Errata to

Determinantal Probability Measures

Publ. Math. Inst. Hautes Études Sci. **98** (2003), 167–212.

by Russell Lyons

The last line of equation (5.4) should read

$$= \sum_{C \subseteq E'} \mathbf{P}^H [A_1 \cup C \subseteq \mathfrak{B}, \big(A_2 \cup (E' \setminus C)\big) \cap \mathfrak{B} = \varnothing] ,$$

where we define

$$E' := E \setminus (A_1 \cup A_2) .$$

The last line of (5.5) should then read

$$= \sum_{C \subseteq E'} \Big(\bigwedge_{e \in A_1 \cup C} P_H e \wedge \bigwedge_{e \in A_2 \cup (E' \setminus C)} P_H^\perp e,\ \theta_{A_1 \cup C} \wedge \theta_{A_2 \cup (E' \setminus C)} \Big) .$$

The proof of Theorem 7.2 uses (6.5), but the proof of (6.5) for E infinite was never explained. First, the second part of Proposition 6.3 is proved for infinite E in the same way that Theorem 7.1 is deduced from Theorem 6.2; then the first part of Proposition 6.3 is deduced by duality. Induction again shows (6.5) for infinite E as long as A and B are finite.

On p. 199, a sentence in the middle of the page is missing a few words. It should read: "Note that we allow A to be infinite; if $A = \bigcup_n A_n$ for an increasing sequence $\langle A_n \rangle$ of finite sets, then $\langle \mathbf{P}^Q(\bullet \mid A_n \subseteq \mathfrak{S}) \rangle$ is a stochastically decreasing sequence of probability measures by Theorem 6.5 and so its limit defines $\mathbf{P}^Q(\bullet \mid A \subseteq \mathfrak{S})$."

On p. 200, the last displayed equation should be

$$\mathbf{1}_{A \cap \{f\}}(e) + (-1)^{\mathbf{1}_A(e)} Q_{e,f} .$$

The last part of Theorem 8.1, which says that if Q_1 and Q_2 are commuting positive contractions and $Q_1 \le Q_2$, then $\mathbf{P}^{Q_1} \preccurlyeq \mathbf{P}^{Q_2}$, has been extended by Borcea, Brändén, and Liggett (*J. Amer. Math. Soc.* **22** (2009), 521–567) to remove the hypothesis that Q_1 and Q_2 commute. (They have also proved a stronger property than stochastic domination.) In fact, the method we used can also be used to remove the hypothesis of commutativity. Clearly, it suffices that there exist projections P_1 and P_2 that are dilations of Q_1 and Q_2

such that $P_1 \le P_2$. As pointed out to me by Hari Bercovici (August 2007), this follows from a more general result, Naĭmark's dilation theorem, which says the following: any measure whose values are positive operators, whose total mass is I, and is countably additive in the weak operator topology dilates to a spectral measure. The measure in our case is defined on a 3-point measure space, with one point having mass Q_1, the second mass $Q_2 - Q_1$, and the third mass $I - Q_2$.

It was never explained how to deduce negative associations for a determinantal probability measure on an infinite ground set E from that (Theorem 6.5) for a finite E. This is achieved through (6.6): Given increasing bounded functions f_1, f_2 measurable with respect to the σ-fields generated by $A \subset E$ and $E \setminus A$, respectively, let $E_n \subset E$ be finite sets that are increasing to E. The conditional expectations $\mathbf{E}[f_1 \mid \mathcal{F}(A \cap E_n)]$ and $\mathbf{E}[f_2 \mid \mathcal{F}(E_n \setminus A)]$ are increasing functions to which (6.6) applies and which converge to f_1, f_2 in L^2.

The last part of the proof of Theorem 7.2 has a more straightforward proof, not needing the event $\mathcal{A}_3$: Suppose that $\mathcal{A}$ is any event and $\mathcal{F}$ is any σ-field. We claim that if $\mathbf{P}(\mathcal{A} \mid \mathcal{F})\mathbf{1}_{\mathcal{A}} = 0$ a.s., then $\mathbf{P}(\mathcal{A}) = 0$. Indeed, it follows that $0 = \mathbf{E}\big[\mathbf{P}(\mathcal{A} \mid \mathcal{F})\mathbf{1}_{\mathcal{A}}\big] = \mathbf{E}\big[\mathbf{P}(\mathcal{A} \mid \mathcal{F})^2\big]$ by conditioning on $\mathcal{F}$, whence $\mathbf{P}(\mathcal{A} \mid \mathcal{F}) = 0$ a.s., and so $\mathbf{P}(\mathcal{A}) = 0$. Now apply this to $\mathcal{A} := \mathcal{A}_1 \cap \mathcal{A}_2 \subseteq \mathcal{A}_1$, $\mathcal{F} := \mathcal{F}(E \setminus \{e\})$ and $\mathbf{P} := \mathbf{P}^H$.

Department of Mathematics, Indiana University, Bloomington, IN 47405-5701
rdlyons@indiana.edu
http://mypage.iu.edu/~rdlyons/

29 May 2014