Lecture 7. A Problem Recovering Tax Policy Behavior

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The Messages

- Draws on work by Bing Li, Indiana U Ph.D. student
- Examines empirical efforts to recover fiscal behavior from regressions of the form

\[ s_t = \gamma_0 + \gamma b_{t-1} + Z_t \Phi' + \varepsilon_t \]

- Parameter of interest is \( \gamma \)
- We seek to test \( H_0 : \hat{\gamma} > \bar{\gamma} \)
- \( \hat{\gamma} \): estimated value of \( \gamma \)
- \( \bar{\gamma} \): some pre-determined critical value of \( \gamma \)
\[ s_t = \gamma_0 + \gamma b_{t-1} + Z_t \Phi' + \varepsilon_t \]

- Typically \( \gamma \) estimated using a partial-information method, such as OLS or VARs
- Examples often cited: Bohn (1998), Canzoneri-Cumby-Diba (2001), many EC papers
- These estimates have been influential, being interpreted as supporting Ricardian-style fiscal rules
  - increases in debt bring forth subsequent increases in surpluses
- OLS justified by the usual argument that \( b_{t-1} \) is predetermined for \( s_t \)
- For purposes of inference about \( \gamma \), OLS is valid
- What could be wrong with this argument?
The Ubiquitous Equilibrium Condition

\[ b_t = E_t \sum_{i=0}^{\infty} \left( \prod_{j=0}^{i} \pi_{t+j} R_{t+j-1}^{-1} \right) \left[ T_{t+i} - G_{t+i} + \frac{M_{t+i} - M_{t+i-1}}{P_{t+i}} \right] \]

- This implies that in any equilibrium \( b_t \) will be positively correlated with future \( s_{t+i} \)
- How do estimates of \( \gamma \) ensure they are estimating
  - the causal channel \( b_{t-1} \rightarrow s_t \) and
  - not the equilibrium channel \( E_t s_{t+i} \rightarrow b_{t-1} \)?
- This is Bing Li’s insight and he documents
  - the OLS bias in a simple analytical example
  - the ability of a fully specified DSGE model to get things right
- Open question: How much structure is necessary to estimate \( \gamma \)?
A Simple Model

- Endowment, MIUF, nominal debt, stochastic rules for MP & FP
- Tax rule
  \[ \tau_t = \gamma_0 + \gamma b_{t-1} + \psi_t \]
  \[ \psi_t = \rho \psi_{t-1} + \varepsilon_{\psi_t} \]
- Monetary policy rule
  \[ R_t = \alpha_0 + \alpha \pi_t + \theta_t \]
  \[ \theta_t = \rho \theta_{t-1} + \varepsilon_{\theta_t} \]

\[ \varepsilon_{\psi,t} \sim N(0, \sigma_{\psi}^2) \& \varepsilon_{\theta,t} \sim N(0, \sigma_{\theta}^2) \]
A Simple Model

• After linearization, system is

$$\Gamma_0 Y_{t+1} = \Gamma_1 Y_t + \Pi \eta_{t+1} + \Psi \epsilon_{t+1}$$

where $Y_{t+1} = [\hat{\pi}_{t+1}, \hat{b}_{t+1}, \theta_{t+1}, \psi_{t+1}]'$, $\epsilon_{t+1} = [\epsilon_{\theta_{t+1}}, \epsilon_{\psi_{t+1}}]'$, $\Pi = [1, 0, 0, 0]'$ and

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \varphi_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} \alpha \beta & 0 & 0 & 0 \\ -\varphi_2 & (\beta^{-1} - \gamma) & -(\varphi_3 \rho \theta + \varphi_4) & 0 \\ 0 & 0 & \rho \theta & 0 \\ 0 & 0 & 0 & \rho \psi \end{bmatrix}, \quad \Psi = \begin{bmatrix} 0 & 0 \\ -\varphi_3 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$
A Simple Model

- Solve as in “Lecture 2. Fiscal Theory of the Price Level”
- Make alternative assumptions on MP & FP
- Focus only on determinate, bounded equilibria
- Under various \((\alpha, \gamma)\) combinations, examine

\[
\hat{\gamma}_{OLS} = \frac{\sum_{t=2}^{T} \hat{b}_{t-1} \hat{r}_t}{\sum_{t=2}^{T} \hat{b}_{t-1}^2} = \frac{\sum_{t=2}^{T} \hat{b}_{t-1} (\gamma \hat{b}_{t-1} + \psi_t)}{\sum_{t=2}^{T} \hat{b}_{t-1}^2} = \gamma + \frac{\sum_{t=2}^{T} \hat{b}_{t-1} \psi_t}{\sum_{t=2}^{T} \hat{b}_{t-1}^2}
\]

\(\hat{b}_{t-1}\) is demeaned & unconditional expectation of \(\psi_t\) is 0 \(\Rightarrow\)

\[
\text{plim}\hat{\gamma}_{OLS} = \gamma + \frac{\text{plim}\sum_{t=2}^{T} \hat{b}_{t-1} \psi_t}{\text{plim}\sum_{t=2}^{T} \hat{b}_{t-1}^2} = \gamma + \frac{\text{cov}(\hat{b}_{t-1}, \psi_t)}{\text{var}(\hat{b}_{t-1})}
\]
Case I. Non-Ricardian

- $\alpha = \gamma = \rho_\theta = \rho_\psi = 0$

$$\hat{b}_t = (\beta^2 \varphi_1 + \beta \varphi_4)\theta_t = (\beta^2 \varphi_1 + \beta \varphi_4)\varepsilon_{\theta_t}$$

$\psi_t$ i.i.d. $\Rightarrow \psi_t = \varepsilon_{\psi_t}$

$$\text{cov}(\hat{b}_{t-1}, \psi_t) = \text{cov} \left[ (\beta^2 \varphi_1 + \beta \varphi_4)\varepsilon_{\theta_{t-1}}, \varepsilon_{\psi_t} \right] = 0$$

plim $\hat{\gamma}_{OLS} = \gamma$
Case II. Non-Ricardian

- \( \alpha = \gamma = \rho_\theta = 0, \rho_\psi \neq 0 \)

\[
\text{cov}(\hat{b}_{t-1}, \psi_t) = \text{cov} \left[ (\beta^2 \varphi_1 + \beta \varphi_4)\varepsilon_{\theta_{t-1}} + \left( \frac{\rho_\psi}{\beta^{-1} - \rho_\psi} \right) \frac{\varepsilon_{\psi_{t-1}}}{(1 - \rho_\psi L)} \right]
\]

\[
= \left( \frac{\rho_\psi}{\beta^{-1} - \rho_\psi} \right) \text{cov} \left[ \frac{\varepsilon_{\psi_{t-1}}}{(1 - \rho_\psi L)}, \frac{\varepsilon_{\psi_t}}{(1 - \rho_\psi L)} \right]
\]

\[
= \left( \frac{\rho_\psi \sigma_\psi^2}{\beta^{-1} - \rho_\psi} \right) \sum_{i=1}^{\infty} \rho_\psi^{2i-1}
\]

\[
= \frac{\rho_\psi^2 \sigma_\psi^2}{(\beta^{-1} - \rho_\psi)(1 - \rho_\psi^2)}
\]

Variance term is

\[
\text{var}(\hat{b}_{t-1}) = \text{cov}(\hat{b}_{t-1}, \hat{b}_{t-1})
\]

\[
= (\beta^2 \varphi_1 + \beta \varphi_4)^2 \sigma_\theta^2 + \frac{\rho_\psi^2 \sigma_\psi^2}{(\beta^{-1} - \rho_\psi)^2(1 - \rho_\psi^2)}
\]
Case II. Non-Ricardian

\[ \alpha = \gamma = \rho_\theta = 0, \rho_\psi \neq 0 \]

Note that \( \text{cov}(\hat{b}_{t-1}, \psi_t) > 0 \)

\[ \text{plim} \hat{\gamma}_{OLS} > \gamma \]

\( \text{plim} \hat{\gamma}_{OLS} \) is upper bound for OLS estimator

This biases inferences toward Ricardian interpretations of fiscal data
**Case III. Non-Ricardian**

- $\gamma = \rho_\theta = 0, \alpha \neq 0, \rho_\psi \neq 0$
- Much messier
- Evaluate bias numerically over a grid for $(\alpha, \rho_\psi)$ of $[.01, .99]$
- This ensures passive MP & stationary tax shock
- See figure
Asymptotic Bias. Non-Ricardian
Biases also arise in the case of active MP/passive FP.

In this case, the bias tends to be small—in this model—and to be negative.

If one simulates data from this model and uses Bayesian MLE methods, it is possible to recover the true $\gamma$.

Why?

- The MLE exercise imposes *much* more structure on the estimation.
  - In particular, it imposes the ubiquitous eqm condition.

Is it possible to get accurate estimates without imposing so much structure?

If one does not pretend to know the true structural model, what are the tradeoffs one faces in estimating $\gamma$?
Why is this an important question?

FTPL is not testable without auxiliary assumptions, such as the nature of the policy rules.

It would be useful to pursue the idea of recovering the correct tax policy rule without pretending to know so much about the economy.

But we cannot do this without imposing the *some* structure.

Would just imposing the intertemporal gbc suffice to recover correct $\gamma$?

Even if you are not interested in the FTPL *per se*, you should be interested in trying to learn actual tax policy behavior.

All other inferences about time series are conditional on some maintained assumption about tax behavior.