Welfare Implications of Uncertain Social Security Reform

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Abstract

Current projections estimate that the Old-Age and Survivors Insurance (OASI) trust fund will be depleted by 2035. In this paper, I develop a general equilibrium overlapping generations model to characterize the welfare implications of uncertainty surrounding the type and timing of an inevitable Social Security reform. I find that uncertainty’s impact on household welfare through the general equilibrium channel is uneven across generations. While older generations prefer tax based reform, the young prefer a cut in Social Security benefits. The main result of this paper is that the welfare implications of uncertainty are sensitive to households’ subjective beliefs over potential reforms.

Key Words: Social Security Reform, Household Beliefs, Policy Uncertainty, Welfare Analysis, Transition Dynamics

JEL Classification: E62, H31, H55, H60

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1 Introduction

From 1975 to 2008 the Social Security program in the United States paid benefits to approximately 30 beneficiaries for every 100 workers. By 2015 the program’s Old Age and Survivors Insurance (OASI) trust fund had accumulated $2.8 trillion in assets; however, as the baby-boomer generation enters retirement the number of beneficiaries per 100 workers is expected to rise to 45 by 2030. Furthermore, due to an increase in life expectancy and a general fall in fertility rates since 1970, the program is expected to support between 40-64 beneficiaries per 100 workers through 2090. As the program has been running a deficit since 2010 the Social Security and Medicare Board of Trustees estimate that the OASI trust fund will be depleted by 2035 under current law (Blauhous III and Reischauer (2015)). A 2015 survey found that 91% of individuals between the ages of 25 and 59 are aware of the forecasted Social Security budget shortfall (Luttmer and Samwick (2015)). Conditional on reform taking place, 24% expect benefits to remain unchanged and taxes to increase, 18% expect taxes to remain unchanged and benefits to be cut, and 58% expect an adjustment on both margins.\(^1\) This paper is part of a growing body of literature concerned with the impact of policy uncertainty on household welfare.

I develop a general equilibrium overlapping generations model in which households are uncertain as to the type and timing of an inevitable Social Security reform. I analyze the welfare implications of policy uncertainty along the transition path from the economy’s initial steady state to the terminal steady state following the implementation of a reform. I allow for two possible timings of reform (early and late) and consider two pure reform types: a benefit cut and a labor tax increase. These combine to form four distinct reforms: (1) early tax increase, (2) late tax increase, (3) early benefits cut, and (4) late benefits cut. I calibrate the model to the US economy and allow households to have subjective beliefs over potential reforms. This paper stresses the importance of household beliefs when determining the welfare implications of policy uncertainty in a general equilibrium setting. I analyze how households’ subjective beliefs surrounding policy reform impact market clearing prices and how those in turn affect welfare across generations. I detail how beliefs affect households’ perceived lifetime wealth and the welfare cost of uncertainty attributable to their innate risk aversion. Crucially I find that the welfare implications of uncertainty are sensitive to households’ beliefs.

A few papers have quantitatively addressed the welfare costs associated with policy uncertainty through empirical exercises and calibrated partial equilibrium lifecycle models. Dusek (2007) empirically measured the welfare cost of risk surrounding the Social Security system in the Czech Republic. He found that the volatile swings in real benefits for retirees between 1988 and 1995 could amount to a welfare cost of 1.0% of lifetime consumption. On average individuals in Luttmer and Samwick (2015)’s survey were willing to forego 6.0% of their currently promised Social Security benefits to live in a world absent Social Security benefit uncertainty. As of October 2015, the

\(^1\)Other papers documenting uncertainty surrounding Social Security and other pension systems include: Dominitz et al. (2003), Van der Wiel (2008), Giavazzi and McMahon (2012), Guiso et al. (2013), Baker et al. (2015).
average monthly Social Security benefits were $1,338.27, implying an annual welfare cost of $963.55 in retirement.\(^2\) Büttler (1999) analyzed how uncertainty over the timing dimension of a Social Security reform in Switzerland could impact the macroeconomy in a calibrated life-cycle model. She found that household expectations could have a significant impact on the macroeconomy and intergenerational consumption and labor profiles. Gomes et al. (2007) studied several sources of uncertainty and found that a failure to alleviate uncertainty over future Social Security benefits could yield welfare costs of 0.6% of annual consumption. Michelangeli and Santoro (2013) considered Medicare policy reform uncertainty in both the type and timing dimension and found that uncertainty amounted to a welfare loss of $10,000 in wealth. Similarly, Caliendo et al. (2015) also considered uncertainty in both the type and timing dimension but instead turned their attention towards Social Security reform. Using a continuous-time partial equilibrium model they found that the welfare cost of uncertainty is generally less than 0.1% of lifetime consumption when households save optimally; however, the impact can be much larger for those households who do not save.

The closest paper to this is Kitao (2017) that addresses the welfare effects of uncertain Social Security reform in a general equilibrium life-cycle framework. Her model incorporates a demographic structure calibrated to Japan’s aging economy. She finds that the general equilibrium channel plays an important role in determining the welfare effects of uncertainty and that the welfare implications vary substantially across generations, ranging from \((-3.4%, +3.1\%)\) of lifetime consumption. This paper differs from Kitao (2017) in four ways. First, Kitao (2017) calibrates her model to Japan’s economy while the model in this paper is calibrated to the US economy. Second, Kitao (2017) focuses on three different timings of a reform that consists of both a cut in benefits and an increase in the tax on consumption. Later reforms correspond to larger increases in the consumption tax in her model. I consider only pure reforms in the form of a labor tax increase or benefit cut. This structure allows for a comparison across policy types and a disentangling of their impact on welfare.

The third difference concerns households’ beliefs. I begin my analysis, like Kitao (2017), by exogenously imposing household beliefs such that households place equal weight on the four reforms. However, I then analyze how the welfare implications change as I vary household beliefs in both the timing and type dimension. I find that the welfare implications of uncertainty are quantitatively sensitive to household beliefs. Through uncertainty’s effect on general equilibrium prices, the future young generation’s welfare as measured by their realized lifetime utility ranges from \((-1.2%, +1.5\%)\) of lifetime consumption relative to a perfect foresight economy. Finally, I introduce a positive measure of households into the model economy that do not have access to capital markets and are ostensively hand-to-mouth (HtM). The introduction of these households partially mitigates the welfare implications of uncertainty by reducing the sensitivity of market clearing prices to household uncertainty. Consistent with Caliendo et. al. (2015) I find that the welfare implications for those households that do not save are larger in magnitude relative to those households who save optimally. I extend this finding by documenting the result’s sensitivity to house-

\(^2\)Source: https://www.ssa.gov/cgi-bin/currentpay.cgi
hold beliefs. The welfare cost attributable to a young HtM household’s innate risk aversion ranges from (-2.15%, -0.01%) of lifetime consumption.

The remainder of the paper is organized as follows: Section 2 formally introduces the model, defines the competitive equilibrium in a perfect foresight economy, and discusses the parameterization of the model. Section 3 discusses how households rank each of the four reforms in accordance with their preferences along the transition path in a perfect foresight economy. Section 4 computes the welfare implications of reform uncertainty across a set of household beliefs and concludes with a discussion on hand-to-mouth households and their impact on equilibrium prices and welfare results. Section 5 concludes.\[3\]

2 Model

The model economy is populated with overlapping generations. Each generation consists of a measure-one of identical households with a certain lifespan of three periods. Households consume and supply their labor elastically in the first two periods of life and retire exogenously in the third period. Firms are perfectly competitive and the government’s sole purpose in the model is to operate a Social Security pension system.

2.1 Firms

Firms are perfectly competitive, hire labor from households at rate $w_t$, rent households’ capital at rate $r_t$, and have access to a constant returns to scale Cobb-Douglas production technology. The firms’ problem is:

$$\max_{K_t, N_t} K_t^\alpha N_t^{1-\alpha} - w_t N_t - r_t K_t$$

(1)

where $K_t$ and $N_t$ denote the aggregate capital stock and labor supply respectively at time $t$.

2.2 Government

The government in the model runs a Social Security pension system. The pension system is a “pseudo” pay-as-you-go Social Security system in which the government levies a tax on the labor earnings of currently working households, $\tau_t$, and pays benefits to retired households as a fraction, $B_t$, of their average lifetime labor earnings, $e_t$. $B_t$ is known as the replacement rate of Social Security. The system can run a surplus, at which point excess revenues in each period are deposited into the Social Security Trust Fund (SSTF), $F_t$. The SSTF has access to a storage technology. The government does not have access to credit markets and cannot finance its Social Security system.

\[3\] Appendix A contains the analytical solution to the households’ problem. Appendix B compares the initial steady state to those that result from each of the four reforms and that of the social planner. Appendix C discusses the numerical algorithms used to solve the model.
through borrowing. Furthermore, the government is mandated to keep the stock value of the SSTF positive in all time periods. Finally, the government does not consume or produce any other goods or services.

Let \( F_t \) and \( A_t \) denote the stock value and present value of the SSTF at time \( t \) respectively. I assume the Social Security Administration discounts future surpluses and deficits at rate \( \beta \):

\[
F_t = F_{t-1} + z_t 
\]

\[
A_t = F_t + \sum_{j=1}^{\infty} \beta^j z_{t+j} 
\]

where \( z_t \) is the period \( t \) Social Security surplus or deficit:

\[
z_t \equiv \tau_t w_t N_t - B_t e^{t-2} 
\]

and \( e^{t-2} \) denotes the average lifetime pre-tax labor earnings of generation \( t-2 \):

\[
e^{t-2} \equiv \frac{w_{t-2} n_{t-2}^{t-2} + w_{t-1} n_{t-1}^{t-2}}{2} 
\]

In steady state, if \( \tau < \frac{B}{2} \) the Social Security program runs a deficit in each period. I assume that the program is running a deficit and that the SSTF’s initial endowment is \( F_0 > 0 \) such that, absent any government intervention, the SSTF’s stock value will hit zero in period \( t = 2 \). As the government is mandated to maintain a weakly positive SSTF, a reform must occur in period \( t = 1 \) (early) or \( t = 2 \) (late). The government can reform the program in two different ways. It could increase labor taxes, \( \tau \), or cut benefits, \( B \). All told the there are four possible reforms: (1) early labor tax increase where \( \tau_t = \tau_e \ \forall t \geq 1 \), (2) late labor tax increase where \( \tau_t = \tau_L \ \forall t \geq 2 \), (3) early benefits cut where \( B_t = B_e \ \forall t \geq 1 \), and (4) late benefits cut where \( B_t = B_L \ \forall t \geq 2 \). Each reform, \( \{ \tau_e, \tau_L, B_e, B_L \} \), ensures the SSTF never falls below zero. In equilibrium, under perfect foresight, each reform yields the same time-zero discounted present value of the infinite sequence of the SSTF’s deficits and surpluses:

\[
A_0(a) = \bar{A} = F_0 + \sum_{t=1}^{\infty} \left[ \beta^t z_t(a) \right] \quad \forall a \in \{ \tau_e, \tau_L, B_e, B_L \} 
\]

As the SSTF experiences an additional period of deficits when the reform occurs late, the late reforms must be larger for equation (6) to hold. Therefore, \( \tau_e < \tau_L \) and \( B_e > B_L \).

\[\textsuperscript{4}\]

\[\textsuperscript{4}\text{A discussion concerning how equivalent policies are computed is in Appendix C.}\]
2.3 Uncertainty and Beliefs

Reform uncertainty is the only source of uncertainty in the model. Once a reform takes place all uncertainty is alleviated and the environment becomes a perfect foresight economy. In period \( t = 0 \) uncertainty exists in both the timing (early or late) and type (taxes or benefits) dimension. For simplicity, I assume households’ beliefs over the type and timing of reform are independent. This allows for a simple belief updating procedure as information concerning the timing of reform is revealed in period \( t = 1 \). When households wake up in period \( t = 1 \) and an early reform did not occur, their belief over the type of reform to be implemented in period \( t = 2 \) remains unchanged.

Let \( \pi_\tau \) denote the subjective probability that households place on the reform being done through a tax increase; furthermore, let \( \pi_e \) denote the probability that households place on reform occurring early in period \( t = 1 \). Using the independence assumption, we can obtain the subjective probabilities households places on each of the four possible reforms in period \( t = 0 \):

\[
\pi_{e,\tau} = \pi_e \pi_\tau \quad \pi_{L,\tau} = (1-\pi_e) \pi_\tau \quad \pi_{e,B} = \pi_e (1-\pi_\tau) \quad \pi_{L,B} = (1-\pi_e)(1-\pi_\tau)
\]

where, for example, \( \pi_{L,B} \) denotes the probability weight that households place on the government cutting benefits late in period \( t = 2 \). If an early reform occurred then all uncertainty is alleviated in period \( t = 1 \). If reform did not occur in period \( t = 1 \), then beliefs are updated as follows:

\[
\pi_{L,\tau} = \pi_\tau \quad \pi_{L,B} = (1-\pi_\tau)
\]

2.4 Households’ Problem

Every household in each generation is born with zero capital stock and is endowed with one unit of time in each period to be divided between labor and leisure. Households born in period \( t \) have time-separable preferences over consumption and leisure, \( \{c_{t+j}, l_{t+j}\}_{j=0}^2 \), and a subjective time discount factor \( \beta \). A young household born into generation \( t \) faces the following problem:

\[
\max_{\{c_{t+j}^t, l_{t+j}^t\}_{j=0}^2} \mathbb{E}_t \left[ \sum_{j=t}^{t+2} \beta^{j-t} u(c_{j}^t, l_{j}^t) \right]
\]

subject to:

\[
c_t^t + k_{t+1}^t = w_t (1-\tau_t)(1-l_t^t)
\]

Footnote: Households internalize their labor supply’s effect on benefits. This is consistent with the recent literature on how well households understand and respond to changes in Social Security: Imrohoroglu and Kitao (2009), Liebman and Luttmer (2008), Liebman et al. (2009), Liebman and Luttmer (2011), Mastrobuoni (2009), and Mastrobuoni (2011).
\begin{align*}
  c^t_{t+1} + k^t_{t+2} &= w_{t+1}(1 - \tau_{t+1})(1 - l^t_{t+1}) + R^t_{t+1}k^t_{t+1} \\
  c^t_{t+2} &= B_t + R^t_{t+2}k^t_{t+2} \\
  e^t &= \frac{w_t(1 - l^t_t) + w_{t+1}(1 - l^t_{t+1})}{2} \\
  k^t_{t+j} &\geq 0 \quad \forall j \in \{0, 1, 2\}
\end{align*}

2.5 Definition of Equilibrium

In a perfect foresight economy, the competitive equilibrium consists of the Social Security structure \( \{\tau_t, B_t\}_{t=0}^\infty \), an allocation \( \{c^t_t, c^t_{t+1}, c^t_{t+2}, k^t_{t+1}, k^t_{t+2}, l^t_t, l^t_{t+1}\}_{t=0}^\infty \), and set of prices \( \{R_t, w_t\}_{t=0}^\infty \) such that:

1. Given the Social Security structure and prices, the households’ allocations solve their optimization problem as described in section 2.4.

2. The capital, labor, and goods markets clear in each period:

\begin{align*}
  K_t &= k^{t-1}_t + k^{t-2}_t \\
  N_t &= (1 - l^t_t) + (1 - l^{t-1}_t) \\
  c^t_t + c^t_{t-1} + c^t_{t-2} + k^{t+1}_t + k^{t-1}_{t+1} &= K^t_t N^{1-\alpha}_t + (1 - \delta)K_t - z_t
\end{align*}

3. Prices are determined competitively in each period:

\begin{align*}
  R_t &= 1 + \alpha \left( \frac{K_t}{N_t} \right)^{\alpha-1} - \delta \\
  w_t &= (1 - \alpha) \left( \frac{K_t}{N_t} \right)^{\alpha}
\end{align*}

4. The Social Security structure satisfies the government’s non-negativity mandate on the SSTF in each period:

\[ F_t > 0 \quad \forall t \]

2.6 Parameterization

Households’ period utility function takes the form:

\[ u(c, l) \equiv \theta \ln c + (1 - \theta) \ln l \]

which when combined with the assumption of a 20-year model period yields closed form decision rules for household allocations in a perfect foresight economy (see Ap-
This saves from having to solve the households’ optimization problem numerically in each period along the transition path under each reform and set of household beliefs. Households’ utility weight on consumption, \( \theta \), is set such that under the initial policy parameters, \( \{\tau, B\} \), households dedicate 38.0% of their time endowment during their career to the labor market. This is consistent with the share of hours allocated to work over the life cycle in the US as discussed in Kitao (2014). The households’ time discount factor, \( \beta \), the capital depreciation rate, \( \delta \), and capital’s share of output, \( \alpha \), are similar to Kitao (2014).

The initial Social Security policy parameters, \( \{\tau, B\} \), are chosen to match the 2015 OASI tax rate and the approximate replacement rate for retirees born in 1980 that made, on average, $45,128 per year during their career.\(^7\) The size of the reforms are computed within the model such that the government’s mandate to maintain a positive SSTF is met indefinitely (see section 2 and Appendix C for details).\(^8\) The initial SSTF endowment is set such that, in a perfect foresight economy, the fund would hit zero at the beginning of period \( t = 2 \) if no reform took place. The initial SSTF endowment as a share of steady state output is 9.12%, which is consistent with the data from 2000 when the OASI trust fund’s assets as a share of GDP were 9.05%. Moreover, the SSTF in the model hits zero absent any reform at the beginning of period \( t = 2 \). This is analogous to the year 2040 and in line with the 2015 projections published by the Board or Trustees of the OASI and DI trust funds in which they expect the OASI fund to be depleted in 2035 (Blauhous III and Reischauer (2015)).\(^9\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Annual time discount factor</td>
<td>0.9815</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Utility weight on consumption</td>
<td>0.3414</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Capital share of output</td>
<td>0.4000</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Annual capital depreciation</td>
<td>0.0820</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>Initial Stock of the Social Security Trust Fund</td>
<td>0.0264</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Initial Social Security tax rate</td>
<td>0.1060</td>
</tr>
<tr>
<td>( \tau_e )</td>
<td>Early Social Security tax reform rate</td>
<td>0.1844</td>
</tr>
<tr>
<td>( \tau_L )</td>
<td>Late Social Security tax reform rate</td>
<td>0.2208</td>
</tr>
<tr>
<td>( B )</td>
<td>Initial Social Security replacement rate</td>
<td>0.3640</td>
</tr>
<tr>
<td>( B_e )</td>
<td>Early Social Security replacement rate reform</td>
<td>0.2110</td>
</tr>
<tr>
<td>( B_L )</td>
<td>Late Social Security replacement rate reform</td>
<td>0.1458</td>
</tr>
</tbody>
</table>


\(^8\)The size of the reforms in this paper are larger in magnitude than the SSA’s actuarial estimates. This is driven primarily by the lack of population growth and mortality in the model.

\(^9\)Due to the inherent uncertainty associated with long-run projections the Board of Trustees report a 95% confidence interval with regards to the date at which the combined OASIDI trust fund is expected to hit zero: 2029-2046.
3 Reform Analysis

As there is no source of growth in the model economy the competitive equilibria converge to a steady state.\footnote{For a detailed discussion on steady state equilibria and the social planner’s problem see Appendix B.} The main welfare analysis concerning uncertainty in section 4 focuses exclusively on the transition path from the initial steady state to each reform’s respective terminal steady state. It is however a useful exercise to first examine how households are affected by policy changes in a perfect foresight economy.

In a perfect foresight economy, every generation is affected differently by the four reforms along the transition path. Table 2 shows how each generation ranks the four reforms based on the total lifetime utility each reform delivers for that generation. In each row, a rank of 1 signals the reform most preferred and a rank of 4 indicates the reform least preferred by that generation. All generations born after period 2 have the same preference ordering as generation 2. Households born between \( t = -2 \) and \( t = 2 \) have different preference rankings over reforms. The main driver of these differences is that not all generations are directly affected by the reforms themselves and are only affected by the equilibrium prices that differ along the transition path for each reform. Understanding how generations along the transition path are affected by the reforms, in a perfect foresight economy, is useful when analyzing how reform uncertainty impacts these generations’ welfare in section 4.2.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Early Tax</th>
<th>Late Tax</th>
<th>Early Benefit</th>
<th>Late Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2 (initial old)</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>-1 (initial middle aged)</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>0 (initial young)</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1 (future gen 1)</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2+ (future gen 2+)</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

The initial old generation is never affected directly by the Social Security reforms as they die at the end of period \( t = 0 \). However, as living generations in period \( t = 0 \) change their labor supply in anticipation of the impending policy change, the market clearing interest rate is affected, which in turn impacts the initial old’s capital income in retirement. As it turns out an anticipated early tax increase yields the largest interest rate in period \( t = 0 \), making this reform the initial old’s most preferred. An early benefit cut results in the lowest interest rate in period \( t = 0 \) making it the least preferred reform from the initial old generation’s perspective.

The initial middle aged are only directly affected by an early benefit cut as they are retired in period \( t = 1 \). This makes an early benefit cut the least preferred reform for the initial middle aged. The remaining three reforms only affect the initial middle aged through the price channel. An early tax increase yields the lowest wage in period \( t = 0 \) and interest rate in period \( t = 1 \), which makes it the second least preferred. A
late tax increase is preferred to a late benefit cut as it generates a relatively higher interest rate in period $t = 1$ when the initial middle aged are retired.

The initial young are directly affected by all the reforms except for a late tax increase, which again makes it this generation’s most preferred reform. Of the remaining three reforms, an early tax increase generates the lowest after-tax wage in period $t = 1$; however, it also generates the largest interest rate in period $t = 2$ when the initial young are retired. This boost in retirement capital income makes this reform the second most preferred for the initial young. As the late cut in benefits is larger than an early benefit cut, the initial young prefer an earlier benefit cut to a later one.

The generation born in period $t = 1$ is directly affected by all four reforms. They prefer an early benefit cut the most as it generates higher wages during their career while still offering a relatively large Social Security benefit in retirement. A late tax increase maintains high benefits in retirement and high wages while young and despite having the lowest after-tax wages in period $t = 2$, ultimately making it the second most preferred reform for households in generation $t = 1$. When comparing an early tax increase to a late benefit cut there is a tradeoff between lower lifetime wages under the tax reform and the large reduction in Social Security benefits in retirement under the benefit cut reform. It turns out that the wage effect dominates their welfare, making an early tax increase this generation’s least preferred reform.

The generations born in periods $t \geq 2$ have identical preference rankings over reforms. Tax style reforms are least preferred as they lower aggregate output, and early benefit cuts are preferred to late benefit cuts as the former returns more resources to the economy through a more generous replacement rate of Social Security.

4 Welfare

In this section I document the welfare implications of uncertainty in three ways. As discussed in section 3 households may be directly affected by the reforms themselves but also by prices through the general equilibrium channel. Recall that in period $t = 0$ households become aware of the Social Security budget shortfall and the necessity of reform. Early reforms occur in period $t = 1$ while late reforms occur in period $t = 2$. Households are uncertain which reform type, taxes or benefits, the government will use and when they plan to implement the change. Once a reform occurs all uncertainty is alleviated and as a result uncertainty exists for at most two periods in the model.

I first explore the implications of policy uncertainty in section 4.1 by examining how the general equilibrium channel affects each generation’s lifetime utility at the end of their life. I do this by computing a retrospective ex-post consumption equivalent variation (CEV) that captures how much in terms of lifetime consumption a household is affected by reform uncertainty after the household has died. This metric ignores the direct affect associated with households’ innate risk aversion and only captures the general equilibrium implications of uncertainty for each type of reform.

Following Kitao (2017), in section 4.2 I use a behind-the-veil ex-ante CEV to
measure the welfare implications of reform uncertainty for each benchmark economy. The ex-ante CEV is identical to the ex-post CEV used in section 4.1 for those generations not directly exposed to uncertainty. The ex-ante CEV captures the amount of lifetime consumption a household would require or need to give up in the perfect foresight economy, under a specific benchmark reform, to be equally well off as in the economy with reform uncertainty. However, as each generation’s lifetime income is affected by each reform differently this measure does a better job at capturing the intergenerational implications of the four policy reforms.

In section 4.3 I vary households’ subjective beliefs and discuss how they impact market clearing prices and the welfare implications of uncertainty. Households’ beliefs concerning which reforms they see as most likely affect their savings and labor supply decisions that in turn have an impact on market clearing prices. The sensitivity of prices to household beliefs generates sensitivity in the welfare implications of uncertainty. Under certain beliefs and benchmark economies the ex-ante CEV can double in magnitude for some generations. Moreover, the qualitative results found in sections 4.1 and 4.2 are not robust to changes in household beliefs.

Section 4.4 discusses how the welfare results are affected when I restrict a fraction of each generation’s population from having access to capital markets. These restricted households are in effect hand-to-mouth (HtM) and consume their income in each period. Accounting for these households alters the magnitude of the general equilibrium effects and ultimately the presence of HtM households in the economy decreases market clearing prices’ sensitivity to household uncertainty. This in turn decreases the magnitude of the welfare implications of reform uncertainty across generations for households that save optimally. I find that the welfare implications of uncertainty are larger in magnitude for HtM households relative to those who save optimally. Finally, in section 4.5 I briefly discuss an alternative CEV metric used to isolate the direct implications of reform uncertainty due to households’ innate risk aversion and compare the results to the existing literature.

4.1 Ex-Post Welfare Analysis

In this section I compute the ex-post CEV for each generation under the four benchmark economies. This exercise offers insight into the complex interactions between household uncertainty and market clearing prices. Moreover, it provides useful intuition as to how uncertainty may affect households’ welfare through the general equilibrium channel. In order to obtain the ex-post CEV I start by computing each generation’s realized lifetime utility under each reform in a perfect foresight economy, $U_t(a)$, and in the economy with uncertainty, $\hat{U}_t(a, \Pi)$:

\[
U_t(a) = \sum_{j=t}^{t+2} \beta^{j-t} u(c^*_j(a), l^*_j(a))
\]

\[
\hat{U}_t(a, \Pi) = \sum_{j=t}^{t+2} \beta^{j-t} u(\hat{c}^*_j(a, \Pi), \hat{l}^*_j(a, \Pi))
\]
where \( a \in \{ \tau_e, \tau_L, B_e, B_L \} \) denotes the actual policy implemented by the government and \( \Pi \equiv \{ \pi_e, \pi_\tau \} \) is the set of household beliefs in period \( t = 0 \). The ex-post CEV, \( \mathcal{V}_t^{\text{expost}}(a, \Pi) \), is defined as:

\[
\sum_{j=t}^{t+2} \beta^{j-t} u\left((1 + \mathcal{V}_t^{\text{expost}}(a, \Pi))c^t_j(a), l^t_j(a)\right) = \sum_{j=t}^{t+2} \beta^{j-t} u\left(\hat{c}^t_j(a, \Pi), \hat{l}^t_j(a, \Pi)\right)
\]

which, under the utility specification defined in equation (21), yields the following expression:

\[
\mathcal{V}_t^{\text{expost}}(a, \Pi) = \exp\left[\frac{U_t(a, \Pi) - U_t(a)}{\theta(1 + \beta + \beta^2)}\right] - 1
\]

As each of the four reforms in a perfect foresight economy generates a different transition path for prices, the CEV welfare metrics differ substantially depending on which perfect foresight economy is being used as the benchmark. For this reason, figure 1 shows the ex-post CEV for each benchmark economy corresponding to the four reforms. For simplicity, I begin by assuming households’ beliefs in period \( t = 0 \) place equal weight on the four reforms (\( \pi_e = 0.5, \pi_\tau = 0.5 \)). This assumption is relaxed and explored in detail in section 4.3. Figure 1 also plots the capital-labor ratio transition path under uncertainty relative to the transition path in the perfect foresight economy. From an ex-post perspective, many generations benefit through the general equilibrium channel as uncertainty raises the capital-labor ratio during their careers resulting in higher lifetime wages. While the wage effect tends to dominate the general equilibrium welfare implications, the interest rate also plays an important role especially for the initial old generation born in period \( t = -2 \). As discussed in section 3 the initial old generation is only affected by the interest rate in period \( t = 0 \) when they are retired. Uncertainty’s effect on prices can make the initial old better off (worse off) if the capital-labor ratio is lower (higher) under uncertainty in period \( t = 0 \) relative to the prevailing interest rate in the perfect foresight economy.
Figure 1: On the left axis the ex-post CEV across generations is plotted under household beliefs: $\pi_e = 0.5$ and $\pi_\tau = 0.5$. An ex-post CEV greater than zero indicates a welfare gain in terms of lifetime consumption. The right axis plots the capital-labor ratio transition path under uncertainty, relative to the transition path under perfect foresight. Early and late reforms occur in periods $t = 1$ and $t = 2$ respectively.

4.2 Ex-Ante Welfare Analysis

This section uses a behind-the-veil ex-ante CEV metric to capture the welfare implications of uncertainty in line with the style of analysis conducted in Kitao (2017). The ex-ante CEV captures the amount of lifetime consumption a household would require or need to give up in the perfect foresight economy to be made as well off as their expected utility in the world with reform uncertainty. As uncertainty is only present in periods $t = 0$ and $t = 1$, the only households that face it directly are those born in periods $t \in \{-1, 0, 1\}$. For those generations that do not face uncertainty directly the ex-ante and ex-post CEV are identical as the only mechanism through which uncertainty impacts their welfare is through the relative price channel documented in section 4.1. For this reason, this section focuses only on those generations that face uncertainty directly during their lifetime.

As before $U_t(a)$ denotes the realized lifetime utility of generation $t$ under reform $a \in \{\tau_e, \tau_L, B_e, B_L\}$ in a perfect foresight economy. A household born in period $t \in \{-1, 0, 1\}$ has an expected lifetime utility $\mathbb{E}_t(U_t|\Pi)$, which depends on household beliefs $\Pi \equiv \{\pi_e, \pi_\tau\}$. Generation $t = 1$ faces type uncertainty only when a reform does not occur early. In this case I compute the household’s expected utility using the two remaining transition paths.

\[
\mathbb{E}_t(U_t|\Pi) = \sum_{k=1}^{4} \left[ \pi_t(a_k) \sum_{j=t}^{t+2} \beta^{j-t} u(\tilde{c}_j^t(a_k, \Pi), \tilde{l}_j^t(a_k, \Pi)) \right]
\]  

(26)
where \( \{c^j_t(a, \Pi), l^j_t(a, \Pi)\}_{j=t}^{t+2} \) denotes generation \( t \)'s allocations in an economy with reform uncertainty in periods \( t \in \{0, 1\} \) but with reform \( a \) begin announced at the end of either period \( t = 0 \) or \( t = 1 \) for generations \( t \in \{-1, 0\} \) and \( t = 1 \) respectively.

The ex-ante CEV, \( \mathcal{V}^\text{ex ante}_t(a, \Pi) \), is defined as:

\[
\sum_{j=t}^{t+2} \beta^{j-t} u((1 + \mathcal{V}^\text{ex ante}_t(a, \Pi)c^j_t(a), l^j_t(a)) = \mathbb{E}_t(U_t | \Pi)
\]

which, under the utility specification defined in equation (21), yields the following expression:

\[
\mathcal{V}^\text{ex ante}_t(a, \Pi) = \exp \left[ \frac{\mathbb{E}_t(U_t | \Pi) - U_t(a)}{\theta(1 + \beta + \beta^2)} \right] - 1
\]

Similar to figure 1, I plot the ex-ante CEV on the left axis and the relative capital-labor ratio on the right axis of figure 2. Once again, I assume that household beliefs in period \( t = 0 \) are such that equal weight is placed on the four reforms (\( \pi_e = 0.5, \pi_r = 0.5 \)). If we compare figure 1 to figure 2 it is immediately clear that the ex-ante welfare implications differ substantially from the ex-post analysis for those households that face uncertainty during their lifetime.

Figure 2: The ex-ante CEV across generations is plotted on the left axis under household beliefs: \( \pi_e = 0.5 \) and \( \pi_r = 0.5 \). An ex-ante CEV greater than zero indicates a welfare gain in terms of lifetime consumption. The capital-labor ratio transition path under uncertainty, relative to the transition path under perfect foresight, is plotted on the right axis.

The ex-ante CEV is positive for some generations. This is not a result of uncertainty loving households but rather a result of two other forces. The first is through the general equilibrium channel. As prices in the economy with uncertainty differ
from those in the perfect foresight economy, some generations are made better off in the current period due to higher wages/interest rates. The second channel is due to changes in households’ perceived lifetime wealth. This results from the fact that each reform generates a different income profile for each generation. The ex-ante CEV may be positive due to the benchmark economy having a less preferred path for a generation’s income profile than the one expected under uncertainty. The general equilibrium effect and perceived wealth effect can work for or against one another depending on the generation, household beliefs, and benchmark reform.

First consider the initial middle-aged born in period $t = -1$. From Table 2 we know that in a perfect foresight economy the initial middle aged ranks the reforms as follows: (1) late tax, (2) late benefits, (3) early tax, and (4) early benefits. When households place a positive probability on an early benefit cut it decreases their expected old age utility significantly. This lowers their ex-ante CEV via the perceived wealth effect when an early tax increase, late tax increase, and a late benefit cut are used as the benchmark. In contrast, the initial middle aged ex-ante CEV is higher when an early benefits cut is used as the benchmark. Households placing a positive weight on the other three reforms increases their expected utility. Furthermore, under uncertainty the wage rate in period $t = 0$ is lower than it would have been in a perfect foresight economy when benefits are cut early. The relatively lower wage results in middle aged households working fewer hours, consuming more, and saving less for retirement in the presence of uncertainty. The increase in realized middle age utility and expected old age utility generates welfare gains for the initial middle aged relative to a perfectly anticipated benefit cut in period $t = 1$. The perceived wealth effect and general equilibrium effect work together to increase the initial middle aged ex-ante CEV when we use an early benefit cut as the benchmark reform.

In a perfect foresight economy, the initial young born in period $t = 0$ ranks the four reforms as follows: (1) late tax, (2) early tax, (3) early benefits, and (4) late benefits. The initial young’s expected utility, and ex-ante CEV, is decreasing in the probability they place on benefits being cut late. Even though wages in period $t = 0$ are lower under uncertainty relative to a perfectly anticipated late benefit cut, the perceived wealth effect dominates the general equilibrium effect. The same story holds when using an early benefit cut as the benchmark economy as the probability weights on tax style reforms increases the initial young’s expected utility, which dominates the negative price effect.

Finally, we consider the generation born in period $t = 1$. Recall that households in this generation only face uncertainty in the case of a late reform, leaving them with only reform type uncertainty. In a world where an early reform did not occur, the future young generation prefers a late tax increase to a late benefit reduction. Therefore, as households place equal weight on the two reforms, expected utility is smaller than the realized utility level in a prefect foresight economy when taxes are increased late. In terms of the general equilibrium channel, under uncertainty the wage in period $t = 1$ is higher (lower) than it would have been in a perfect foresight economy when taxes were increased late (benefits were cut late). In the end, the perceived wealth effect dominates the general equilibrium mechanism for the future young born in period $t = 1$ when households’ beliefs place equal weight on all reform
possibilities.

4.3 Varying Household Beliefs

In this section I relax the assumption on household beliefs whereby households place equal weight on the four reforms. I now consider the following set of beliefs:

\[ \Pi \equiv \{ \pi_e, \pi_T \} \in \{0.0, 0.2, 0.5, 0.8, 1.0\} \times \{0.0, 0.2, 0.5, 0.8, 1.0\} \]  \hspace{1cm} (29)

which consists of 25 different belief sets. I first explore how the relative capital-labor ratio along the transition path is affected by household beliefs and its implication for households’ welfare. I then discuss how beliefs impact households’ ex-ante CEV for those born into generations \( t \in \{-1, 0, 1\} \).

Figure 3 shows how the realized capital-labor ratio transition path under uncertainty, relative to the perfect foresight economy, varies across household beliefs. Critically the qualitative nature of the relative capital-labor ratio isn’t robust to the specification of household beliefs. For example, consider the capital-labor ratio under uncertainty when we use an early tax increase as the benchmark economy. When households place equal weight on each of the four reforms, \( \{\pi_e = 0.5, \pi_T = 0.5\} \), the capital-labor ratio is 0.6% lower under uncertainty in period \( t = 1 \) relative to the perfect foresight economy. However, across the set of household beliefs outlined in equation (29) the relative capital-labor ratio in period \( t = 1 \) ranges from -3.2% to +0.8%.

The variation in the relative capital-labor ratio stems from how household beliefs affect households’ savings and labor supply decisions. Continuing to use an early tax increase as the benchmark economy, as household beliefs move closer towards expecting a late tax increase they save less in period \( t = 0 \) because reform isn’t expected to occur for another period. This in turn yields a capital-labor ratio 3.2% lower in period \( t = 1 \) relative to the benchmark economy. When beliefs move towards early reforms, aggregate saving in period \( t = 0 \) increases in an attempt to compensate for lower after-tax labor income and/or an expected cut in retirement benefits in period \( t = 1 \). As beliefs move closer towards an early benefit cut the initial middle-aged households save more in period \( t = 0 \) to better consumption smooth in the face of lower Social Security benefits. The end result, is that the capital-labor ratio is 0.8% higher in period \( t = 1 \) when household beliefs are tilted towards an early benefit cut relative to the benchmark economy. Ultimately, the realized prices along the transition path are sensitive to the specification of household beliefs.
Figure 3: The dotted lines shows the capital-labor ratio under uncertainty, relative to the perfect foresight economy, across household beliefs (see equation (29)). The solid line shows the relative capital-labor ratio under household beliefs: \( \{ \pi_e = 0.5, \pi_\tau = 0.5 \} \).

The sensitivity of market clearing prices to beliefs has implications for households’ ex-post CEV. As discussed in section 4.1, households tend to benefit from uncertainty, ex-post, when uncertainty generates a higher wage (i.e. a higher capital-labor ratio) during their career relative to the perfect foresight economy. To better understand the interplay between household beliefs and their ex-post welfare I plot households’ ex-post CEV in figure 4 for three sets of beliefs:

\[
\Pi_1 = \{ \pi_e = 0.5, \pi_\tau = 0.5 \}
\]

\[
\Pi_2 = \begin{cases} 
\{ \pi_e = 0.8, \pi_\tau = 0.8 \} & \text{if benchmark = early tax increase} \\
\{ \pi_e = 0.2, \pi_\tau = 0.8 \} & \text{if benchmark = late tax increase} \\
\{ \pi_e = 0.8, \pi_\tau = 0.2 \} & \text{if benchmark = early benefit cut} \\
\{ \pi_e = 0.2, \pi_\tau = 0.2 \} & \text{if benchmark = late benefit cut} 
\end{cases}
\]

\[
\Pi_3 = \begin{cases} 
\{ \pi_e = 0.2, \pi_\tau = 0.2 \} & \text{if benchmark = early tax increase} \\
\{ \pi_e = 0.8, \pi_\tau = 0.2 \} & \text{if benchmark = late tax increase} \\
\{ \pi_e = 0.2, \pi_\tau = 0.8 \} & \text{if benchmark = early benefit cut} \\
\{ \pi_e = 0.8, \pi_\tau = 0.8 \} & \text{if benchmark = late benefit cut} 
\end{cases}
\]

The first set is used as a reference whereby households place equal weight on all four reforms. The second set of beliefs is structured such that households place 0.8 probability on the type and timing of the reform used in the benchmark economy. The third set of beliefs is structured such that households place 0.2 probability on the type and timing of reform used in the benchmark economy. I will refer to the second and third sets of beliefs as being more ‘aligned’ and ‘misaligned’ than belief
set one respectively. Under belief sets two and three households are equivalently uncertain; however, the difference between the two sets of beliefs is driven by their degree of misalignment. The misalignment of household beliefs is the extent to which household beliefs are at odds with the Social Security reform ultimately enacted by the government. The degree of misalignment matters because beliefs play an important role in determining the transition path of market clearing prices under uncertainty. When households put more weight (belief set (2), circle line in Figure 4) on the reform ultimately implemented by the government the welfare implications of uncertainty, through the general equilibrium channel, are diminished for all generations under each reform. In contrast, as households put less weight (belief set (3), diamond line in Figure 4) on the reform ultimately implemented by the government the welfare implications generated by the general equilibrium channel grow in magnitude.

Figure 4: The solid line shows the ex-post CEV under household beliefs: $\pi_e = 0.5$ and $\pi_\tau = 0.5$ (the same as Figure 1). The circle line shows the ex-post CEV under household beliefs that place 0.8 probability on the reform used in the benchmark economy. The diamond line shows the ex-post CEV under household beliefs that place 0.2 probability on the reform used in the benchmark economy.

Finally, this section concludes with a look at how the ex-ante CEV varies across household beliefs for each generation that faces uncertainty directly ($t \in \{-1, 0, 1\}$). Figure 5 shows the ex-ante CEV across generations for each benchmark economy across a large set of household beliefs (see equation (29)). Interpreting how beliefs impact generations born in periods $t \in \{-1, 0, 1\}$ follows the analysis conducted in section 4.2 and comes down to how each generation ranks the four reforms in a perfect foresight economy. For example, the initial middle aged and initial young generations believe themselves to be best off when households’ beliefs place 100% probability on a late tax increase ($\{\pi_e = 0, \pi_\tau = 1\}$), regardless of which benchmark economy is used. This is because these generations expected lifetime income profile is higher under a certain late tax increase than the other four reforms. This is driven by the fact that they never pay the tax increase because they are either dead or retired by period.
Moreover, they still receive the full amount of their promised Social Security benefits in retirement. Placing weight on other reforms lowers their expected future utility, thereby decreasing their ex-ante CEV via the perceived wealth channel.

When an early tax increase is used as the benchmark economy and when household beliefs place equal weight on each of the four reforms, the initial young generation’s ex-ante CEV is -0.5% in terms of lifetime consumption. However, as beliefs are varied the initial young’s ex-ante CEV ranges from (-1.8%,+2.6%). The initial young are never directly exposed to a late tax increase making it their most preferred reform. As beliefs move closer towards a late tax increase the initial young’s ex-ante CEV increases as their expected utility rises. In contrast, the initial young generation’s least preferred reform is a late benefit cut; therefore, as households place a higher likelihood on benefits being cut late the initial young’s ex-ante CEV falls and becomes negative. While the wage rate in period \( t = 0 \) is higher under uncertainty when household beliefs are heavily tilted towards an early benefit cut, its positive effect on the initial young’s welfare is small relative to the perceived wealth effect.

4.4 Hand-to-Mouth Households

In this section I introduce hand-to-mouth households (HtM) into the economy and explore their impact on general equilibrium prices. I then examine how this impacts the welfare implications of uncertainty for non-HtM households. For simplicity HtM households are assumed to not have access to capital markets and consume their income in each period. Kaplan et al. (2014) documented that approximately 30% of the US population is hand-to-mouth. For this reason, I set the measure of HtM households within each generation to 0.30. The HtM households’ problem is:
Welfare Implications of Uncertain Reform

\[
\max_{\{c_j^i, l_j^i\}_{j=t}^{t+2}} \mathbb{E}_t \left[ \sum_{j=t}^{t+2} \beta^{j-t} u(c_j^i, l_j^i) \right]
\]  

subject to:

\[
c_t^i = w_t (1 - \tau_t)(1 - l_t^i)
\]  

\[
c_{t+1}^i = w_{t+1} (1 - \tau_{t+1})(1 - l_{t+1}^i)
\]  

\[
c_{t+2}^i = B_{t+2} e^t
\]  

\[
e^t = \frac{w_t (1 - l_t^i) + w_{t+1} (1 - l_{t+1}^i)}{2}
\]

In steady state HtM households supply 40% of their time endowment to the labor market over the course of their career in contrast to households with access to capital markets that supply 36.2%. This stems from the HtM households’ desire to increase old age consumption by increasing their Social Security benefits through an increase in their average lifetime labor earnings. Under the utility specification defined in equation (21) the Social Security policy parameters: \(\{\tau, B\}\) have no direct impact on HtM households’ labor supply as they do not show up in their problem’s first-order conditions. The wage rate affects their labor supply and is the only mechanism through which HtM households are affected by uncertainty ex-post.\(^{11}\)

As non-HtM households make up 70% of the population their capital savings has a smaller impact on the relative aggregate capital-labor ratio. While increasing the share of HtM households mechanically reduces the aggregate capital stock, the welfare implications of uncertainty stemming from general equilibrium effects are always in reference to the perfect foresight economy prices. This means that the increase in the equilibrium wage coming from the presence of HtM households is netted out of the ex-post and ex-ante CEV calculations. Moreover, as changes in the aggregate capital stock are the primary driver of the relative capital-labor ratio, the smaller measure of households with access to capital markets reduces variation in the relative capital-labor ratio across beliefs (see figure 6). The introduction of HtM households leaves the qualitative results from sections 4.1 and 4.2 unaffected, but lowers the magnitude of the ex-post and ex-ante CEV metrics across optimally saving households.

\(^{11}\)See Appendix A for details.
Figure 6: The dotted lines show the capital-labor ratio under uncertainty, relative to the perfect foresight economy, across household beliefs (see equation (29)). The solid line shows the relative capital-labor ratio under household beliefs: \( \{\pi_e = 0.5, \pi_{\tau} = 0.5\} \). Population is 30% HtM.

Figure 7: The dotted lines show the ex-ante CEV for non-HtM households across all beliefs: \( \Pi \in \{0.0, 0.2, 0.5, 0.8, 1.0\} \times \{0.0, 0.2, 0.5, 0.8, 1.0\} \). The solid line shows the ex-ante CEV for non-HtM households under household beliefs: \( \pi_e = 0.5 \) and \( \pi_{\tau} = 0.5 \). Population is 30% HtM.
Table 3 displays HtM households’ preferences over the four reforms in a perfect foresight economy. The initial old HtM households are unaffected by the reforms as they hold zero capital in retirement and are un-exposed to the change in the interest rate in period $t = 0$. The initial middle aged HtM households are only directly affected by an early benefit cut, making it their least preferred reform. The remaining three reforms are ranked in accordance with their impact on the after-tax wage in period $t = 0$. The initial young generation prefers a late tax increase as the increase coincides with their retirement. The initial young are directly exposed to both benefit cut reforms and despite the lower after-tax wages during their career under an early tax increase, they prefer it to a cut in retirement benefits as Social Security is their only source of income in retirement.

The HtM households born in period $t = 1$ are directly affected by all four reforms. They prefer tax increases to benefit cuts due to the sensitivity of their old age consumption to changes in the replacement rate of Social Security. Regarding tax increases, they prefer a late tax increase over an early one as they can work under the lower initial tax rate when they are young thereby increasing their young consumption. Finally, all HtM households born in periods $t \geq 2$ have identical preference rankings. An early tax increase is most preferred as it maintains the higher level of Social Security benefits while having a smaller impact on wages relative to a late tax increase. Surprisingly an early benefit cut is preferred to a late tax increase as the reduction in benefits is counter balanced by higher after-tax wages during their careers. However, the magnitude of a late benefit cut dominates the after-tax wage effect resulting in HtM households preferring a late tax increase to a late benefit cut.

Table 3: Reform Preferences by Generation (HtM)

<table>
<thead>
<tr>
<th>Generation</th>
<th>Early Tax</th>
<th>Late Tax</th>
<th>Early Benefit</th>
<th>Late Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1 (initial middle aged)</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>0 (initial young)</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1 (future gen 1)</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2+ (future gen 2+)</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 8 shows how the ex-ante CEV differs for HtM households as we vary households’ subjective beliefs at time zero. The qualitative results for HtM households are similar to those depicted in figure 7 for optimally saving households; however, the magnitudes are significantly larger for HtM households. This extends the results found in Caliendo et al. (2015) to a general equilibrium framework regarding the welfare implications of policy uncertainty for those households that do not save. Regardless of which reform is used as the benchmark the ex-ante welfare implications of uncertainty for HtM households are highly sensitive to household beliefs.
Figure 8: The dotted lines show the ex-ante CEV for HtM households across all beliefs: \( \Pi \in \{0.0, 0.2, 0.5, 0.8, 1.0\} \times \{0.0, 0.2, 0.5, 0.8, 1.0\} \). The solid line shows the ex-ante CEV for HtM households under household beliefs: \( \pi_e = 0.5 \) and \( \pi_t = 0.5 \). Population is 30% HtM.

### 4.5 An Alternative Welfare Analysis

This section uses an alternative metric to discuss the welfare implications of uncertainty, analogous to the one used in Caliendo et al. (2015). Specifically, I compare households’ expected utility to the utility they would receive in a perfect foresight economy should they consume the same amount, in terms of consumption and leisure, as they do in expectation in the world with reform uncertainty (\( \bar{U}_t \)). This metric captures the welfare implications coming directly from households’ innate risk aversion and accounts for the perceived wealth effects discussed in section 4.2.

Expected utility is defined by equation (26). The alternative CEV, \( V_{alt}^t(\Pi) \), is defined as:

\[
\sum_{j=t}^{t+2} \beta^{j-t} u((1 + V_{alt}^t(\Pi))E_t(c_j^t|\Pi), E_t(l_j^t|\Pi)) = E_t(U_t|\Pi) \quad (38)
\]

which, under the utility specification defined in equation (21), yields the following expression:

\[
V_{alt}^t(\Pi) = \exp \left[ \frac{E_t(U_t|\Pi) - \bar{U}_t}{\theta(1 + \beta + \beta^2)} \right] - 1 \quad (39)
\]

In an economy with a zero-measure of HtM households, young non-HtM households born in period \( t = 0 \) have an alternative CEV ranging from -0.004% to -0.027% of lifetime consumption depending on household beliefs. As for HtM households born in period \( t = 0 \), their alternative CEV can be much larger and ranges from -0.01% to -
2.15% of lifetime consumption. While the levels should be interpreted with caution as household preferences exhibit a relatively low level of risk aversion (log preferences), the relative welfare costs between non-HtM and HtM households are consistent with the results found in Caliendo et al. (2015). A higher degree of risk aversion would yield quantitatively larger welfare costs associated with uncertainty relative to the findings in Caliendo et al. (2015), but more in line with the empirical literature. Finally, the main result of this paper continues to hold as the alternative CEV exhibits sensitivity to household beliefs.

5 Concluding Remarks

This paper develops a general equilibrium overlapping generations model to characterize the welfare implications of uncertainty surrounding the type and timing of an inevitable Social Security reform. As households face uncertain future prices and policies their optimal savings and labor supply decisions deviate from those in the perfect foresight economy. These individual-level decisions aggregate to affect market clearing prices, which in turn affect the welfare of all generations along the transition path from the economy’s initial steady state to its terminal steady state following the implementation of the reform. Generations born after reform has taken place face no uncertainty directly; however, they are still affected by uncertainty in the past through its effect on market clearing prices. When uncertainty generates a higher market clearing wage, relative to the perfect foresight economy, younger households benefit through higher career earnings while older households are made worse off through the lower rate of return on their savings. Ultimately, the welfare implications of uncertainty stemming from the general equilibrium channel are uneven across generations.

This paper extends the existing literature by investigating how the welfare implications of uncertainty are impacted by household beliefs and the presence of hand-to-mouth households in a general equilibrium setting. The introduction of a positive measure of hand-to-mouth households into the model economy lowers the sensitivity of prices to reform uncertainty. This in turn lowers the magnitude of the welfare implications of uncertainty for those households with access to a savings technology, but leaves the qualitative results unaffected. I find that households’ subjective beliefs over the four reforms considered in this paper have a significant impact on the quantitative and qualitative welfare results. While the quantitative results over a large set of beliefs are consistent with those found in the literature, they should be interpreted carefully given a 20-year model period and a low degree of risk aversion. This paper highlights the importance of the specification of household beliefs when conducting general equilibrium analysis surrounding policy uncertainty.
References


Appendices

A Households’ Problem

Consider the households’ problem from section 2.4. Under logged Cobb-Douglas preferences there are closed form decision rules for households in a perfect foresight economy. In equilibrium the solution to a young household’s problem can be written down as a system of seven equations and seven unknowns: \{c_t^t, c_{t+1}^{t+1}, c_{t+2}^{t+2}, l_t^t, l_{t+1}^{t+1}, k_t^t, k_{t+1}^{t+1}\}. For notational convenience let: \(T_{t+2} = \frac{R_{t+2}}{2}\).

\[
\begin{align*}
\frac{1}{c_t^t} &= \frac{\beta R_{t+1}}{c_{t+1}^{t+1}} \quad (1) \\
\frac{1}{c_{t+1}^{t+1}} &= \frac{\beta R_{t+2}}{c_{t+2}^{t+2}} \quad (2) \\
1 - \theta &= \frac{\theta w_t(1 - \tau_t)}{c_t^t} + \frac{\theta \beta^2 w_t T_{t+2}}{c_{t+2}^{t+2}} \quad (3) \\
1 - \theta &= \frac{\theta w_{t+1}(1 - \tau_{t+1})}{c_{t+1}^{t+1}} + \frac{\theta \beta w_{t+1} T_{t+2}}{c_{t+2}^{t+2}} \quad (4) \\
c_t^t + k_t^t &= w_t(1 - \tau_t)(1 - l_t^t) + R_t k_t^t \quad (5) \\
c_{t+1}^{t+1} + k_{t+1}^{t+1} &= w_{t+1}(1 - \tau_{t+1})(1 - l_{t+1}^{t+1}) + R_{t+1} k_{t+1}^{t+1} \quad (6) \\
c_{t+2}^{t+2} &= \left[ w_t(1 - l_t^t) + w_{t+1}(1 - l_{t+1}^{t+1}) \right] T_{t+2} + R_{t+2} k_{t+2}^{t+2} \quad (7)
\end{align*}
\]

where equations (1)-(4) are Euler equations and equations (5)-(7) are budget constraints. The analytical solution utilizes the following objects for algebraic convenience:

\[
\begin{align*}
\Omega_0 &\equiv \frac{1 - \theta}{\theta} \cdot \frac{R_{t+1} R_{t+2}}{w_t(1 - \tau_t) R_{t+1} R_{t+2} + w_t T_{t+2}} \\
\Omega_1 &\equiv \frac{1 - \theta}{\theta} \cdot \frac{R_{t+2}}{w_{t+1}(1 - \tau_{t+1}) R_{t+2} + w_{t+1} T_{t+2}} \\
\alpha_0 &\equiv \frac{\beta R_{t+2}(w_t(1 - l_t^t) + w_{t+1}) T_{t+2}}{\beta R_{t+2} + w_{t+1} \Omega_1 T_{t+2}} \\
\alpha_1 &\equiv \frac{\beta R_{t+2} R_{t+2}}{\beta R_{t+2} + w_{t+1} \Omega_1 T_{t+2}} \\
b_0 &\equiv 1 + w_t(1 - \tau_t) \Omega_0 \\
b_1 &\equiv 1 + w_{t+1}(1 - \tau_{t+1}) \Omega_1 \\
\phi &\equiv 1 + \frac{w_t \Omega_0 T_{t+2}}{\beta^2 R_{t+1} R_{t+2}} + \frac{w_{t+1} \Omega_1 T_{t+2}}{\beta R_{t+2}}
\end{align*}
\]
\[\xi = \frac{R_{t+2}}{\phi} + \frac{\beta R_{t+2}}{b_1}\]
\[\psi_0 = \frac{\beta R_{t+2}w_{t+1}(1 - \tau_{t+1}) - (w_t + w_{t+1})T_{t+2}}{\xi b_1}\]
\[\psi_1 = \frac{\beta R_{t+1}R_{t+2}}{\xi b_1}\]
\[\eta = \frac{R_{t+1} - \psi_1}{b_1} + \frac{\beta R_{t+1}}{b_0}\]

The young household’s optimal allocation \(\{c^t_k, l^t_k, k^t_{t+1}\}\) is characterized by:
\[k^t_{t+1} = \frac{\beta R_{t+1}(w_t(1 - \tau_t) + R_t k^t_k) - w_{t+1}(1 - \tau_{t+1}) - \psi_0}{\eta b_0}\] (8)
\[c^t_k|k^t_{t+1} = \frac{w_t(1 - \tau_t) + R_t k^t_k - k^t_{t+1}}{b_0}\] (9)
\[l^t_k|c^t_k = \Omega_0 c^t_k\] (10)

The middle aged household’s optimal allocation \(\{c^t_{t+1}, l^t_{t+1}, k^t_{t+2}\}\) is characterized by:
\[k^t_{t+2} = \frac{\beta R_{t+2}(w_{t+1}(1 - \tau_{t+1}) + R_{t+1} k^t_{t+1}) - \alpha_0 b_1}{\alpha_1 b_1 + \beta R_{t+2}}\] (11)
\[c^t_k|k^t_{t+2} = \frac{w_{t+1}(1 - \tau_{t+1}) + R_{t+1} k^t_{t+1} - k^t_{t+2}}{b_1}\] (12)
\[l^t_{t+1}|c^t_{t+1} = \Omega_1 c^t_{t+1}\] (13)

The old household’s optimal allocation \(\{c^t_{t+2}\}\) is given by its period budget constraint (equation (7)).

Next let us consider households who must make decisions in the face of uncertainty. For notational simplicity, from this point on I will omit the superscript \(t\) denoting the household’s period of birth. Furthermore, let \(\{c_t(h), c_{t+1}(h), c_{t+2}(h), l_t(h), l_{t+1}(h), k_{t+1}(h), k_{t+2}(h)\}\) and \(\{w_t(h), R_t(h)\}\) denote the equilibrium allocations and market clearing prices in a perfect foresight economy under reform \(h \in \{\tau_e, \tau_L, B_e, B_L\}\). Finally let \(\{\hat{c}_t(a), \hat{c}_{t+1}(a), \hat{c}_{t+2}(a), \hat{l}_t(a), l_{t+1}(a), k_{t+1}(a), k_{t+2}(a)\}\) and \(\{\hat{w}_t(a), \hat{R}_t(a)\}\) denote the equilibrium allocations and market clearing prices under uncertainty when the actual reform implemented by the government is of type \(a \in \{\tau_e, \tau_L, B_e, B_L\}\).

A young household’s allocation under uncertainty must satisfy their two Euler equations and period budget constraint:
\[\frac{1}{\hat{c}_t(a)} = \mathbb{E}_h \left[ \frac{\beta R_{t+1}(h)}{c_{t+1}(h)} \right]\] (14)
\[
\frac{1 - \theta}{l_t(a)} = \frac{\theta \hat{w}_t(a)(1 - \hat{l}_t(a))}{\hat{c}_t(a)} + \hat{w}_t(a) \mathbb{E}_h \left[ \frac{\theta \beta T_{t+2}(h)}{c_{t+2}(h)} \right] 
\]

\[
\hat{c}_t(a) + k_{t+1}(a) = \hat{w}_t(a)(1 - \hat{l}_t(a))(1 - \hat{l}_t(a)) + \hat{R}_t(a)k_t 
\]

A middle aged household’s allocation under uncertainty must satisfy their two Euler equations and period budget constraint:

\[
\frac{1}{c_{t+1}(a)} = \mathbb{E}_h \left[ \frac{\beta R_{t+2}(h)}{c_{t+2}(h)} \right] 
\]

\[
\frac{1 - \theta}{l_{t+1}(a)} = \frac{w_{t+1}(a)(1 - \tau_{t+1}(a))\theta}{c_{t+1}(a)} + w_{t+1}(a) \mathbb{E}_h \left[ \frac{\theta \beta T_{t+2}(h)}{c_{t+2}(h)} \right] 
\]

\[
c_{t+1}(a) + k_{t+2}(a) = (1 - \tau_{t+1}(a))w_{t+1}(a)(1 - l_{t+1}(a)) + R_{t+1}(a)k_{t+1}(a) 
\]

The routine used to solve these systems can be found in Appendix C.

An old household’s allocation simply follows their budget constraint:

\[
c_{t+2}(a) = \left[ \hat{w}_t(a)(1 - \hat{l}_t(a)) + w_{t+1}(a)(1 - l_{t+1}(a)) \right] \hat{T}_{t+2}(a) + R_{t+2}(a)k_{t+2}(a) 
\]

### A.1 Hand-to-Mouth Households

In section 4.4 we introduced hand-to-mouth households that do not have access to a savings technology. After plugging in the household’s period budget constraints, a young HtM household’s optimization problem can be written as:

\[
\max_{\{l_t, l_{t+1}\}} \mathbb{E}_t \left[ \ln (1 - l_t) + \chi \ln l_t + \beta \left( \ln (1 - l_{t+1}) + \chi \ln l_{t+1} \right) \right.
\]

\[
\left. \quad + \beta^2 \ln \left( w_t(1 - l_t) + w_{t+1}(1 - l_{t+1}) \right) + C \right] 
\]

where \( \chi \equiv \frac{1 - \theta}{\theta} \) and \( C \equiv \ln \left( w_t(1 - \tau_t) + \beta \ln \left( w_{t+1}(1 - \tau_{t+1}) \right) + \beta^2 \ln \left( B_{t+2} \right) \right) \).

In a perfect foresight economy the HtM households’ Euler equations when young and middle aged respectively are as follows:

\[
\frac{\chi}{l_t} = \frac{1}{1 - l_t} + \beta^2 \frac{w_t}{w_t(1 - l_t) + w_{t+1}(1 - l_{t+1})} 
\]

\[
\frac{\chi}{l_{t+1}} = \frac{1}{1 - l_{t+1}} + \beta \frac{w_{t+1}}{w_t(1 - l_t) + w_{t+1}(1 - l_{t+1})} 
\]

When HtM households face uncertainty the young and middle aged households’
Euler equations are:

\[
\frac{\chi}{\ell_t(a)} = \frac{1}{1 - \ell_t(a)} + \beta^2 \mathbb{E}_h \left[ \frac{\hat{w}_t(a)}{\hat{w}_t(a)(1 - \ell_t(a)) + w_{t+1}(h)(1 - l_{t+1}(h))} \right] \tag{24}
\]

\[
\frac{\chi}{\ell_{t+1}(a)} = \frac{1}{1 - l_{t+1}(a)} + \beta \frac{\hat{w}_{t+1}(a)}{\hat{w}_t(a)(1 - \ell_t(a)) + w_{t+1}(h)(1 - l_{t+1}(h))} \tag{25}
\]

The routines used to solve these systems can be found in Appendix C.
The main welfare analysis concerning uncertainty in section 4 focuses exclusively on the transition path from the initial steady state to each reform’s respective terminal steady state. It is however a useful exercise to examine how the four reforms affect the economy in the long-run to better understand the underlying mechanisms in the model. I solve the social planner’s problem and compare the planner’s optimal allocation to those from the competitive equilibria across a set of six policy parameterizations. I first compare the planner’s allocation to the competitive allocation from an economy absent the Social Security system. The five remaining allocations consist of the steady states under the Social Security program’s initial parameterization and those resulting form each of the four reforms. The social planner maximizes households’ welfare according to their welfare weights subject to the economy’s aggregate resource constraint. The planner’s problem can be written as:

$$\max_{\{c_t^i, n_t^i\}_{j=t+1}^{\infty}, K_{t+1}} \sum_{t=0}^{\infty} \omega_t \sum_{j=t}^{t+2} \beta^{j-t} \left[ \theta \log c_t^j + (1 - \theta) \log (1 - n_t^j) \right]$$

subject to:

$$c_t^t + c_t^{t-1} + c_t^{t-2} + K_{t+1} = K_t^\alpha (n_t^t + n_t^{t-1})^{1-\alpha} + (1 - \delta)K_t \quad \forall t$$

I assume $\omega_t = \omega^d$ for $\omega < 1$ in order to obtain a steady state allocation. Furthermore, I focus attention on the limiting case in which $\omega \to 1$ that yields the welfare maximizing planner allocation. Using the allocation chosen by the planner as a point of reference I compare the model’s competitive equilibrium allocations under different policy parameterizations. Table 4 shows the allocation chosen by the planner (column 1) along with the allocations resulting from the competitive equilibria under the initial Social Security program ($\tau = 0.1060$, $B = 0.3640$), and the four reforms (columns 3-7). The second column in table 4 contains the equilibrium allocation of the model absent the Social Security program (i.e. $\tau = 0$ and $B = 0$). Following Balasko and Shell (1981), the model absent a public pension system is dynamically efficient as $\sum_{t=0}^{\infty} R_t = +\infty$ in steady state. Therefore the introduction of a Social Security system into this model is not addressing any inherent dynamic inefficiency as commonly observed in overlapping generations models.

Under the competitive equilibria households’ accumulate less capital, in aggregate, relative to the output maximizing level chosen by the planner. This increases the return on capital, which dictates the consumption and labor profile for households. For all equilibrium interest rates $R > \frac{1}{\beta} = 1.4528$ the consumption profile slopes upwards, and labor profiles slope downwards, in contrast to the planner’s allocation when $\omega = 1$. By comparing the terminal steady states to the initial steady we see that a cut in benefits, holding taxes constant, increases the aggregate capital stock.
labor supply, and output. As households save optimally in the model, a reduction in benefits incentivizes households to save more for retirement. Moreover, as resources stored in the SSTF are unproductive and do not enter the capital stock, a cut in benefits leaves more productive resources in the economy that boosts aggregate output. Increasing the OASI tax magnifies the Social Security program’s distortion on the economy and increases the amount of resources stored in the unproductive SSTF. The end result is a fall in aggregate capital, labor supply, and output.

Table 4: Steady State Analysis

<table>
<thead>
<tr>
<th>Social Planner</th>
<th>NO Social Security</th>
<th>Initial Policy</th>
<th>Early Tax</th>
<th>Late Tax</th>
<th>Early Benefits</th>
<th>Late Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_t )</td>
<td>0.2880</td>
<td>0.4259</td>
<td>0.4463</td>
<td>0.4527</td>
<td>0.4557</td>
<td>0.4420</td>
</tr>
<tr>
<td>( n_{t+1} )</td>
<td>0.5099</td>
<td>0.3549</td>
<td>0.3137</td>
<td>0.2989</td>
<td>0.2914</td>
<td>0.3230</td>
</tr>
<tr>
<td>( N_t )</td>
<td>0.7980</td>
<td>0.7808</td>
<td>0.7600</td>
<td>0.7516</td>
<td>0.7471</td>
<td>0.7651</td>
</tr>
<tr>
<td>( k_{t+1} )</td>
<td>–</td>
<td>0.0326</td>
<td>0.0291</td>
<td>0.0268</td>
<td>0.0299</td>
<td>0.0303</td>
</tr>
<tr>
<td>( k_{t+2} )</td>
<td>–</td>
<td>0.0585</td>
<td>0.0386</td>
<td>0.0349</td>
<td>0.0333</td>
<td>0.0435</td>
</tr>
<tr>
<td>( K_t )</td>
<td>0.2415</td>
<td>0.0911</td>
<td>0.0677</td>
<td>0.0625</td>
<td>0.0601</td>
<td>0.0735</td>
</tr>
<tr>
<td>( c_t )</td>
<td>0.1373</td>
<td>0.0756</td>
<td>0.0619</td>
<td>0.0543</td>
<td>0.0509</td>
<td>0.0630</td>
</tr>
<tr>
<td>( c_{t+1} )</td>
<td>0.0945</td>
<td>0.0849</td>
<td>0.0804</td>
<td>0.0732</td>
<td>0.0699</td>
<td>0.0785</td>
</tr>
<tr>
<td>( c_{t+2} )</td>
<td>0.0651</td>
<td>0.0954</td>
<td>0.1044</td>
<td>0.0988</td>
<td>0.0961</td>
<td>0.0979</td>
</tr>
<tr>
<td>( Y_t )</td>
<td>0.4947</td>
<td>0.3306</td>
<td>0.2889</td>
<td>0.2780</td>
<td>0.2726</td>
<td>0.2997</td>
</tr>
<tr>
<td>( w_t )</td>
<td>0.3720</td>
<td>0.2540</td>
<td>0.2281</td>
<td>0.2219</td>
<td>0.2189</td>
<td>0.2350</td>
</tr>
<tr>
<td>( R_t )</td>
<td>1.0000</td>
<td>1.6325</td>
<td>1.8869</td>
<td>1.9588</td>
<td>1.9953</td>
<td>1.8124</td>
</tr>
</tbody>
</table>

B.1 Solving the Social Planner Problem

The planner’s optimal allocations must satisfy the following six equations coming from their problem’s first-order-conditions and the aggregate resource constraint:

\[
c^t + c^{t-1} + c^{t-2} + K_{t+1} = K^\alpha_t (n_t + n^{t-1})^{1-\alpha} + (1-\delta)K_t
\]

\[
c^t_{t+1} = \beta \left(1 + \alpha K^\alpha_{t+1} (n_{t+1}^{t+1} + n_{t+1}^{t})^{1-\alpha} - \delta \right) c^t_t
\]

\[
c^t_{t+2} = \beta \left(1 + \alpha K^\alpha_{t+2} (n_{t+2}^{t+2} + n_{t+2}^{t+1})^{1-\alpha} - \delta \right) c^t_{t+1}
\]

\[
\frac{c^t_t}{1-n_t} = \frac{\theta}{1-\theta} (1-\alpha) K^\alpha_t (n_t + n^{t-1})^{-\alpha}
\]

\[
\frac{c^{t-1}_t}{1-n^{t-1}_t} = \frac{\theta}{1-\theta} (1-\alpha) K^\alpha_t (n_t + n^{t-1})^{-\alpha}
\]

\[
1 + \alpha K^\alpha_{t+1} (n_{t+1}^{t+1} + n_{t+1}^{t})^{1-\alpha} - \delta = \frac{\omega_t}{\omega_{t+1}} \frac{c^{t+1}_{t+1}}{c^t_t}
\]
In order for the social planner’s allocation to admit a steady state \( \omega_t \) must be a constant for all \( t \). I assume \( \omega_t = \omega \) \( \forall t \) where \( \omega < 1 \) and \( \sum_{t=0}^{\infty} \omega^t = \frac{1}{1-\omega} < \infty \). Then after imposing the steady state such that \( \frac{c_{t+1}}{c_t} = 1 \), I use equation (6) to obtain an expression for \( K_{t+1} \):

\[
K_{t+1} = \left( \frac{\omega \alpha}{1 - \omega(1 - \delta)} \right)^{1/1-\alpha} (n_{t+1} + n_t)
\]  

Note that as \( \omega \to 1 \):

\[
K_{t+1} \lim_{\omega \to 1} \left( \frac{\alpha}{\delta} \right)^{1/1-\alpha} (n_{t+1} + n_t)
\]

which is the planner’s welfare maximizing capital choice such that \( R_t = 1 \) \( \forall t \). For this reason I focus on the limiting case where \( \omega \to 1 \) above.

After plugging in equations (6) and (7) into equations (1) through (5) we obtain the social planner’s optimal allocation:

\[
c_t = \left[ \frac{2\phi}{1 + \rho + \rho^2 + \frac{1+\rho}{\phi} \psi} \right] \quad \forall t
\]  

\[
c_{t+1}^t = \rho \left[ \frac{2\phi}{1 + \rho + \rho^2 + \frac{1+\rho}{\phi} \psi} \right] \quad \forall t
\]  

\[
c_{t+2}^t = \rho^2 \left[ \frac{2\phi}{1 + \rho + \rho^2 + \frac{1+\rho}{\phi} \psi} \right] \quad \forall t
\]  

\[
n_t = 1 - \frac{1}{\phi} \left[ \frac{2\psi}{1 + \rho + \rho^2 + \frac{1+\rho}{\phi} \psi} \right] \quad \forall t
\]  

\[
n_{t-1}^t = 1 - \frac{\rho}{\phi} \left[ \frac{2\psi}{1 + \rho + \rho^2 + \frac{1+\rho}{\phi} \psi} \right] \quad \forall t
\]  

\[
K_{t+1} = \eta \left( 2 - \frac{1 + \rho}{\phi} \left[ \frac{2\psi}{1 + \rho + \rho^2 + \frac{1+\rho}{\phi} \psi} \right] \right) \quad \forall t
\]

where

\[
\eta \equiv \left( \frac{\omega \alpha}{1 - \omega(1 - \delta)} \right)^{1/1-\alpha}
\]

\[
\rho \equiv \beta(1 + \alpha \eta^{\alpha-1} - \delta)
\]
\[
\psi \equiv \eta^\alpha - \delta \eta \\
\phi \equiv \frac{\theta}{1 - \theta}(1 - \alpha)\eta^\alpha
\]
C Numerical Solution Method

This section outlines the numerical algorithm used to compute the transition path from the economy’s initial steady state to its terminal steady state following a reform. I consider a numerical time-horizon of 10 periods. Prior to period $t = 0$ the economy is in its initial steady state. Early reforms occur in period $t = 1$ and late reforms in period $t = 2$. The numerical time horizon is large enough such that increasing the horizon has no impact on the transition path.

As the log utility specification admits closed form expressions for households’ decisions in a perfect foresight economy (see Appendix A), I bypass solving the households’ optimization problem numerically. I use these rules to solve for the transition paths under each reform in a perfect foresight economy. To solve for the transition path in a world with uncertainty I iterate on households’ decision rules until convergence conditions are met. The solution algorithm requires an initial guess of the capital-labor ratio and the equilibrium set of household allocations is robust to changes in these initial guesses.

The algorithm used to solve for the transition path from this initial steady state to the terminal steady state, in a perfect foresight economy, is as follows:

(i) Guess a sequence of capital-labor ratios, $\{K^G_t\}_{t=1}^{10}$, and generate the corresponding prices $\{R^G_t, w^G_t\}_{t=1}^{10}$.

(ii) Given the sequence of prices and policies, $\{R^G_t, w^G_t, \tau_t, T_t\}_{t=1}^{10}$, use the household decision rules under certainty to obtain household level capital and labor choices: $\{\{n^t_j, k^t_{j+1}\}_{j=0}^{11}\}_{t=1}^{10}$.

(iii) Aggregate the optimal allocations obtained in step (ii) to arrive at the implied capital and labor supply: $\{K^I_t, N^I_t\}_{t=1}^{10}$.

(iv) Using the implied capital and labor supplies, generate the implied capital-labor ratio, and define a gap variable as: $gap = \max_t \left[ |K^G_t - K^I_t| \right]$.

(v) If $gap < toler$ then stop, market clearing prices have been found. If not then let: $K^G_t = \gamma K^G_t + (1 - \gamma)K^I_t$ for $\gamma \in (0, 1)$ and return to step (i).

The algorithm used to solve for the transition path from this initial steady state to the terminal steady state, under uncertainty, is as follows:

(i) Guess $K^G_0$ and generate the corresponding prices $\{R^G_0, w^G_0\}$.

(ii) Given the past capital-labor ratio realizations and the perfect foresight transition paths under each reform, the guess on period 0’s capital-labor ratio, and household beliefs. Solve for household decisions under uncertainty to obtain household level capital and labor choices: $\{n^t_j, k^t_{j+1}\}_{j=0}^{11}$.

\footnote{Plugging in equation \{24\} into \{22, 23\} from Appendix A yields two equations and two unknowns ($\{\hat{c}(a), \hat{l}(a)\}$) that can be solved via a fixed point argument.}
(iii) Aggregate the optimal allocations obtained in step (ii) to arrive at the implied capital and labor supply in $t = 0$: $\{K_0^I, N_0^I\}$

(iv) Using the implied capital and labor supplies, generate the implied capital-labor ratio, and define a gap variable as: $\text{gap} = \left| KN_0^G - KN_0^I \right|$ for $t = 0$

(v) If $\text{gap} < \text{toler}$ then continue to step (vi), else let: $KN_0^G = \gamma KN_0^I + (1 - \gamma)KN_0^I$ for $\gamma \in (0, 1)$ and return to step (i).

(vi) The next part of the algorithm depends on if the actual reform occurred early in period $t = 1$ or late in period $t = 2$. If the reform occurred early then:

(viia) Take the initial steady state prices and allocations as given along with the realized outcomes in period $t = 0$. Following an early reform in period $t = 1$ all uncertainty is alleviated and we can use the algorithm described above in the perfect foresight economy case to solve for the remainder of the transition path.

otherwise,

(viib) Type uncertainty persists and we must solve for the sequence of prices under both a late tax increase and an early benefits cut in a perfect foresight economy.

(viii) Once the certainty transition paths are obtained we guess $KN_1^G$ and generate the corresponding prices $\{R_1^G, w_1^G\}$.

(ix) Take past prices and the perfect foresight transition paths under each reform, the guess on the capital-labor ration in period $t = 1$, and household beliefs as given. Solve for household decisions under uncertainty to obtain household level capital and labor choices:

$\{n_{1+j}^t, k_{1+j+1}^t\}_{j=0}^1$.

(x) Aggregate the optimal allocations obtained in step (ii) to arrive at the implied capital and labor supply for $t = 1$: $\{K_1^I, N_1^I\}$

(xi) Using the implied capital and labor supplies, generate the implied capital-labor ratio, and define a gap variable as: $\text{gap} = \left| KN_1^G - KN_1^I \right|$ for $t = 1$.

(xii) If $\text{gap} < \text{toler}$ then continue to step (xiii), else let: $KN_1^G = \gamma KN_1^I + (1 - \gamma)KN_1^I$ for $\gamma \in (0, 1)$ and return to step (viii).

(xiii) Take the realized market clearing prices and household allocations from periods $t = 0 : 1$ as given. Following a late reform in period $t = 2$ all uncertainty is alleviated and we can use the algorithm described above in the perfect foresight economy case to solve for the remainder of the transition path.
C.1 Equivalent Policy Reforms

In equilibrium, under perfect foresight, each reform yields the same infinite horizon discounted present value of the government’s Social Security budget at time $t = 0$ (see equation 6 in section 2.2). Under each reform the economy will eventually converge to within a finite tolerance of its terminal steady state by period $\bar{t}$, as a result I can re-write the above expression as:

$$\bar{A} = F_0 + \sum_{j=1}^{t-1} \left[ \beta^j z_j(a) \right] + \frac{\beta^t}{1-\beta} z(a) \quad \forall a \in \{\tau_e, \tau_L, B_e, B_L\}$$

I exogenously set $\tau_e$ such that the smallest steady state surplus is zero (it happens to be an early benefits cut that runs a balanced budget in steady state) while all other reforms yield a positive steady state surplus. This ensures that the SSTF will never go negative following a reform. Once the $\tau_e$ is set, finding the other three equivalent policy reforms amounts to a one-dimensional root finding problem. I implement the standard Newton’s bisection method until the $A_0(a) = \bar{A}$ for each reform.

C.2 Hand-to-Mouth Households

Introducing Hand-to-Mouth households into the economy is relatively straightforward. Once the measure of HtM households within each generation has been set exogenously, the aggregation step in the routines above adjust accordingly such that the aggregate capital stock and labor supply are linear combinations of the aggregates of each sub-group. When the measure is set to zero the HtM households have no effect on general equilibrium outcomes. After some algebra, obtaining HtM households’ equilibrium allocations entails applying Newton’s bisection root-finding algorithm to the households’ Euler equations in Appendix A.