Contact Patterns and Aggregate Opinion Levels - Results from a Simulation Study

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Abstract: In the last ten years, sociologists, social psychologists and physical scientists have developed opinion change models where influence is exerted through social ties. The goal of our paper is to investigate the effect of contact patterns on the number of individuals within a group who share the same opinion. We present an opinion formation model, simulate opinion formation in eight different kinds of networks and present the results. We find that contact patterns can have large effects on agreement levels. For example, individuals in tree networks display low levels of agreement. Basic statistical descriptors of network structure, such as density, correlate with group agreement in our simulated data. We also found that dummy variables indicating type of network have statistically significant effects, suggesting that additional unmeasured features of the network contribute to agreement in our simulations.

Introduction

For decades, sociologists have been concerned with social contact patterns and their effects on opinions. Sociologists have recently explored the effects of contact patterns on opinions through computer simulation and formal theory building. In these models, individuals influence each other through their social ties. Individuals are linked to a handful of other individuals and change their opinion in response to the influence exerted by their neighbors. We ask: how much does the contact pattern affect the aggregate opinion level in a group? In our review of the literature, we find that some authors have addressed this question. We extend their work by simulating agreement levels in a wider variety of networks. What we do is fairly simple: we vary contact patterns, keep other parameters of the model constant and record the number of actors who share an opinion. We compare simulated aggregate opinion levels in artificial networks as well as in observed networks. The comparison allows us formulate testable hypotheses about social structure and opinion.

Opinion Formation Models and Sociological Theory

Sociologists have long shown an interest in opinion formation. According to early sociological theories, influence exerted by some members of a community over others leads to opinion change. Janowitz (1975) summarized this line of thought when he argued that interpersonal influence leads to “social control,” the ability of a community to regulate the activities of its own members through the establishment of accepted rules of conduct. Contemporary sociology remains interested in opinion formation. For example, diffusion theories can be understood as application of opinion formation models because they describe how an idea or practice spreads throughout a population via personal contacts. Cultural theorists also show a continued interest in interpersonal influence. Jepperson and Swidler (1993) argue that opinion change and influence are important dimensions of culture, while other cultural theorists have tried to map out opinions in a historical era (see Mohr 1998 for a review) or have tried to relate group structure to opinions within a group (Martin 2001).

Social psychologists have also shown a great deal of interest in opinion formation. Bibb Latane (1981, 1996) argues that human beings form their opinions in response to each other and that perceived immediacy, intensity of the contact and the number of contacts. Subsequent researchers have empirically verified Latane's model (Vallacher and Nowak 1994). Lewenstein, Nowak and Latane (1992) review this research.

Inspired by both the sociological and social psychology literatures, social network theorists and mathematicians have developed a set of mathematical tools for describing and analyzing patterns of influence and opinion. French (1956) introduced a formal theory of social power describing how interpersonal influence resulted in opinion change. Harary (1959) and De Groot (1974) extended French's work using linear algebra techniques associated with Markov process models. Friedkin and Johnsen (1990, 1999, 2002), building on Harary's work, present a social network model of agreement formation in which each person is influenced by their associates, a process leading to equilibrium. Their model includes a description of opinion change and the flows of interpersonal influence, combining a theory of individual opinion formation with a concern for network effects.

Sociologists who study influence and agreement through formal modeling tend to work with a model that shares the basic features of the Friedkin-Johnsen model. Their models tend to have actors who exert influence through social ties. Friedkin and Johnsen list the assumptions behind their model and others like it: (1) There is "cognitive weighted averaging." Future opinions are combinations of existing opinions and the opinions of influential group members. According to Friedkin and Johnsen (1999), "flows of interpersonal influence are established by repeated responses of actors to the (possibly changing) influential opinions on the issue." (2) Fixed social structure: The pattern of influence remains fixed during agreement formation. (3) Determinism: The matrices describing the initial the state of the system completely determine the outcome. (4) Continuance: "The process of opinion formation in the group continues until all changes of opinion that may occur have played themselves out." (5) Decomposability into time periods. (6) Simultaneity: Freidkin's model is a simultaneous system of linear equations.

A related class of influence models are the Sznajd models (Stauffer 2001; Sznajd-Weron and Sznajd 2000; Holyst, Kacperski and Schweitzer 2001). In early models, individuals are arrayed in a rectangular grid. Each actor influences their neighbor (e.g., Ising 1925). The more recent Sznajd models stipulate a distance between every pair of actors. There is usually a rule in Snazd models requiring an actor to change his opinion if the number of disagreeing neighbors reaches a certain number. The Sznajd model resembles the Friedkin-Johnsen model in many respects; both models have a fixed social structure and interactions continue until opinion distributions stabilize. The Sznajd model differs in some important respects from the Friedkin-Johnson model. There is usually a very specific interaction structure (the grid) and many models allow influence at a distance while the Freidkin-Johnsen models usually focus on influence between immediate neighbors. Another key difference is that the Friedkin-Johnson network influence models focuses on continuous opinions, while Sznajd models and their relatives focus on dichotomous yes/no opinions.

Later contributors in this tradition include Bahr and Passerini (1998a, 1998b) who model opinion change in a grid, Stocker, Cornforth and Bossomaier (2002) and Stocker, Green and Newth (2001) who examine opinion dynamics of individuals arrayed in a rectangular lattice, Krackhardt (1997) who examines the emergence of unanimity within organizational networks and Butts (1998), who analyzes the spread of panic in crowds via a network influence model. Like Friedkin and Johnsen and the Sznajd, these authors view opinion as changing in discrete time periods as a response to influence exerted through network ties.

The Research Question

This mathematical opinion formation literature has been motivated by a desire to link social structure and opinion. Krakhardt (1997), for example, shows that some individuals can impose their opinion on an entire group through social ties. Carley (1992) shows how network structures emerge from random mixing and social interaction. However, what this literature has yet to provide is a more systematic approach to assessing the relationship between contact patterns and opinion formation. For example, there is yet no research comparable to Faust and Skvoretz (2002), which analyzes a large number of empirically observed network structures to discover common structural features. Similarly, there is no paper in the sociological literature that systematically looks at the effects of model parameters across a number of network structures. In this paper, we simulate agreement in eight different kinds of networks and compare in-group agreement levels. We also examine the effects of model parameters on opinion levels in eight different network structures. This knowledge would help social scientists understand the link between social structures and commonly shared beliefs and it would also help applied social scientists how to design communication networks so that they might lead to agreement.

The model we present utilizes properties of both the Friedkin-Johnsen and the Sznajd family of models. We see it as a modest extension of both these classes of models. From the Friedkin-Johnsen model, we adopt a network structure for influence. And we see disagreement in an individual's egocentric neighborhood as the event leading to opinion change. From the Sznajd family of models, we adopt a probabilistic opinion change rule and a focus on dichotomous opinions. The model we present has a logistic opinion change rule: the probability of opinion change increases when the individual disagrees with her neighbors, but the effect decreases when disagreement becomes large. We also incorporate a feature commonly found in Snazd models – the presence of an external factor J that affects all actors. J might correspond to a commonly experienced event that encourages a certain opinion, such as advertising. The opinion change rule is:

$$\Pr(y_{T,i} = -1 \cdot y_{T-1,i}) = e^{-\sum_{j \neq i} w_{j,i} y_{T-1,i} y_{T-1,j} + J y_{T-1,i}} \Big/ 1 + e^{-\sum_{j \neq i} w_{j,i} Y_{T-1,i} y_{T-1,j} + J y_{T-1,i}}$$

This rule allows external events and interpersonal influence to affect opinion. It has the properties just mentioned: the probability of opinion change is a log-linear weighted sum of the disagreement in an individual's egocentric neighborhood; opinion change correlates with disagreement with the egocentric neighborhood; it has the diminishing influence assumption; and it is nondeterministic: individuals are allowed to spontaneously change their mind.

We examine special case of this general model, in which network ties determine influence and each individual exerts the same influence on her neighbors. The simplification allows us to focus on the role that network features have on agreement, because we can vary the network structure but keep constant the degree of influence individuals have on each other. In the simplified model, if two actors have no influence on each other, $w_{ij} = w_{ji} = 0$. If every person has the same degree of influence on each other, then $W = EG$, where G is the network's adjacency matrix and E is a number indicating the strength of interpersonal influence.

Simulating the Model

We simulate our model and statistically analyze the output. We offer a few words regarding our modeling philosophy. Computer simulations tend to embody one of two approaches, which Carley (2002) describes as "veridical" and "transparent." Veridical simulations try to include as many relevant variables as possible. The goal of a veridical simulation is to mimic observed social phenomena through the use of large numbers of model parameters. Such an approach is useful for policy makers, engineers, and others who desire accurate forecasting of specific states. It is difficult for these types of models to offer the user much theoretical insight because it is hard to isolate key relationships leading to the model's output. These models are also hard to interpret because they are created by large groups of researchers, and few, if any, of them have knowledge of all the components of the simulation. Another difficulty is that the user of such a complex model is not the same person who created the model, so it can be hard to understand the sequence of calculations from inputs to results.

Transparent modeling employs few parameters. Such models have the virtue of simplicity. The researcher can easily communicate how the model's inputs lead to the outputs. A mathematical proof can be produced linking outcome and initial conditions. Succinct behavioral rules can lead to unexpected outcomes, which can then be directly attributed to the rules governing the model. Schelling's (1969) residential segregation model, perhaps the most famous example of this style of modeling, employs simple and intuitive assumptions and demonstrates that unexpected outcomes can yield theoretical insights.

The model that we present falls within the "transparent" tradition of computational modeling. Rather than include every possible source of opinion change, we start with a specific agreement formation model and then slightly modify it. The model contains only three variables—a measure of social structure (the adjacency matrix G), a measure of interpersonal influence (E), and a measure of external influence (J).

In the simulations described in the rest of this paper, we further simplify the model in order to isolate specific effects and clearly communicate the theoretical importance of our model. Our transparent approach also has the virtue of reproducibility. Other researchers can write their own code and verify our results, or download our code and execute it on their own computer.

We start with a group of N actors and randomly assign each an opinion of +1 or –1 with a probability $p = 0.5$. In each time period, actors change their opinion according to a modification of equation (6). Opinion change comes mostly from an individual's disagreement with their neighborhood. When actors in an individual's egocentric neighborhood are evenly split (½ are +1 and ½ are –1) then according to equation (6), they would change their opinion half the time. We find this to be implausible—a model with no social connections would then have half the population changing their opinion in every time period. We weight the probability in equation (6) so that an individual whose neighborhood is evenly split has a 5% chance of changing their mind. We experimented with this weight and found that increasing the weight eliminates the distinction between different contact patterns.

This process is repeated for five thousand time periods. At each time period, we record the number of persons with opinion +1. As a result, each simulation of agreement formation produces a time series of opinion levels. Additional simulations vary the pattern of interactions between actors and the parameters describing interpersonal and external influence. The E and J variables were varied from 0 to 1 by 0.05 increments. After each simulation, we saved data on the percentage of actors who hold opinion +1, the parameters E and J, and the adjacency matrix. Using this data, we can estimate the relationship among network structure, influence, and agreement. Diagram 1 illustrates an example of opinion fluctuations over time.

[Diagram 1 here.]

More details about our simulation: First, we are interested in how network structures promote or suppress agreement. To address the question, we varied the network structure between individuals. We selected eight types of networks to examine in our simulation. We first created samples of "artificial" networks, that is, networks created by the application of mathematical rules. For example, we examined undirected "random density" networks, in which social links are assigned randomly based on a given density of the resulting network. Density is defined as the number of existing links per number of possible links. We then performed the simulation with individuals interacting in a "naturally" occurring network. These are empirically observed network structures.

We examine three classes of theoretically interesting artificial networks: tree structures, "small world" networks (Watts 1999), and random density networks, also known as homogeneous Bernoulli networks (Erdos and Renyi 1960). Tree structures are created by starting with a single actor and connecting them to a finite number of "descendants," each of whom has descendants, and the process is continued. Tree structures are interesting because they embody the idea of authority in organizations, where each person has a small number of subordinates. Small worlds networks were introduced into sociology by Watts (1999), who argued that many observed networks have the small world property, that is, the average distance between pairs of individuals is bounded by the logarithm of the network size: $l(G) < \log(N)$, where $l(G)$ is the least upper bound on the smallest distance between any pairs of points in a network G. Watts shows that many interesting social behaviors in groups are made possible by the small world property.Examples of small worlds networks are obtained by randomly reassigning ties in a cyclic pattern where each actor is connected not only to their immediate neighbors, but also to their neighbor's neighbors. The random density network, also known as a homogeneous Bernoulli graph (1960), is created by starting with N actors and assigning a tie between actors with probability p .

We produced artificial networks of size $N = 100$, 16, and 21. The larger networks were included to see if differences in agreement effects could be attributed to network size. The smaller simulated networks were included so we could compare groups with the same number of individuals but different contact patterns. Following Watts, we created examples of small worlds networks by starting with 100 actors in a 10x10 grid and then randomly moving 5% of the ties to new pairs of individuals. Tree structures are generated by starting with a single individual who is connected to D other persons, where D has a Poisson distribution with mean 3. Each subsequent person is then tied to more people, with distribution D. The tie formation probability for our random density networks is set to $p = .05$. All of these artificially generated networks are undirected—if person i influences j, then j will influence i. The adjacency matrix is symmetrical: $G = G^t$.

We examine five examples of "natural" networks: the business and marriage networks of Renaissance Florentine families as reported by Padgett and Ansell (1993), and the friendship, reporting, and advice networks in a group of engineers as reported by Krackhardt and Porter (1985). These networks were chosen because of their prominence in the social network analysis literature. Padgett found the Florentine networks interesting because the de Medicis were able to mobilize political resources through social ties and because of the conversion of social capital in one network into social capital in another network; the Krackhardt networks are interesting because they represent different sets of ties within the same social group, and the corresponding networks have markedly different topologies. If agreement exists within these groups and simulations show how different network structures produce differing levels of agreement, then one would have a strong a priori reason to believe that group cohesion is due to the structure of the network, not just the type of tie.

A few notes about these natural networks. Unlike the artificial networks, the natural networks may be directed. It is frequently the case that influence is exerted by only one person in the dyad. In all of our simulations, we take into account the direction of influence. We do not assume that $G = G^t$ for these networks. Also, these networks vary in size. The Krackhardt networks both have size $N = 21$, and the Padgett networks have size $N = 16$. The network data can be found in Wasserman and Faust (1993, Appendix B).

Finally, we assess the robustness of our results by modifying the artificial and observed networks in the simulation. First, we "rewire" each network and repeat the simulation. By rewire, we mean the reassignment of a fixed proportion of ties to new pairs of actors. After performing the simulation with the original networks, we then performed the same simulation but rewired 5% of the ties in each network. Separately, we evolve each natural occurring network in our study. An "evolution" of a network is a procedure proposed by Jin, Girvan, and Newman (2001) where individuals form relationships at a higher rate if they have one or more mutual friends, relationships between individuals who rarely see each other decay over time and individuals have an upper limit on the number of relationships they can maintain at a given time. Each network that we examined was rewired and/or evolved a total of ten times.

The simulated data set that we constructed treats each network as a unit of analysis, with the E and J variables for each simulation; dummy variables for type of network (tree vs. Padgett marriage network, for example); measures of the network structure such as density, symmetry, and the number of connected components; and the dependent variables in our analysis, “agreement” and “volatility.” Agreement (A) in a time period T is defined to be

$$A_T = 2 \cdot |P_T - .5|, \tag{7}$$

where P_T is the proportion of the individuals in time T who hold the +1 opinion. When $A_T = 1$, all individuals hold the same opinion. When $A_T = 0$, the group is evenly split between two opinions. The dependent variable in the analysis is the average agreement for a simulation over the entire run ($T = 5000$):

$$A = \frac{\sum_{i=1}^{T} A_i}{T}. \tag{8}$$

For a given network structure, we average agreement over the simulation because the model has fluctuating opinion. We also include the standard deviation of agreement, called volatility to assess how a given network structure creates instability in agreement levels over time. See Diagram 2 for an example of single network and its agreement level time series for 200 time periods. We discovered that “burn-in” was not a significant issue. Calculating A with and without the first 250, 500, and 1000 time periods shows little difference; each pair of variables has a correlation of over 0.90. Table 1 presents the descriptive statistics in our analysis.

[Table 1 here.]

Comparison of Means

The first question we address is: how much does the contact pattern of affect the proportion of people who hold the same opinion? Table 2 presents the average agreement level and volatility for each graph type. The mean is estimated by averaging the agreement variable from all simulations, including evolved and rewired versions of the network. The estimates presented in Table 2 were computed according to the following formula:

$$Mean(A \mid Network\ Type = M) = \frac{\sum_{E,J \in I} A(E,J)}{T}. \qquad (9)$$

I is the set of all *E* and *J* values, and *M* is a categorical variable denoting the kind of network (small worlds, random density, etc.). $A(E,J)$ is the value of the agreement variable for given levels of interpersonal and external influence. We also estimated the mean volatility via the same method. By averaging over values of *E* and *J,* the results in Table 2 show how agreement levels and agreement variation differ by the type of network. Diagram 2 plots the mean agreement levels for each category of networks.

[Table 2 about here.]

[Diagram 2 about here.]

The ranking of network structures according to their mean agreement and volatility reveals some striking patterns. First, the network structure with the highest level of agreement is the Krackhardt friendship network, with an average agreement level of 0.78, by far the highest among the seven different classes of graphs examined. Using the definition of the agreement measure, we calculate that 89% of the actors in the Krackhardt friendship network come to hold the same opinion in a given time period. In contrast, the network structure with the lowest agreement is the Krackhardt reporting network. The agreement measure is 0.269, which leads to 63% of actors sharing an opinion in a given time period. Note that a group of completely disconnected individuals (E=0) with no external influence (J=0) would have about 50% of the individuals split between two opinions. The reporting network does produce more agreement than a null network, but it does not fare well when compared to some of the network structures considered in this study. For example, the random density network, where 5% of pairs of actors are connected, has a agreement measure of 0.366—higher than the Krackhardt reporting network.

The reader should also note the great difference between multiple networks within the same group. Networks with the highest and lowest agreement levels are both Krackhardt networks. The two networks measured by Padgett had much more similar mean agreement levels. The difference between the Krackhardt and Padgett networks is easy to explain. The Krackhardt networks were formed on completely different principles (work authority vs. friendship), while both Padgett networks were shaped by the de Medicis and ties in one network were used to create ties in another. Thus we expect both to be fairly similar in structure and thus in simulated agreement levels. In contrast, we would expect the Krackhardt networks to behave quite differently.

These results lead to two observations. First, individuals rarely think about large-scale interaction patterns when they select friends, spouses, or business partners. Social scientists frequently assert that macrosocial patterns arise from “local” decisions and that any explanation of collective behavior must include a “microsociological” account (Coleman 1990). The results in Table 2 show that the networks arising from such local decisions can have dramatic, indirect effects on the formation of opinions, effects that are not attributable to the strength of interpersonal ties and of exogenous events framing opinions. Agreement is not just a matter of strong personal bonds or threatening events that unite a group; it is the outcome of global group structure, which in turn emerges from individual decisions. Aggregate opinions depend on social structure, which in turn depends on rules that actors use to create social ties.

Second, we note that our model provides a way of theorizing about multiple social networks within the same group. It is frequently the case that individuals will use different sets of social ties to achieve various goals. The huge variation in simulated agreement levels shows that the decision to mobilize individuals through certain ties will probably affect the outcome of the mobilization. This is not because some ties are stronger than others; we have already controlled for that. It is because the network created by individuals forming ties according to different rules, as noted above, results in different interaction patterns, which in turn shape aggregate opinion levels. A researcher studying a specific group could simulate the agreement levels of two different networks that she has observed, and that would help shape the researcher's belief about which kinds of ties were used to shape the distribution of opinions in the group.

Effects of Interpersonal Influence and External Events

Our next question is: given a particular pattern of contacts in our model, how well do interpersonal influence and the external influence predict agreement ? The answer to this question suggests how much the contact structure magnifies the effects of interpersonal influence and the external influence.

We answer this question by estimating the effects of the interpersonal influence (*E)* and external influence (*J)* parameters on agreement for each type of network. That is, we treat all the simulations that use a given network and its rewired and evolved variants (small worlds, for example) as a single data set. We can then compare the coefficient for *E* and *J* across networks. Table 3 presents the results of applying a regression model to the agreement and volatility measures on the strength of interpersonal influence (*E*) and the strength of the exogenous event (*J*) for each type of graph. Estimating the effects for *E* and *J* separately assumes that the error terms are uncorrelated. This assumption is not true because agreement and volatility are both functions of the same distributions. Therefore, we employ a seemingly unrelated regression (SUR) model to jointly estimate the effects of *E* and *J* on agreement and volatility. The SUR model is

$$Agreement = \beta_0 + \beta_1 E + \beta_2 J + \varepsilon \, , \tag{10}$$

$$Volatility = \beta_0^{'} + \beta_1^{'} E + \beta_2^{'} J + \varepsilon^{'} \, , \tag{11}$$

and

$$V(\varepsilon,\varepsilon^{'}) = v\ ,\ Mean(\varepsilon) = 0\ ,\ Mean(\varepsilon') = 0\ . \qquad (12)$$

Not surprisingly, the networks with the smallest *E* effect on agreement are the small worlds network (β_1 =0.031) and the Krackhardt reporting network (β_1 =0.029). For the small worlds network, the small *E* effect is probably a consequence of the fundamental property of the small worlds network, the small upper bound on the shortest length between any two actors. Most actors are close to most other actors because most actors are close to an actor with connections to distant parts of the network. Regardless of the magnitude of *E*, opinion change in one part of the network quickly spreads to other parts of the network, suppressing the *E* effect. The Krackhardt reporting network probably has a small *E* effect because it is so asymmetrical. Opinion change in one individual will not cause others to change because the influence is frequently unreciprocated. Later, we test the hypothesis that symmetry within the network has positive effects on the agreement variable.

The other network structures have large *E* effects on agreement (β_1 >0.30). The random density graph has β_1 =0.316, which is still less than most other networks. This is an interesting point. Of the two networks studied by Padgett, one network has *E* effects about as strong as the network created when individuals are randomly paired 5% of the time. This means that a change in the strength of influence between individuals will have a larger effect on agreement in the random density network than in the Florentine business network. In Padgett's study, he found that the de Medici family relied more on marriage ties than business ties. While this may be attributable to the content of the ties—marriages are more durable than business ties—it may also be due to the fact that the marriage network is more able to project the de Medicis' influence. A central actor who changes his opinion or increases his influence in the marriage network will have more effects on agreement than a central actor in the business network.

A second result is that the effects of *J* on agreement (β_2) are often much larger than the effects of interpersonal influence on agreement (β_1) in most of the examined networks. The most extreme example is the Krackhardt reporting network, where the *J* effect on agreement (β_2) is 16 times as large as the *E* effect (β_1). Two exceptions to this pattern are the tree branching structure and Krackhardt friendship network. These two exceptions may occur because of their unusual structures. The next section examines this possibility in more detail.

Why is $\beta_2 > \beta_1$ for most of the networks in this study? This is probably due to the opinion change model. The external force *J* affects all individuals equally, while *E* has an effect only when an actor finds himself with an opinion very different than those in the egocentric neighborhood. Consider the case in which individuals are randomly assigned +1 with probability equal to 50%. The average actor will not change their opinion as a response to disagreement with network neighbors because disagreements among neighbors cancel each other out. The external influence starts opinion change and sets a trend modified by later social network interactions.

Agreement and agreement variation in tree structures are not modeled well by *E* and *J*. The reason probably lies in the peculiar structure of the tree graph. The tree structure is acyclic, unlike the other networks in this study. Also, tree structures are much bigger than similar networks, such as the Krackhardt reporting network, which also has few cycles. That might be another possible explanation for the difference between the reporting and the tree. In the next section, we control for size and the presence of cycles, which we measure with the Watts-Strogatz clustering coefficient.

The effects of *J* on volatility ($\beta_2^{'}$) are almost all negative, and the effects of *E* on volatility ($\beta_1^{'}$) are small but positive. The first estimate is intuitive: if there is an event pulling all individuals toward a single opinion, then that will decrease the opinion's variance. The small but positive *E* effects are more interesting: when individuals increase the strength of their ties, they slightly increase opinion volatility. This has novel implications. The results show that strong interactions decrease agreement, but the effect can often be countered by exposure to an external event. For example, those interested in stabilizing agreement will, according to this model, have more success persuading individuals through mass communication than by increasing the strength of ties between individuals. According to our model, a person who changes their opinion will have a strong effect on others, which in a "high-*E*" environment will start to undermine agreement.

Network Effects

The next question we address is: in our formal model, how much agreement within a group can be attributed to features of the network, such as density or group centralization? To address this question, we estimate a series of models in which agreement and volatility are the dependent variables and the independent variables are *E*, *J*, and a few standard network measures. We include four measures of network structure: network density, the clustering coefficient, the number of connected components, and the symmetry of the graph. Networks may have similar density measures, for example, but may have unmeasured features that contribute to aggregate opinion levels. To account for this possibility, the models include dummy variables indicating the type of network.

We include density and cluster coefficients because networks in which actors are frequently connected to others seem likely to have high agreement. Density is simply the number of ties divided by the total possible number of ties. A clique has density 1, and a group of isolates has density 0. Watts and Strogatz (1998) introduced clustering as a measure of how frequently transitive triads appear in the network. Their clustering coefficient is 6 times the number of triads in the network divided by number of paths of length 2. A large number of connected components should decrease agreement, while highly symmetric networks should also exhibit agreement because they are networks where the links of influence are undirected. We also included the number of actors in each network.

As in the previous discussion, we do not assume that the errors for agreement and volatility are uncorrelated. We model agreement and volatility via a seemingly unrelated regression:

$$Agreement = \beta_0 + \beta_1 E + \beta_2 J + \beta_3 Density + \beta_4 Cluster + \beta_5 Symmetry + \beta_6 Components + \sum_{i \in \{1,\ldots,7\}} \alpha_i Type_i +_i \varepsilon,$$

$$Volatility = \beta_0' + \beta_1' E + \beta_2' J + \beta_3 Density + \beta_4 Cluster + \beta_5 Symmetry + \beta_6 Components + \sum_{i \in \{1,\ldots,7\}} \alpha_i Type_i +_i \varepsilon', \quad (14)$$

and

$$V(\varepsilon, \varepsilon^{'}) = v\,,\;\; E(\varepsilon) = 0\,,\;\; E(\varepsilon') = 0\,. \qquad (15)$$

Table 4 shows the results of estimating the full model specified above and models with some of the omitted variables. Model 1 is the model with only *E* and *J*. Models 2 and 3 exclude either network statistics or dummy network-type variables. Model 4 includes all the variables.

The effects of *E* and *J* on agreement are predictable. They are both positive and relatively large. Furthermore, they do not change much when other variables are included in the analysis. The effects of *J* on volatility are negative, which is also predictable. A strong external influence should reduce variance in the level of agreement. However, interpersonal influence (*E*) has a very small effect on agreement variance, which is consistent with the earlier analyses presented in Table 3. This small *E* effect was found when the models were estimated separately for different kinds of networks and when the simulation data was pooled together.

If network statistics, such as density, are not included in the analysis, all dummy variables have significant effects. The inclusion of network statistics causes the dummy variable for the Krackhardt reporting network not to be statistically significant at the α =0.05 level in model 4. The dummy variable for the Krackhardt reporting network is significant at the α =0.10 in model 4, while it is significant at the α =0.001 level in model 3. This should not be surprising. Tree structures are the omitted category in the analysis, and the reporting network is constructed in a fashion similar to the tree. This is a nice assurance that the computational model produces plausible results—networks created from similar decision rules are statistically indistinguishable in their simulated agreement levels.

Compared to the omitted category (tree structures), all the dummy variables have positive effects on agreement. The inclusion of variables for density, clustering, size, and symmetry increase the magnitude of these effects. For example, the effect of the dummy variables for the Padgett business network increases from $\beta_3 = .080$ to $\beta_3 = .122$. Using the definition of the agreement variable, model 4 predicts that 56% of the actors in the Padgett business network will hold the same opinion, without including the effects of the other variables. The analogous effect in model 3 is 52%. The results are similar for other graph types.

Model 4 allows for the comparison of *E*, *J*, and various network structure effects. The first observation is that *E* and *J* have larger effects on agreement than every other variable, except for the dummy variable for the Krackhardt friendship network. This suggests that many common network structures, by themselves, do not increase agreement more than tie strength or external influence. However, this does not mean that network structure has no effect—in fact, two networks have effects larger than 0.05 and the rest mildly contribute to agreement, with effects between 0.01 and 0.05. There is a consistent story with network statistics. Except for density and clustering, network statistics tend to have very small effects in both models 3 and 4. We conclude that density and clustering contribute a great deal to agreement, while other measures of network structure do not.

The story is somewhat different for agreement variation. Table 4 shows that the variables with largest positive effects on agreement variance are density, clustering and the dummy variable for the Krackhardt friendship structure. This last variable almost completely suppresses variance, suggesting that the friendship network is highly idiosyncratic. Density and clustering have positive effects, which indicate that when highly connected individuals change their mind, it reverberates through the network and creates change. One might expect density to stabilize agreement, but in a nondeterministic model density seems to contribute to instability.

Summary and Concluding Remarks

We found evidence that interaction patterns strongly affect simulated aggregate opinion levels. In our simulation, a group's average aggregate opinion level dramatically changes as we alter the network structure. We present less obvious and more interesting findings. First, we found that some structures have particularly higher opinion levels than others and that some network structures did not encourage much more agreement than the random density network in which 5% of pairs of individuals were tied. This is an important finding from the perspective of organizational design—networks in which individuals influence each other through lines of authority, e.g. the Krackhardt reporting network, exhibited agreement levels below the network in which 5% of potential dyads are connected. Thus, if agreement among workers is a desirable goal, then management theorists should consider incorporating informal relationships into theories of organizational design.

We also found evidence that interpersonal influence effects vary greatly by network structure. For some networks, the estimated effect of E on the aggregate opinion level is less than 0.05; this coefficient is much greater for other networks. This means that even if individuals in certain networks, such as the Krackhardt reporting network, strongly influence each other, in-group agreement does not necessarily follow. This is an interesting finding—some networks are able to almost completely suppress interpersonal influence as a factor in group cohesion. Further theorizing on the relationship between social structure and opinion formation might take into account the possibility that social structures by themselves mitigate the effects of "strong ties."

The effect of the external influence (*J)* is large and consistently larger than the effect of interpersonal influence (*E)* on agreement. Only in one network, the tree structure, did *J* cease to have a positive effect on aggregate opinion levels. We found little evidence that network structure mitigates the effects of external influences. This lack of an effect might be due to a fundamental assumption of the opinion change model—*J* influences all individuals in every time period. In contrast, the *E* parameter has an effect only when there is a great deal of disagreement between an individual and those in his network neighborhood. Thus, the system of neighborhoods probably determines the *E* effect on agreement, while the effect of *J* is not changed except for the most unusual network structures. The effects of *E* and *J* on agreement variance are predictable—mostly positive for *E* and negative for *J*, with the tree structure once again having anomalous effects.

We tested the hypothesis that agreement effects are due to unmeasured attributes of the graph itself by estimating models with dummy variables for each type of network as well as standard network measures such as density. The model with both sorts of variables was the best-fitting model and many of the network dummy variables retained their statistical significance. Our analysis of the model simulations suggests the following hypotheses:

> Hypothesis 1: The rankings reported in Table 2 show which contact patterns have the highest effects on in-group agreement, controlling for other factors. Some contact patterns have extremely low agreement levels when compared to the network produced by linking 5% of all possible dyads.

> Hypothesis 2: Increases in the average strength of interpersonal influence do not always lead to increases in the number of individuals who hold the same opinion. The increase is affected by the pattern of contacts within the network. For example, the Krackhardt friendship network displays a large interpersonal influence effect while the small worlds network does not.

> Hypothesis 3: External vs. internal influence effects – The effect of external influence on agreement will be larger than the effect of interpersonal influence for the networks examined in this study.

The model simulations suggest questions for future research. The previous section presented evidence indicating that unmeasured features of an influence network might account for some shared opinions. Future research could investigate a wider range of network measures, aside from measures of connectedness and density, centralization. For example, we found relatively difference among networks in volatility, the tendency for aggregate opinion to swing. This stability might be due to the presence of groups that anchor opinion within the larger group. Another avenue for research would be to apply the model to the study of situations where individuals influence each other through a given pattern of ties. For example, focus groups might be usefully described a group of individuals talking to each other who are all exposed to a moderator, who might have a fixed uniform effect on the participants. The hypotheses here suggest how the patterns of contact affect the outcome of a focus group session.

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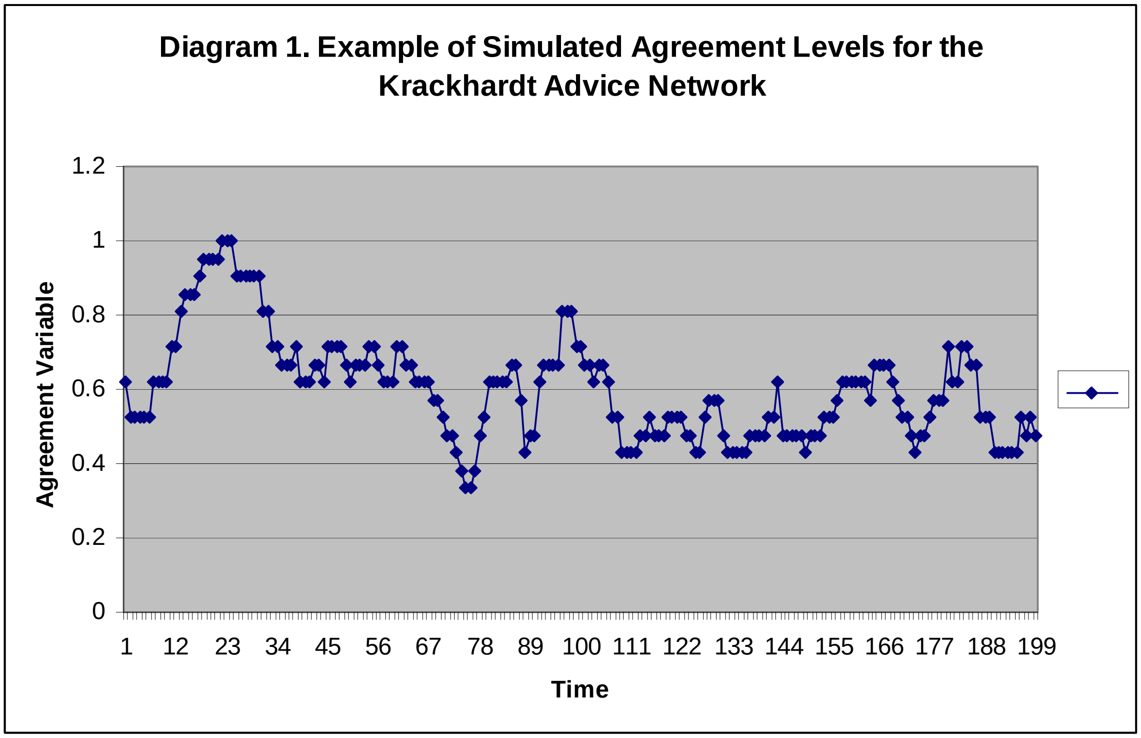
Diagram 1. Example of Simulated Agreement Levels for the
Krackhardt Advice Network
Agreement Variable
1.2
1
0.8
0.6
0.4
0.2
0
1 12 23 34 45 56 67 78 89 100 111 122 133 144 155 166 177 188 199
Time

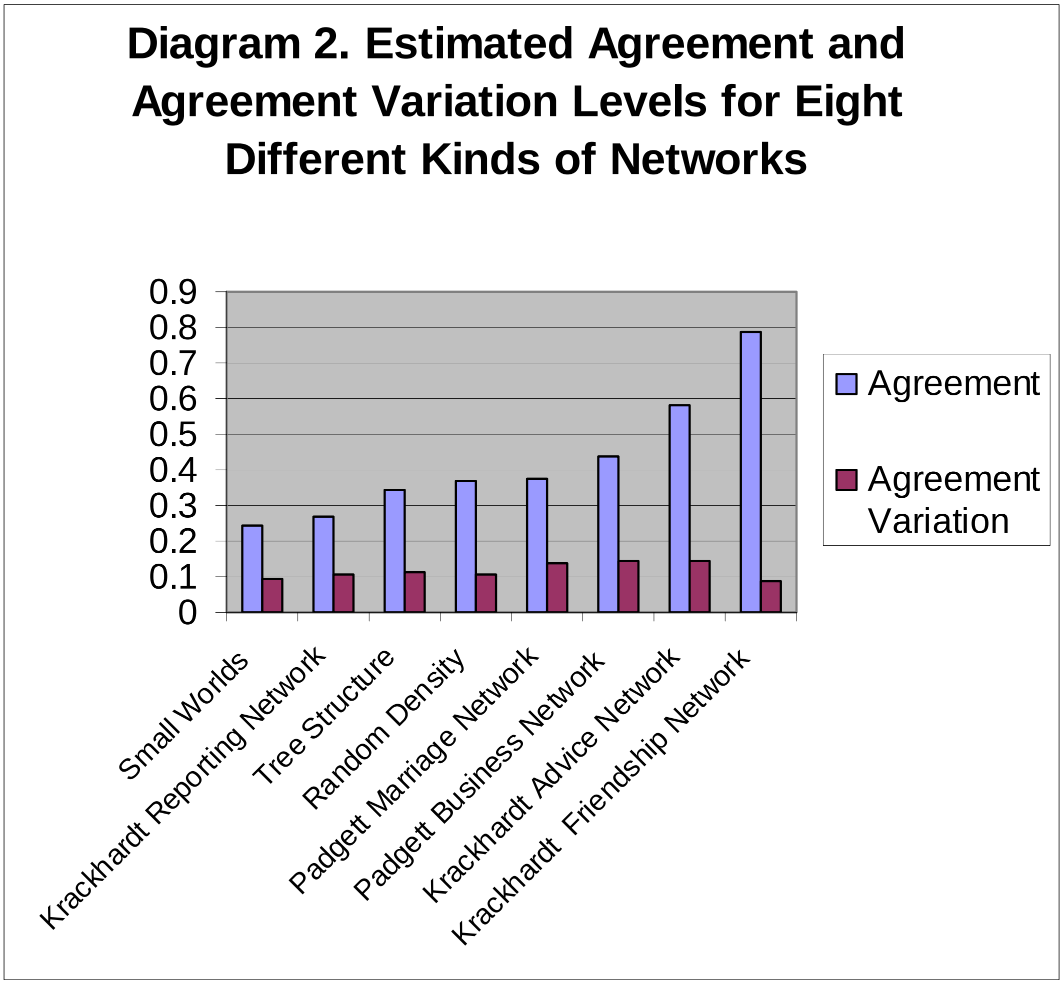
Diagram 2. Estimated Agreement and Agreement Variation Levels for Eight Different Kinds of Networks
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Agreement
Agreement Variation
Small Worlds
Krackhardt Reporting Network
Tree Structure
Random Density
Padgett Marriage Network
Padgett Business Network
Krackhardt Advice Network
Krackhardt Friendship Network

Table 1. Descriptive Statistics of Simulated Data

Variable	Observations	Mean	Standard Deviation	Min	Max
Agreement Measure	53520	0.378	0.256	0.000	0.996
Standard Deviation of Agreement	53520	0.110	0.049	0.013	0.469
External Influence (*J*)	53520	0.489	0.283	0.000	1.000
Interpersonal Influence (*E*)	53520	0.489	0.283	0.000	1.000
Number of Actors	53520	33.724	29.786	16.000	100.000
Clustering Coefficient	53520	0.091	0.151	0.000	0.541
Number of Connected Components	53520	6.393	6.801	1.000	27.000
Density of Network	53520	0.126	0.118	0.003	0.431
Symmetry	53520	0.626	0.430	0.000	1.000

Table 2. Mean Simulated Agreement Level for Each Type of Network (*N*=6690)

Name of Network	Agreement	Agreement Variance
Padgett Marriage Network	0.378	0.135
Padgett Business Network	0.438	0.141
Krackhardt Friendship Network	0.785	0.088
Krackhardt Advice Network	0.584	0.145
Krackhardt Reporting Network	0.269	0.109
Small Worlds	0.243	0.093
Random Density	0.366	0.105
Tree Structure	0.345	0.111

Table 3. Effects of *E* and *J* on Agreement and Agreement Variance by Type of Network (*N*=6690 each category)

Type of Network	Agreement			Agreement Variance		
	E	*J*	R^2	*E*	*J*	R^2
Padgett Marriage Network	0.310 *** (.002)	0.624 *** (.002)	0.951	0.024 *** (.009)	-0.083 *** (.0009)	0.684
Padgett Business Network	0.440 *** (.003)	0.661 *** (.003)	0.940	0.028 *** (.001)	-0.111 *** (.001)	0.648
Krackhardt Friendship Network	0.720 *** (.009)	0.333 *** (.009)	0.653	-0.134 *** (.004)	-0.108 *** (.004)	0.303
Krackhardt Advice Network	0.616 *** (.006)	0.681 *** (.006)	0.841	0.046 *** (.003)	-0.222 *** (.003)	0.488
Krackhardt Reporting Network	0.029 *** (.0006)	0.489 *** (.0006)	0.994	0.006 *** (.0001)	-0.018 *** (.0001)	0.812
Small Worlds	0.031 *** (.0008)	0.497 *** (.0008)	0.971	0.002 *** (.001)	-0.017 *** (.001)	0.030
Random Density	0.316 *** (.001)	0.638 *** (.001)	0.956	0.022 *** (.001)	-0.055 *** (.001)	0.178
Tree Structure	0.598 (.005)	-0.230 (.005)	0.530	-0.012 (.001)	0.014 *** (.001)	0.018

*** - P<0.001. Standard Errors in Parentheses.

Table 4. Effects of *E*, *J*, and Network Characteristics on Agreement and Agreement Variation (*N*=53520)

	Model 1		Model 2		Model 3		Model 4	
Agreement	Coef.	*P*	Coef.	*P*	Coef.	*P*	Coef.	*P*
Interpersonal Influence (*E*)	0.359 (.003)	0.000	0.350 (.003)	0.000	0.351 (.002)	0.000	0.351 (.002)	0.000
External Influence (*J*)	0.395 (.003)	0.000	0.386 (.002)	0.000	0.387 (.002)	0.000	0.387 (.002)	0.000
Padgett Marriage Network	-	-	-	-	0.020 (.002)	0.000	0.100 (.009)	0.000
Padgett Business Network	-	-	-	-	0.080 (.002)	0.000	0.122 (.005)	0.000
Krackhardt Friendship Network	-	-	-	-	0.427 (.002)	0.000	0.481 (.020)	0.000
Krackhardt Advice Network	-	-	-	-	0.226 (.002)	0.000	0.292 (.016)	0.000
Krackhardt Reporting Network	-	-	-	-	-0.089 (.002)	0.000	-0.030 (.017)	0.090
Small Worlds	-	-	-	-	-0.103 (.002)	0.000	-0.043 (.0193)	0.028
Random Density	-	-	-	-	0.020 (.002)	0.000	0.048 (.003)	0.000
Density of Network	-	-	0.557 (.010)	0.000	-	-	0.242 (.019)	0.000
Clustering Coefficient	-	-	0.494 (.006)	0.000	-	-	-0.197 (.028)	0.000
Symmetry	-	-	-0.009 (.002)	0.000	-	-	0.019 (.017)	0.282
Number of Actors	-	-	0.001 (.00002)	0.000	-	-	0.00001 (.00003)	0.000
Number of Connected Components	-	-	-0.003 (.0002)	0.000	-	-	-0.0001 (.0003)	0.696
Constant	0.009 (.002)	0.000	-0.110 (.0008)	0.000	-0.011 (.002)	0.000	-0.087 (.018)	0.000
R^2	0.348		0.612		0.634		0.636	
χ^2	0.077		84474.800		92744.120		93445.790	
Agreement Variance								
Interpersonal Influence (*E*)	0.002	0.011	0.004	0.000	0.001	0.232	0.004	0.000

	(.0007)		(.0005)		(.0006)		(.0005)	
External Influence (*J*)	-0.048 (.0005)	0.000	-0.046 (.0005)	0.000	-0.049 (.0006)	0.000	-0.046 (.0005)	0.000
Padgett Marriage Network	-	-	-	-	0.025 (.0008)	0.000	0.007 (.002)	0.003
Padgett Business Network	-	-	-	-	0.030 (.0008)	0.000	0.014 (.001)	0.000
Krackhardt Friendship Network	-	-	-	-	-0.022 (.0008)	0.000	-0.117 (.004)	0.000
Krackhardt Advice Network	-	-	-	-	0.035 (.008)	0.000	-0.009 (.003)	0.013
Krackhardt Reporting Network	-	-	-	-	-0.001 (.008)	0.176	0.016 (.004)	0.000
Small Worlds	-	-	-	-	-0.018 (.005)	0.000	0.025 (.007)	0.000
Random Density	-	-	-	-	-0.006 (.005)	0.000	0.020 (.004)	0.000
Density of Network	-	-	-0.031 (.002)	0.000	-	-	0.187 (.006)	0.000
Clustering Coefficient	-	-	0.004 (.001)	0.018	-	-	0.076 (.004)	0.000
Symmetry	-	-	0.014 (.0005)	0.000	-	-	-0.005 (.004)	0.245
Number of Actors	-	-	-0.001 ($6.02x10^{6}$)	0.000	-	-	-0.001 ($8.09x10^{6}$)	0.000
Number of Connected Components	-	-	0.000 (.0004)	0.000	-	-	-0.001 (.00007)	0.000
Constant	0.133 (.0005)	0.000	0.158 (.008)	0.000	0.134 (.006)	0.000	0.125	0.000
R^2	0.077		0.383		0.202		0.455	
χ^2	4439.220		33172.880		13530.220		44609.310	