regime DSGE
- Accurate quantitative estimates $\hat{\alpha}_\pi, \hat{\gamma}_b$

<table>
<thead>
<tr>
<th>All Regimes</th>
<th>AM/PF</th>
<th>PM/PF</th>
<th>PM/AF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\alpha}_\pi$</td>
<td>0.723</td>
<td>1.308</td>
<td>0.595</td>
</tr>
<tr>
<td>$\hat{\gamma}_b$</td>
<td>0.002</td>
<td>0.016</td>
<td>0.018</td>
</tr>
</tbody>
</table>

- Inaccurate qualitative inferences
SOME EMPIRICAL IMPLICATIONS

“Fixed”: All Regimes parameters in fixed-regime DSGE
Some Empirical Implications

- “All regimes” implies PM/AF: fiscal theory equilibrium
  - correct inference about policy impacts
- Conditioning on regime gives incorrect inferences
  - AM/PF: Taylor principle & Ricardian
  - PM/PF: Indeterminacy & sunspots
- Most accuracy from full sample and averaging across regimes
  - quantitative predictions close
  - qualitative inferences correct
Fiscal theory can break down Ricardian equivalence
  • may be quantitatively important in U.S.
  • likely still more important in other countries
If fiscal theory important, need to modify models
Misleading to study MP (or FP) in isolation
  • models must be consistent with evidence on both MP & FP
Need a serious integration of MP & FP
  • tax distortions
  • other sources of non-neutrality
  • GBC met non-trivially
Wrap Up

• Empirical complications
  • identification: disentangling monetary and fiscal impacts
  • unobserved fiscal state: foreknowledge of fiscal policy

• Understanding source of regime change
  • optimal policy response?

• Holy Grail
  • joint estimation of policy and private parameters in DSGE with switching
  • some work with just MP switching (Zha et al.) and with everything switching (Svensson-Williams)
  • no work with MP & FP switching