COGS Q580 Assignment #1
due 9/11/18

Your written solution should be turned in as a printed Mathematica notebook. It should follow the structure of each question (that is, make a separate section for each problem and separate subsections for each subpart; see the solution template on the course web page). Be sure to show and document all of your work and results and to clearly explain your reasoning. Do not include temporary work and errors that are not part of your final solution.

For this assignment, consider the following differential equation:

\[ \dot{x} = \sin(ax + 2) - 8x^3 + bx \]

1. Suppose \( a = 10, \ b = 3 \). Without using Dynamica except for part (d), carry out the following analysis of this dynamical system.
   a. Plot a set of trajectories over time for this system. What can you conclude about the dynamics of this system just from this plot?
   b. Find all of the equilibrium points of this system
   c. Determine the stabilities of all equilibrium points you found in (b)
   d. Use Dynamica to display the phase portrait of this system. How accurate were your conclusions from (a)?

2. Now let’s explore the effect of the parameters \( a \) and \( b \).
   a. Without using Dynamica or any similar package, plot the bifurcation diagram of this system as \( a \) varies across the range \([6, 15]\) with fixed \( b = 3 \). Indicate the stability of each branch of equilibrium points.
   b. Without using Dynamica or any similar package, determine the parameter values at which any bifurcations occur and their types.
   c. Use Dynamica to plot the bifurcation diagram and compare it to the results that you obtained in parts (a) and (b).
   d. With \( a \) varying across the range \([6, 15]\) and \( b \) varying across the range \([-5, 7]\) use Dynamica to plot the parameter chart of this system. Explain the pattern of bifurcation curves that you see. Hint: explore the effects that \( a \) and \( b \) individually have on \( f \) and then relate those effects to the bifurcation diagrams you see at fixed \( a \) or fixed \( b \) slices through the parameter chart.
   e. Plot the equilibrium surface defined by this system. Discuss how the shape of this surface relates to your results from parts (c) and (d).