



Minimal length in quantum gravity and the fate of Lorentz invariance

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ABSTRACT

The paper highlights a recent debate in the quantum gravity community on the status of Lorentz invariance in theories that introduce a fundamental length scale, and in particular in deformed special relativity. Two arguments marshaled against that theory are examined and found wanting.

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1. Introduction

The construction of new physics is among the most fascinating scientific activities, for it requires an extremely delicate balance between conservatism and innovation. The new theory, of course, cannot start out “from scratch”, and so, apart from demonstrating internal mathematical consistency it should, in general, conform to well-established phenomenological principles and to current empirical data. But, arguably, explaining “old evidence” should not be its *final* goal, especially when rival contenders offer equally plausible explanations. In order to be counted as more than a mere possibility proof—yet another consistent formalism that can reproduce all known predictions of earlier theories—the new theory should also generate some predictions in the new domain it purports to describe, that can—at least in principle—be experimentally confirmed.

Theoretical physicists often anticipate the final stage of this process and come up with heuristic arguments that are sufficient to uncover the empirical consequences of new theories even before precise predictions can be stated. For philosophers who defy Otto von Bismarck’s famous quip (“Laws are like sausages, it

is better not to see them being made”), this state of affairs allows an interesting glimpse into the actual practice of physics.

Such an opportunity presents itself today in the domain of quantum gravity (QG), where different theoretical frameworks struggle to unify quantum mechanics with general relativity. In this paper I focus on one of the features that many of these frameworks share, namely the notion of a minimal—or fundamental—length, and on the lively debate that has ensued on the phenomenological consequences thereof. In particular, I shall examine two theses that have emerged in this context: (a) the idea that minimal length ultimately entails deviations from exact Lorentz invariance and (b) the idea that such deviations inevitably indicate a violation of the principle of relativity by singling out a preferred frame. In recent years a possible surrogate has surfaced, (Amelino Camelia, 2000, 2001; Magueijo & Smolin, 2002; Smolin, 2005), which re-interprets the consequence of the fundamental length in QG as a non-linear deformation of the action of the Lorentz group (hence the name, DSR, for deformed special relativity), and in so doing accepts (a) and rejects (b).

Modifying the standard energy–momentum dispersion relations to include an additional observer-independent scale, DSR has been developed so far to little more than a speculative kinematical structure, a non-trivial Minkowski limit of a hypothetical solution to the quantum gravity problem based on a discrete space(time) picture in which the Planck length plays a

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fundamental role. As such it is highly controversial. First, the question of how such a structure may fit as a research program within QG is still open. Second, although attempts have been made to translate it into a consistent field-theoretic framework, (Judes & Visser, 2003; Kimberly et al., 2004; Hossenfelder, 2006, 2007), no such generally accepted and unique translation exists, each attempt leading to different phenomenological consequences. Finally, current experiments, in the highest energies we can manage (See, e.g., Adamson et al., 2008), show no sign of departure from exact Lorentz symmetry.

While so far there seems to be little physical motivation for deforming the standard energy–momentum dispersion relations (apart from the fact that there are good reasons to think that a fundamental QG theory will involve spatial discreteness), from the methodological perspective I am interested in here the attitude within the QG community towards DSR exemplifies nicely the aforementioned delicate balance between conservatism and innovation. On one hand one would like to establish “quantum gravity phenomenology”, and thus to transcend the old pessimistic dimensional argument (e.g., Isham, 1995), that says we would never be able to test quantum gravity effects, as the only physical regime where these effects might be studied directly is in the immediate post big-bang era of the universe—not the easiest thing to probe experimentally; on the other hand one would like to keep intact as much “old” physics as possible, Lorentz invariance being one of the pillars of quantum field theory. In this paper I flesh out this intriguing situation by focusing on two challenges DSR faces: the first aims to undermine (a) by arguing for a compatibility between discrete spatial geometry and Lorentz invariance (Rovelli & Speziale, 2003), the second aims to establish (b) by arguing that *any* non-linear energy-dependent deformation of the Lorentz symmetry group will single out a preferred frame (Schüzhold & Unruh, 2003).

The plan of the paper is as follows. In Sections 2 and 3 I briefly set the stage, presenting the intuition that has led to DSR. In Section 4 I analyze the two challenges DSR faces, pointing in Section 5 at a methodological symptom these challenges share that makes them unconvincing as arguments against DSR. Limited conclusions are given in Section 6.

2. Why fundamental length?

Is there any theoretical reason to expect space to be discrete rather than continuous? Apart from the recent “computational” view of nature (e.g., Wolfram, 2001), that seems to require such a fundamental discreteness, in theoretical physics the view that space, or geometry, are “quantized” can be found in some of the attempts to solve the quantum gravity problem. Recall that this problem, at least in its scientific facet,¹ is one of producing predictions, in a logically consistent way, for situations in which *both* gravitational (general relativistic) effects and particle physics (quantum field-theoretic) effects cannot be neglected.²

¹ Since in the particle physics approach to this problem gravity is subsumed as just another gauge interaction, the above distinction between particle physics and gravity may seem artificial. However, I take it that even in this approach, e.g., string theory, theoreticians are interested in producing logically consistent predictions.

² For reasons that would become clear below, in what follows I shall consciously ignore an opposite view, that has emerged since the early 1980s, according to which the problems of quantizing gravity within the experimentally accessible situations are similar to those which arise in a host of other non-gravitational applications throughout physics where non-renormalizable theories are involved. On this view, namely effective field theory (EFT), the “quantum gravity problem” is defined differently, as the size of quantum corrections in gravitational effects is estimated to be extremely small, practically negligible in most of the present experimental tests of gravity. For an exposition see, e.g., Burgess (2004).

Early in the 1940s, when Heisenberg was developing his S-matrix theory, he observed that in order to avoid divergences (e.g., infinite self-energy of the electron or infinite polarization of the vacuum), the theory of elementary particles must contain a fundamental length scale.³ This intuition, shared by several other physicists at that time (Kragh, 1995), was later formulated more precisely by Mead who, while elaborating on a famous thought experiment (Bohr & Rosenfeld, 1933), showed that the source of the QFT divergences could be traced to the twofold assumption that arbitrary small particles (or point particles) can exist “in principle”, and that they can be localized with infinite precision and thus serve (via local interactions) as test bodies for measuring the various field quantities. Yet if one assumes that no particle can be localized more closely than its Compton wavelength (without losing its identity as a single particle), one could postulate that no elementary particle can exist with a radius smaller than a certain fundamental length, which would lead to natural cutoff for the divergent integrals. The introduction of a fundamental length scale amounts to, then, a limitation on the possibility of measurement. If the mass of the particle is m , its position would be uncertain by $\Delta x \geq 1/m$, hence the fundamental length scale (assuming an order of magnitude of the mass of the heaviest known baryon) would be $l \sim 10^{-16}$ cm (Mead, 1964, pp. B849–B850).

The success of the renormalization program overshadowed the interest in fundamental length within QFT, but this interest was revived when it was realized that fundamental length could also arise when one considers generalizations of general relativity from a quantum field-theoretic perspective.⁴ General relativity involves a characteristic length, namely the Schwarzschild radius, that tells us (via Einstein’s field equations) when energy densities and masses, concentrated in a given region, reach the point at which gravitational effects cannot be ignored. Fundamental length appears here when one considers, say, a massive particle whose Compton wavelength is near to its Schwarzschild radius. To describe such a case one would require *both* general relativity and quantum field theory, hence, presumably, quantum gravity. In such a theory where both QFT and gravitational interactions are taken into account, or so the argument goes,⁵ the fundamental length would be operationally equivalent to fluctuations in the gravitational field.

These theoretical considerations introduce the fundamental length into QG as both a consistency requirement (i.e., divergences cutoffs) and as an operational restriction on measurements of spatio-temporal relations (when Planck’s length is regarded as an absolute lower limit for the operational distinguishability of spacetime locations). Remarkably, many contenders to the solution of the quantum gravity problem do, in fact, involve this notion (Garay, 1995). One such theory is loop quantum gravity (LQG): combining the two fundamental principles of general

³ Cushing (1982, p. 46); for more references on the notion of fundamental length in field theory see Prugovecki (1995, pp. 9–12).

⁴ E.g., Pauli, commenting on a talk by Klein (1956) in the 50th anniversary of relativity theory, is quoted as saying:

It is possible that this new situation [indeterminacy in spacetime of the light cone—AH] so different from quantized theories invariant with respect to the LORENTZ group only, may help to overcome the divergence difficulties which are so intimately connected with a c-number for the light-cone in the latter theories.

I thank an anonymous referee for this reference. Other famous landmarks in this approach are Deser (1957) and Wheeler (1957).

⁵ See, e.g., Mead (1964, pp. B857–B860). Note that the putative fundamental length need not be necessarily the Planck length ($L_p = \sqrt{\hbar G/c^3} \sim 1.6 \times 10^{-33}$ cm). However, by combining the two characteristic lengths of quantum field theory and general relativity one obtains the fundamental length at that order. See, e.g., Calmet et al. (2004) and Calmet (2008).

relativity—background independence and diffeomorphism invariance—with the standard techniques of quantum mechanics, LQG derives spatial discreteness by predicting that some operators representing geometrical quantities have discrete spectra (Rovelli & Smolin, 1995; Rovelli, 2004, pp. 249–259). These operators are not themselves gauge-invariant,⁶ but it has been argued that under plausible assumptions their spectra provide evidence for physical discreteness of space.⁷

Given the tiny order of magnitude of this feature,⁸ and the fact that current experiments are known not to provide anywhere near the correct resolution to reveal it, the obvious question is whether such a prediction has any observable consequences. It is one thing (and a truly admirable one at that) to give a consistency proof for a putative theory of QG by reproducing classical general relativity as an approximation, (Binachi et al., 2006), or by precisely calculating Bekenstein's and Hawking's results for black hole entropy and radiation, respectively (Ashtekar et al., 1998; Rovelli, 2004, pp. 308–312); it is quite another to point at novel empirical tests that would lend support to the ontological claim regarding discrete spatial geometry, over and above the mere plausibility of such a claim in a consistent theory. This is just another way of saying that spatial discreteness is, on final account, a contingent matter of fact, and not a matter of methodological predilections, in this case, a predilection about the unification of quantum mechanics with general relativity and an urge to quantize the gravitational field (See, e.g., Rosenfeld, 1963; Callender & Huggett, 1999, pp. 5–13).

Whence the idea, explored next, that discrete spatial geometry might entail a departure from exact Lorentz invariance at energy regimes close to the Planck scale, that may be indirectly probed in cosmological contexts.

3. Does physical discreteness break local Lorentz invariance?

3.1. The basic intuition

As everybody knows, “[m]oving clocks run slowly; moving sticks shrink” (Mermin, 2005, chap. 6). What happens, however, when a stick has the minimal length, hence it cannot shrink any further when set in motion relative to an observer? Put differently, does local Lorentz invariance break down when space becomes discrete?

Prima facie, a lattice breaks continuous rotational and translational symmetries. To see this think of a cube. It has many rotational symmetries. It can be rotated by 90°, 180°, or 270° about any of the three axes passing through the faces. It can be rotated by 120° or 240° about the corners, and by 180° about an axis passing from the center through the midpoint of any of the 12 edges. Now think of a sphere. It can be rotated by *any* angle. In this sense the sphere respects rotational invariance: all directions are on a par. The cube, on the other hand, is an object which breaks rotational invariance: once the cube is there, some directions are more equal than others. In a similar vein, a lattice breaks translation invariance: it is easy to tell if a lattice is shifted sideways, unless one shifts it by a whole number of lattice units.

This intuition seems to imply that the introduction of a fundamental lattice structure to space will result in violations of local Lorentz symmetry, as any particular choice of lattice would favor the spatio-temporal directions directed by the adopted lattice points. For example, we would expect Lorentz boost symmetry violations to be manifest in small-scale corrections (or deformation) of dispersion relations $E^2 = c^2 p^2 + m^2 c^4$, just as the atomic structure of matter modifies continuum dispersion relations once the wavelength becomes comparable to the lattice size. From a phenomenological perspective, under such deformed dispersion relations the velocity of light might become energy (or energy-density) dependent.

Does this mean a return to the notion of a preferred frame? Does LQG take us back to the 19th century ether? Early calculations of such deviations from Lorentz symmetry indeed endorsed such a notion (See, e.g., Gambini & Pullin, 1999). It was also admitted that the predicted deviation was mediated through the breaking of CPT symmetry.⁹

Since the predicted modifications of the dispersion relations depend on (energy/length) scale, current experiments, which confirm exact Lorentz invariance and CPT symmetry and give no clue to deviation therefrom, are insufficient to rule them out.¹⁰ However, there is an ongoing attempt within the framework of effective field theory to derive possible constraints that can be imposed on violations of Lorentz invariance and CPT symmetry (See, e.g., Myers & Pospelov, 2003).

One argument that seems to emerge in this context is that if there were some deviations from exact Lorentz invariance and CPT symmetry, they should have appeared in energy levels much lower than those predicted (Collins et al., 2004). In particular, or so this argument goes, the calculations of the deviations from exact Lorentz invariance in theories that predict its breakdown, that locate the observational window for these effects beyond current accessible data are erroneous and misleading as they are dealing only with free particles and neglecting standard interaction terms. However, the argument continues, once one includes known elementary interactions in the calculation, and in particular self-energy, the predicted violations are enormously amplified to levels which have already been probed, and in which Lorentz invariance was found to be exact. Since we have seen no departure from exact Lorentz invariance in those low-energy regimes, the argument concludes, theories that predict the breakdown of Lorentz invariance and CPT symmetry have either been already falsified, or require an *ad hoc* fine-tuning that makes them methodologically suspect. Other constraints that are cited as sufficient to dampen enthusiasm for improved searches for Lorentz invariance violations and that purport to lend support to the claim that the existing unsuccessful searches suffice by many orders of magnitude come from astrophysics (Jacobson, Liberati, & Mattingly, 2003). Combined with the above EFT-based constraints, these considerations seem to suggest that Lorentz invariance and CPT violations suppressed by the ratio E/E_p (where E_p is Planck energy) are already ruled out by experiments.¹¹

The thrust of these considerations, however, is still under dispute. First, as we shall see below, interaction is a vastly open issue in QG; it is hard to see how one can make a decisive argument based on it. Second, while it should conform to the

⁶ For a recent debate on this issue see Dittrich and Theimann (2007) vs. Rovelli (2007).

⁷ Rovelli (1993), Smolin (1994), and Rovelli (2007, pp. 4–5). Let me emphasize that for the purpose of this paper (which is to highlight a specific *methodological* practice), it is immaterial whether, on final account, these geometrical operators are *truly* physical. What's important is that some physicists who believe so come up with heuristic arguments the aim of which is to uncover the phenomenological consequences of this prediction.

⁸ The smallest possible nonzero volume being about a cube of the Planck length, i.e., 10^{-99} cm³.

⁹ For more on this connection see the research program of A. Kosteletsky, e.g., Kosteletsky (2001). Recall that the CPT theorem, e.g., Bell (1955), says that violations of CPT symmetry imply violations of Lorentz invariance, but not vice versa.

¹⁰ Analogously, one would not think of ruling out special relativity simply because one's bare eyes fail to detect time-dilation.

¹¹ Quadratic modifications, however, are not yet ruled out by these considerations. See, e.g., Amelino Camelia (2003b) for a proposal.

predictions of EFT in the low-energy regime, any theory of QG also purports to describe nature in a particularly confined scale of extremely high energy, exactly where EFT is inapplicable (Weinberg, 2009; see also Mattingly, 2008, pp. 5–6):

None of the renormalizable versions of [electrodynamics, the electroweak theory, quantum chromodynamics and even General Relativity] really describes nature at very high energy, where the non-renormalizable terms in the theory are not suppressed ... All of these theories lose their predictive power at a sufficiently high energy. The challenge for the future is to find the final underlying theory, to which the effective field theories of the standard model and General Relativity are low-energy approximations.

Finally, even more important is the fact that the two central features of this interesting debate on the appropriate energy scale for probing QG effects are unacceptable from the perspective of LQG. As a background independent theory, LQG admits no frames, preferred or otherwise. Moreover, if the goal is to point at an empirical test of an underlying discrete space, then one should look for signatures of this feature that are not masked by additional mechanisms such as CPT violations (Jacobson et al., 2003, p. 1021; Smolin, 2005).

3.2. Deformed special relativity

Enter DSR (Amelino Camelia, 2000, 2001; Magueijo & Smolin, 2002, 2003). At its base lies the premise that special relativity (characterized by Minkowski spacetime, or equivalently, by the Lorentz covariance of the fundamental non-gravitational interactions) is only an approximate theory and may not be applicable when (quantum) gravitational interactions *are* present. What DSR offers is a modification of special relativity that is not committed to a preferred frame, and links deviations from exact Lorentz invariance to the existence of a fundamental length by introducing the latter as an additional observer-independent scale in the kinematical structure, over and above c (the velocity of light).

A historical analogy might prove instructive here (Amelino Camelia, 2000). One could describe the transition from Galilean spacetime to Minkowski spacetime as a consequence of the introduction of an observer-independent velocity scale, namely c . Once we accept c as such (“the light postulate”), the kinematical structure employed by the theory changes, and with it the symmetry group that characterizes this structure. Consequently, the simple velocity addition law in Galilean relativity (which involves no velocity scale) $v' = v_0 + v$ is “deformed” and is replaced with the velocity addition law that takes c into account: $v' = (v_0 + v)/(1 + v_0 v/c^2)$. Clearly, what this means is that even if the mathematical structure that best characterizes our spacetime is Minkowskian, in small enough velocities we could still approximate it as Galilean, while keeping in mind, of course, that there are no preferred frames (i.e., that simultaneity is still relative).

DSR involves a similar kinematical shift. It introduces yet another observer-independent scale, namely a fundamental length scale of the order of Planck length.¹² By accepting this length scale, we could still maintain the principle of relativity (i.e., that the laws of physics take the same form in all inertial frames) and just modify the transformation rules between frames to preserve the new scale (as we did in the shift from Galilean to Lorentz invariance). Once more, this means that while the mathematical structure that best describes our space is discrete, in low enough energies (or large enough wavelengths) we could

still approximate it as continuous. In order to accommodate a fundamental length (or, equivalently, a maximum energy) as an observer-independent scale in the theory, DSR suggests a non-linear modification of the action of the Lorentz group on the momentum space. As one would expect, the consequences of this deformation for conservation and composition laws are highly non-trivial (Magueijo & Smolin, 2003; Judes & Visser, 2003; Girelli & Livine, 2005).

At this stage there are several strategies to translate the kinematical structure of DSR into a consistent field-theoretic framework that can generate testable predictions. While all these strategies agree on the starting point, namely a Planck scale modification of the dispersion relation between energy and momentum, they disagree on the possible consequences of these modifications, e.g., possible variations in the velocities of massless particles, and, if so, whether these variations depend on energy or energy-density (See Hossenfelder, 2007). While both these approaches lead to in-principle falsifiable predictions, in forthcoming experiments with gamma-ray bursts (GLAST) the former proves testable in practice while the latter does not. Be that as it may, since these predictions are *different* from a straightforward breakdown of Lorentz invariance (and at least in the former case, also from exactness thereof), proponents of DSR see it as a promising route to the much sought for “quantum gravity phenomenology” (Smolin, 2005 and Amelino Camelia & Smolin, 2009).

From a broader perspective, one may ask what role DSR should play in the QG research program. One way to answer this question is, again, with the analogy to special relativity. The latter may be described as a kinematical constraint on the non-gravitational interactions, quantum-field-theoretic laws being Lorentz covariant. It is thus plausible to assign DSR a similar role in constraining the dynamics of future QG theories which extend QFT-interactions to include gravity. Such an analogy fits well with one of the major requirements of DSR, namely the requirement to approximate special relativity (and the Lorentz group) in regimes where gravitational interactions can be neglected. Compelling as it may seem, however, this analogy must be handled with care; while not fatal to the project, there is an important disanalogy here which should be pointed out. Since all observers agree on the physics once the fundamental length scale is factored into their measurements, DSR establishes a deformation of special relativity without giving up the principle of relativity itself. But how, in the framework of this theory, can we probe the ontological claim that space is fundamentally discrete, or that the Planck length is *physically* meaningful? After all, the fact that by combining other physical scales we can construct L_p with dimensions of a length is immaterial to its meaningfulness. In particular, while c enjoys a robust body of data suggesting its physical interpretation as the speed of light, so far we have no hint on the physical interpretation of L_p .

This disanalogy underlines what some see as the main problem with DSR, namely the lack of physical motivation for deforming special relativity over and above the theoretical intuition regarding spatial discreteness and the hope to turn QG testable. Here one should note that in its early days DSR did in fact enjoy such a motivation, namely anomalous observations in energy thresholds of cosmic rays.¹³ However, discouragingly for its followers, it is now acknowledged that DSR cannot (and does not) predict these anomalies (Amelino Camelia, 2007a). Other try to motivate the theory from cosmological considerations, (Magueijo, 2003), but these attempts also lack an empirical underpinning. On the other

¹² Thus some interpret DSR to read *Doubly* Special Relativity.

¹³ The anomaly, known as ultra-high-energy cosmic ray, was first observed by Linsley (1963).

hand, special relativity has been tested again and again, and still remains “in remarkably rude health” (Amelino Camelia, 2007b, p. 802).

Note, however, that the absence of empirical motivation is not unique to DSR; in fact *all* competing QG programs currently share this problem. Neutral with respect this issue as I am, my sole intention in what follows is to defend DSR against a different type of criticism, one which relies not on (missing) empirical grounds but rather on *a priori* reasoning.

4. Conservatism vs. innovation

Taking stock, a number of theoretical considerations from relativistic quantum mechanics and general relativity seem to indicate that a consistent theory of QG should involve a fundamental length scale, either as an operational limit on the distinguishability of spacetime locations, or as a physically meaningful feature arising from the spatial discreteness. One possible and highly controversial consequence of this feature is a non-linear deformation of the action of the symmetry group of the special theory of relativity, that allows Lorentz invariance to emerge as an exact symmetry in the low energy limit. By keeping the additional length scale introduced into the theory observer-independent, such a deformation is still consistent with the principle of relativity: it does not single out a preferred frame. Nevertheless, it still has phenomenological consequences, such as energy (or energy-density) dependence of the velocity of massless particles. The more enthusiastic theoreticians believe the situation serves to demonstrate that the field of quantum gravity has matured to the point it can make contact with experiments (Smolin, 2006 and Amelino Camelia & Smolin, 2009).

Since its conception, DSR has generated a heated debate within the QG community, that can be nicely characterized according to the following two theses: (a) minimal length implies deformation of Lorentz invariance, and (b) these deformations single out a preferred frame hence imply a violation of the principle of relativity. As explained, DSR accepts (a) and rejects (b). In what follows we shall discuss two arguments to the contrary: the first rejects (a); the second aims to demonstrate that if one accepts (a) then (b) *must* follow. The claim I would like to defend here is that these two arguments are unconvincing as they both tacitly assume what DSR denies, namely the universal applicability of the kinematical structure of QFT, and in particular its applicability in a regime that has not yet been tested.

4.1. Violations of Lorentz invariance are not implied by LQG

There are several options open for one who would like to reject thesis (a). For example, one might agree with the general intuition about the incompatibility of a lattice structure with a continuous symmetry but argue that this intuition fails for some specific cases, supplying a counterexample; or one might try to undercut this intuition altogether, arguing that violations of Lorentz invariance are simply uncalled for in a theory like LQG, hence not to be expected.

Taking the first route there exists at least one candidate for a solution of the quantum gravity problem, namely the causal set approach, (Dowker et al., 2004), that predicts a lattice structure of spacetime but nevertheless entails no *observable* violations of Lorentz invariance. Such peaceful coexistence is achieved with the help of the familiar maneuver of coarse graining: violations of Lorentz invariance are deemed possible but also highly improbable in suitable limits. Philosophers of science conversant in the foundations of statistical mechanics would recognize this strategy, that goes back to the founding fathers of the kinetic theory of

gases who argued for the compatibility of time-reversal-invariant dynamics with macroscopic irreversibility (Hagar, 2005).

The second route leads to a direct attack on the basic intuition underlying thesis (a). Taking it one may argue that LQG, while predicting spatial discreteness, does not imply violations of Lorentz invariance, hence these will not be observed if the theory is true. An argument to this end is defended in great length and detail in (Rovelli & Speziale, 2003). A shorter version of this argument reappears in Rovelli’s monograph *Quantum Gravity* (2004, pp. 316–318).

The argument begins by recalling the apparent inconsistency between discrete geometry and exact Lorentz invariance (see Section 3.1 above): since length transforms continuously under a Lorentz transformation, a minimal length such as L_p (or a minimal area $A_p \approx L_p^2$, which is the corresponding measure in LQG) is going to get Lorentz contracted. If an observer at rest measures the minimal length L_p , a boosted observer will then observe the Lorentz contracted length $L' = \gamma^{-1}L_p$ (here $\gamma = 1/\sqrt{1-v^2/c^2}$ is the Lorentz–FitzGerald contraction factor) which is shorter than L_p , and therefore L_p cannot be the *minimal* length. This “simple minded” (Rovelli & Speziale, 2003, p. 1). intuition is then deemed “wrong, because it ignores quantum mechanics” (Rovelli, 2004, p. 316).

Rovelli’s idea to resolve this apparent conflict can be summarized as follows: (1) Minimal length is not a fixed property of the geometry of space. (2) It is the lowest non-zero eigenvalue in the discrete spectrum of a quantum operator. (3) Boosted observers will measure the same spectrum, with the same minimal eigenvalue. (4) Boost operations will change the probability distribution over the eigenvalues (but the minimal length remains untouched). (5) Therefore, the minimal length does not change, and there are no “sub-minimal” contractions.

In order to analyze this argument we need to unpack it first. (1) and (2) amount to saying that in LQG length (and area, and volume—everything said here holds *mutatis mutandis* for all three) are not classical quantities, but rather quantum observables with discrete spectra which are, according to (3) shared by boosted and non-boosted observables, and for which the minimal length (area, volume) is the lowest positive eigenvalue. But in order for (4) to hold, boosted observables should *not* commute with the original observables measured by an observer at rest, i.e., if the system is in an eigenstate of A (the area operator at rest), it must not be in an eigenstate of A' (the “boosted” area operator). Only then would (5) follow.

The gist of the argument hinges on a famous counterexample to the intuitive conflict between discrete spectra and continuous symmetries (Snyder, 1947). That counterexample shows that spatial rotation symmetries of angular momentum in non-relativistic quantum mechanics are not violated by the discreteness of spectra of the angular momentum components due to the non-vanishing commutation relations between these components. The non-commutativity prevents the theory from making verifiable predictions about the continuous symmetry, in a manner similar to the way in which measuring a position of a particle in non-relativistic quantum mechanics prevents us from sharply predicting its momentum. Hence, it is improper to say that the symmetry breaks when acting on operators with discrete spectra.

To complete his argument, Rovelli must show that the area observable at rest A and its Lorentz transform A' do not commute, i.e., that $[A, A'] \neq 0$. This is done by noticing that due to the relativity of simultaneity, the two observers (“at rest” and “boosted”) measure the gravitational field on their surface with a timelike separation (Rovelli & Speziale, 2003, p. 2; Rovelli, 2004, pp. 317–318), and since no quantum field operator commutes with its time derivative, the two functions of the gravitational field A and A' do not commute.

Rovelli's argument has been given a lot of credit in the LQG community, and many cite it side by side with the above counterexample (e.g., Ashtekar & Lewandowski, 2004, p. R136). Further analysis reveals, however, that at least in its current form, the argument is incomplete, and when completed, is still too weak to rule out DSR.

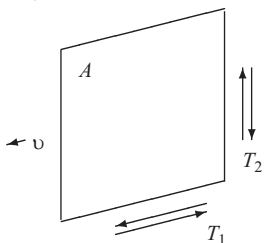
The initial step in this analysis was taken in a couple of penetrating papers by one of the first proponents of DSR (Amelino Camelia, 2002, 2003a). To understand the crucial problem here it is instructive to return to compatibility example Rovelli relies on. There it was argued that because a measurement of one component of the angular momentum, say L_x , introduces (in general) a significant uncertainty concerning the other components, L_y and L_z , one can claim that quantum theory gives no verifiable predictions on the fate of the continuous rotational symmetry. For this result to hold, however, the necessary requirement is that at least *some* of the procedures that are suitable for a sharp measurement of L_x are not such that they depend on sharp information of L_y and L_z (Amelino Camelia, 2003a, pp. 25–28).

But while this operational independence *does* hold for angular momentum, it is not proved, let alone mentioned, in Rovelli's argument!

In fact, when one inquires further into the operational meaning of area measurement, one discovers that the non-commutativity that Rovelli's compatibility argument requires is actually a non-commutativity between the area operator and the velocity operator of the surface, namely $[A, v] \neq 0$.¹⁴ Consequently, the operational independence that holds in the case of angular momentum becomes highly suspect in Rovelli's case.

To see why, let us imagine that a sharp measurement of A , which induces an unsharpness on the measurement of v , *required* a sharp value of v in order to be performed in the first place. Now recall that the non-commutativity result ($[A, v] \neq 0$) is necessary and sufficient to establish premise (4) in Rovelli's compatibility argument. But how can this non-vanishing commutation relation between the area and its velocity be physically meaningful when the area measurement itself requires sharp knowledge of the velocity?

Think, for example, of the following attempt to devise a suitable area measurement procedure that uses only time measurement of light bursts (assume for simplicity that we have previously established that the surface is rectangular, so that by measuring two sides one can obtain the area). The area of the surface should be obtained from two time-of-flight measurements T_1 and T_2 (see figure). (Amelino Camelia, 2002, pp. 46–48). However, it is not sufficient to measure T_1 and T_2 in order to obtain an area measurement: it is also necessary to know the velocity v of the surface! For if the surface is at rest the area will be deduced from the (T_1, T_2) measurement as $A = T_1 \cdot T_2 \cdot c^2/4$. But if the surface is moving with speed v along the direction of the T_1 measurement-procedure one would instead deduce from the (T_1, T_2) measurement that $A = T_1 \cdot T_2(c^2 - v^2)/4$.



¹⁴ This arises from the fact that, at least in the case which is investigated here of a flat surface with an area A in a flat spacetime, the Lorentz boost transforms the area A into the area A' in a way that is continuous and *depends* on the velocity v of the surface. See Amelino Camelia (2003a, p. 30).

In order to establish premise (4) in his argument, Rovelli must present us with an area measurement procedure that is operationally independent of the knowledge of the surface velocity. This, however, is a very difficult task, as, at least in the flat spacetime situation considered here, “all the *commonly considered* length-measurement-procedures do require sharp knowledge of the velocity of the ruler in order to achieve a sharp measurement of its length” (Amelino Camelia, 2003a, p. 30; my emphasis). Note also that in the absence of such an alternative measurement procedure the only area one could measure is one's own “rest-area” (where v is known to be 0).

Here I would like to suggest a further step in the analysis. My claim is that *even if one could come up with an alternative area measurement procedure that is operationally independent of the knowledge of the velocity of the surface, Rovelli's argument is still too weak to rule out DSR*.

Consider two observers (“at rest” and “boosted”), each holding a surface with minimal area in his rest frame. As long as the two observers do not compare any quantity (including their respective surface “rest-areas”), there should be no inconsistency between spatial discreteness and exact Lorentz invariance. And yet, agreement between different frames requires a comparison; an observer in one frame cannot actually “see” the surface in the other frame without probing it. If such a comparison is to be done, however, it must involve some interaction process (Hossenfelder, 2006). The crucial question is the following: what is the appropriate mathematical structure one should use to describe this interaction that also maintains the minimal length observer-independent?

Take a putative measurement procedure that uses three identical clocks, two of which are sent to each of the far ends of the surface and back with an arbitrary velocity. Now the length of each one of the two dimensions of the surface can be calculated from the time-discrepancy between each of the two clocks and the third one, similar to the twin paradox, using a relevant transformations group. But what guarantees us (kinematically) that the *correct* transformations group in this regime is the Lorentz group, or that (dynamically) the dispersion relations that underlie such an interaction between the clocks and the surface are the standard special-relativistic ones?

This is exactly the point of contention.

Recall that underlying DSR is the assumption that any interaction between the clocks and the surface would be constrained by kinematics *different* from standard QFT (since in this case both QFT effects *and* gravitational effects are relevant). This means that when completing Rovelli's argument with a measurement procedure that is independent of the knowledge of the velocity, one must be careful not to treat the interaction required by such a procedure as *a priori* Lorentz covariant; doing so will obviously render the whole argument question begging.¹⁵

But while Rovelli's argument, when completed, is too weak to rule out DSR, as it stands in its current form it is also too strong: some deviations from exact Lorentz invariance might have phenomenological consequences, and while it is still unclear whether these predictions are testable in practice (see below), the issue of practical testability is immaterial. After all, the uncertainty principle is a fundamental limitation on prediction, not a practical one; by using it to claim agnosticism with respect to the fate of Lorentz invariance in LQG, Rovelli's argument addresses the

¹⁵ Note that one cannot defend Rovelli's argument from this attack by arguing that both dynamics and interactions are so poorly understood in LQG, as doing so will *ipso facto* make Rovelli's argument too weak to rule out DSR.

theoretical motivation for deforming the latter, and not its testability in practice.

The upshot is that the thought experiment proposed in Rovelli and Speziale (2003) and Rovelli (2004), while releasing the tension between spatial discreteness and exact Lorentz invariance, remains unconvincing. In its incomplete form it is too strong, and when completed, it is too weak to rule out a theory such as DSR that holds the principle of relativity intact but deforms the Lorentz group by introducing an additional observer-independent fundamental length scale.

4.2. Deformations of Lorentz invariance single out a preferred frame

The main idea behind DSR is to replace the usual linear Lorentz boost transformation with some non-linear function thereof which reduces to the identity for low energies. It is noteworthy that a similar proposal for a non-linear deformation of the Lorentz group was made by Fock (Fock, 1964, pp. 6–7, 369–375), who was motivated by the search for a general symmetry group that would preserve the principle of relativity without assuming the constancy of c . But while Fock modified the action of the transformations at large distances, in DSR the action is modified at large momentum. Note also that the group structure of these deformed transformations is the same as the ordinary Lorentz group (which seems reasonable, as we want to retain the spatial rotations and to reproduce the full set of Lorentz transformations in low energies).

So far DSR relies mostly on the particle picture, and at this stage there is no *unique* and well-defined field-theoretic formulation of the theory that would allow one to translate the particle behavior in momentum space (E, p) into position space (t, x) . As mentioned in Section 3, different attempts to do so lead to different composition rules (i.e., they differ on the question which properties are extensive), and also to different predictions regarding the varying speed of massless Planckian particles (i.e., they differ on whether this velocity varies, and if so, whether this variation depends on energy or on energy-density (see e.g., Magueijo & Smolin, 2003; Kimberly et al., 2004; Girelli & Livine, 2005; Hossenfelder, 2006, 2007), and, respectively, on whether or not the *in principle* testable predictions of LQG are indeed so *in practice*) (Lamon et al., 2008, p. 1733).

A detailed analysis of these different methodologies and the predictive price they carry is deferred to future work. Here I would like to concentrate instead on another challenge to DSR which aims to expose the allegedly dire consequences of *any* translation of DSR from momentum space to position space. This argument (Schützhold & Unruh, 2003) aims to show that while there is still no clear formulation of DSR in field-theoretic terms, when translated into position space the non-linear deformation of the action of the Lorentz group on momentum space leads, *in general*, to some bizarre macroscopic consequences that are presumably inconsistent with our current knowledge of physics.

The motivation for this claim comes from an analogy between the modified dispersion relations suggested in DSR and the modified dispersion relations one finds in the propagation of sound in high energies (as in both cases the modifications are non-linear and energy-dependent).¹⁶ To translate from momentum space (E, p) to position space (t, x) one first does a Fourier transform \mathcal{F} (assuming $ip \leftrightarrow \hbar \partial/\partial x$), then applies the non-linear Lorentz transformation and finally transforms back to get the position space rotated:

$$\phi(t, x) \rightarrow \tilde{\phi}(E, p) = \mathcal{F} \phi,$$

$$\tilde{\phi}(E, p) \rightarrow \tilde{\phi}(E', p'),$$

$$\phi'(t, x) = \mathcal{F}^\dagger \tilde{\phi}(E', p').$$

Now, apart from difficulties in moving from the single-particle case to the multi-particle picture that arise from the non-linearity of \mathcal{F} (difficulties that have been dubbed “the soccer ball problem” and that have been addressed—along with their predictive price—more or less satisfactorily in the aforementioned suggestions for field-theoretic formulations of DSR), the fact that \mathcal{F} and its inverse \mathcal{F}^\dagger are non-polynomial will lead to breakdown of translation invariance and to non-local effects, *both on large scales*.

The former effect is derived by following the evolution of two wave packets. The non-linear correction to the Lorentz transformation is shown to entail that in one coordinate system the wave packets collide, while in another they miss each other. The latter effect is derived by arguing that if one could follow the trajectory of a Planckian particle (i.e., a particle with a Planck scale energy) over a macroscopic distance and time (i.e., larger than the respective Planck scales), non-local effects would emerge on these scales: if the speed of light is, e.g., supposed to decrease with the energy, a massless Planckian particle would eventually stop moving, and if one could localize it for a finite time duration (much longer than the Planck time) within a few Planck lengths, this would clearly single out a preferred frame. The upshot of the argument is that, contrary to the aspirations of its proponents, DSR is no different than those theories that break Lorentz invariance and violate the principle of relativity.

But does DSR generate these arguably fatal empirical consequences? Cannot one “deform” Lorentz invariance without ultimately abandoning the principle of relativity? My claim is that the above argument by Schützhold and Unruh is unconvincing. The problem with their twofold challenge is that it tacitly presupposes several assumptions which are explicitly denied in the DSR framework (see also Arzano, 2003).

For example, in order to establish the first effect, Schützhold and Unruh assume that the Fourier transform is defined by integrating over the *whole* range of energies (or, equivalently, lengths), but DSR, contrary to standard QFT, introduces as a key feature an upper (lower) bound for energy (length). This feature must thus be taken into account when defining the Fourier transform that allows one to translate DSR into position space. In a similar vein, in order to establish the second effect, Schützhold and Unruh assume the standard operational definition of a “localization of a particle”, but note that this definition must be modified in the regime DSR purports to describe. In fact, in order to localize a Planckian particle in this regime one needs *other* Planckian particles which are the only ones that can give the resolution one requires. And since the interaction picture in this regime is still missing, in the current state of the theory it is quite premature to make any *dynamical* predictions of the sort DSR is challenged with here. Arguably, DSR holds that interactions in that regime will be constrained by a structure *different* from Minkowski’s spacetime, and there seems to be no *a priori* reason to think that no possible translation of DSR into position space would be immune to this objection.

5. Constructing the principles

The two arguments mounted against DSR differ in their strategies: the first aims to undercut the intuition behind thesis (a), attempting to explain why the discrete geometry predicted by LQG does not imply violations of Lorentz invariance, hence there is no reason to expect it if LQG is true; the latter aims to demonstrate that thesis (b) must hold, claiming that *if* discreteness of geometry *entailed* deformation of Lorentz invariance, it

¹⁶ Note that the latter case explicitly involves a preferred frame!

would also single out a preferred frame on macroscopic scales, a consequence presumably inconsistent with our current understanding of physics.

The two arguments, however, share a common tacit presupposition which DSR denies: both implicitly impose Lorentz covariance on dynamical interactions whose kinematical constraints are assumed by DSR to be different from the standard QFT ones. While this presupposition is stated explicitly in neither of the two arguments, it reveals itself upon close analysis. In the first case it appears when one tries to complete the (originally incomplete) argument with a measurement procedure that is independent of knowledge of the velocity of the surface whose area is minimal; in the second it lurks behind the assumption that the standard (QFT) interaction picture holds also at the Planck scale.

On final account, there is no *a priori* reason to suppose that a certain physical regime cannot be described by a mathematical structure whose phenomenological consequences are consistent with current experimental evidence but that is nevertheless *different* from QFT. Contrary to the underlying assumption behind the arguments analyzed here, the fact that QFT interactions are Lorentz covariant does *not* entail that QG interactions are so, hence the kinematical structure that may constrain the latter may as well be different from Minkowski spacetime (as long as it reduces to the latter in the appropriate limit). Moreover, the fact that no departure from exact Lorentz invariance has been observed *so far* is consistent with almost any result that may arise when the Planckian regime is probed, and, in particular, with the predictions made within DSR, as long as the latter are made in a logically consistent way.¹⁷

The case of DSR nicely exemplifies the process of devising new theories that aim to predict new physics. In this process the theoretician is not operating in a void or in a complete absence of constraints, but rather starts from some “old” physics and carefully extends it to new regimes. It is natural, therefore, and by far more productive, to use the best confirmed principles of the “old” physics as constraining principles in the construction of the new one. But one should be careful in one’s choice which principle to keep and which to let go (Feynman, 1965, p. 166):

We have all these nice principles and known facts, but we are in some kind of trouble: either we get the infinities, or we do not get enough description—we are missing some parts. Sometimes that means that we have to throw away some idea; ... To guess what to keep and what to throw away takes considerable skill. Actually it is probably merely a matter of luck, but it looks as if it takes considerable skill.

The principle of relativity has endured the transition from Newtonian mechanics to special relativity, and DSR, no matter how speculative, keeps it intact. What is being “thrown away”, as it were, is a certain transformation rule, which DSR replaces with a more general one. Only time (and experiment) will tell whether this was a matter of skill, luck, or just plain wrong.

6. Concluding remarks

In this paper I have investigated an intriguing open question that exists today in the domain of QG, namely the status of Lorentz invariance and its relation to the notion of minimal length, and used it as a case study for highlighting the delicate

balance between conservatism and innovation that characterizes the process of constructing new physics.

Admittedly, Lorentz invariance is one of the pillars of modern physics, and it is hard to imagine what kind of physics one would end up with if it turned out to be non-fundamental. In fact, physicists actually have used Lorentz invariance in constructing our best and (at least so far) most fundamental theory of matter, namely QFT, that dynamically accounts for the contraction of rods and the dilation of clocks (see, e.g., Mermin, 2005, p. 184). And yet, most physicists will agree that even this theory could turn out to be only an approximation (Bjorken & Drell, 1965, Introduction):

There is nothing but positive evidence that special relativity is correct in the high-energy domain, and, furthermore, there is, if anything, positive evidence that microscopic causality is a correct hypothesis. Since there exists no alternative theory which is any more convincing, we shall restrict ourselves to the formalism of local, causal fields. It is indoubtably true that a modified theory must have local field theory as an appropriate large-distance approximation or correspondence. However, we again emphasize that the formalism we develop may as well describe only the large distance limit (that is, distance $> 10^{-13}$ cm) of a physical world of a different submicroscopic properties.

Theoretical arguments that aim to support the quantization of gravity are sometimes accused of “putting physics upside down” since, or so the accusation goes, if the gravitation field is to be quantized, it should be so because of empirical evidence, and not because of some methodological predilection.¹⁸ Imposing Lorentz covariance *a priori* on interactions that involve gravity by arguments that presuppose the universal applicability of QFT is thus tantamount to committing the same sin twice. The following, famous words, it seems, are as relevant today as they were almost half a century ago (Rosenfeld, 1963, p. 356):

It is nice to have at one’s disposal such exquisite mathematical tools as the present methods of quantum field theory, but one should not forget that these methods have been elaborated in order to describe definite empirical situations, in which they find their only justification. Any question as to their range of application can only be answered by experience, not by formal argumentation. Even the legendary Chicago machine cannot deliver the sausages if it is not supplied with hogs.

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¹⁷ Recall that the Planck scale ($\sim 10^{-33}$ cm) is 17 orders of magnitude from presently available experimental information.

¹⁸ See, e.g., Rosenfeld (1963), and recently also Mattingly (2006) and Albers et al. (2008).

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