Dynamical Analysis of the Fitzhugh-Nagumo Model
This is Your Brain

Diagram of the human brain with labeled parts:
- central sulcus
- parietal lobe
- occipital lobe
- frontal lobe
- temporal lobe
- pons
- medulla
- cerebellum
- Sylvian fissure

Diagram of a neuron with labeled parts:
- Dendrite
- Cell body
- Mitochondrion
- Nissl substance
- Axon hillock
- Nucleus
- Neurofibrils
- Collateral branch
- Axon terminal
- Schwann cell
- Nodes of Ranvier

Additional image of a neuron's network.
The Ionic Basis of the Action Potential

The Hodgkin-Huxley Model

\[ \dot{n} = \frac{n_\infty(V) - n}{\tau_n(V)} \]

\[ \dot{m} = \frac{m_\infty(V) - m}{\tau_m(V)} \]

\[ \dot{h} = \frac{h_\infty(V) - h}{\tau_h(V)} \]

\[ CV \dot{V} = g_L (V - E_L) + \bar{g}_K n^4 (V - E_K) + \bar{g}_Na m^3 h (V - E_{Na}) + I \]
A Hodgkin-Huxley Action Potential

The Full Hodgkin-Huxley Model

\[ C \frac{dV}{dt} = g_L (V - E_L) + g_K n^4 (V - E_K) + g_Na m^3 h (V - E_{Na}) + I \]

\[ \dot{n} = \frac{n_\infty (V) - n}{\tau_n (V)} \quad \dot{m} = \frac{m_\infty (V) - m}{\tau_m (V)} \quad \dot{h} = \frac{h_\infty (V) - h}{\tau_h (V)} \]

\[ n_\infty (V) = \frac{\alpha_n (V)}{\alpha_n (V) + \beta_n (V)} \quad m_\infty (V) = \frac{\alpha_m (V)}{\alpha_m (V) + \beta_m (V)} \quad h_\infty (V) = \frac{\alpha_h (V)}{\alpha_h (V) + \beta_h (V)} \]

\[ \tau_n (V) = \frac{1}{\alpha_n (V) + \beta_n (V)} \quad \tau_m (V) = \frac{1}{\alpha_m (V) + \beta_m (V)} \quad \tau_h (V) = \frac{1}{\alpha_h (V) + \beta_h (V)} \]

\[ \alpha_n (V) = \frac{0.01(V + 55)}{1 - e^{-(V+55)/10}} \quad \alpha_m (V) = \frac{0.1(V + 40)}{1 - e^{-(V+40)/10}} \quad \alpha_h (V) = 0.07 e^{-(V+65)/20} \]

\[ \beta_n (V) = 0.125 e^{-(V+65)/80} \quad \beta_m (V) = 4 e^{-(V+65)/18} \quad \beta_h (V) = \frac{1}{e^{-(V+35)/10} + 1} \]
Observation 1

\[ \tau_m \ll \tau_n, \tau_h \]

We can replace \( m(t) \) with \( m_\infty(V) \)
Observation 2

\[ h(t) + n(t) \approx k \]

We can replace \( h(t) \) and \( n(t) \) with a single \( U(t) \)
Phenomena

- Resting potential
- Subthreshold response
- Action potential generation
- Threshold
- Accommodation
- Absolute and relative refractory periods
- Post-inhibitory rebound
- Repetitive firing
- Excitation block
The Fitzhugh-Nagumo Model

\[ \dot{v} = v - \frac{v^3}{3} - w + J \]

\[ \dot{w} = \varepsilon (v + a - bw) \]

Jacobian Matrix:

\[
\begin{pmatrix}
1 - v^2 & -1 \\
\varepsilon & -b\varepsilon
\end{pmatrix}
\]

For now, set \( \varepsilon = 0.1 \) and \( a = b = 1 \)
Vector Field

\[ v \]

\[ w \]
Nullclines

\[ \dot{v} = v - \frac{v^3}{3} - w + J = 0 \]

\[ w = v - \frac{v^3}{3} + J \]

\[ \dot{w} = \varepsilon(v + a - bw) = 0 \]

\[ w = \frac{v + a}{b} \]
Resting Potential and Subthreshold Response

![Graph showing resting potential and subthreshold response over time. The graph includes axes labeled as v and w, with t on the x-axis and the potential on the y-axis. The graph also includes a region labeled J.](image_url)
Resting Potential and Subthreshold Response

\((\bar{v}, \bar{w}) \approx (-1.44, -0.44)\)

\((\lambda_1, \lambda_2) \approx (-0.96, -0.22)\)
Action Potential Generation
Quasithreshold
Quasithreshold
Accommodation
Accommodation
Absolute and Relative Refractory Periods
Absolute and Relative Refractory Periods
Post-Inhibitory Rebound
Repetitive Firing
Excitation Block
Repetitive Firing and Excitation Block
(a, b) Parameter Chart

\[ a \quad b \]

\[ v \quad w \]

\[ BT \quad GH \quad C \]

\[ BT \quad GH \quad BT \]

\[ BT \quad GH \quad C \]

\[ BT \quad GH \quad BT \]