Errata to

Group-Invariant Percolation on Graphs

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On p. 36, the last displayed equation contains "$\iota(X)$", which should be "$\iota_E(X)$".

On p. 54, Remark 5.4 should have the hypothesis "at least one of them has a nonamenable automorphism group", rather than that one of the graphs is itself nonamenable.

On p. 56, the proof of Corollary 6.2 has "$H \subseteq G$", which should be "$G \subseteq H$".

The page numbers for the article by Grimmett and Newman are incorrect. They should be 167–190.

The displayed inequality in Theorem 4.4 should be numbered and referred to in place of (4.1) on the next line.

The proof that (iii) implies (ii) in Theorem 7.2 needs a little more explanation. Remark 4.3 can be extended to a mixed percolation and the expectation made conditional on o being kept. We apply this to the percolation obtained from ω where only infinite components are kept.

The citation of [BuKe1] in the proof of Theorem 1.3 on p. 61 should have been a citation to Newman, C.M. and Schulman, L.S. (1981) Infinite clusters in percolation models, *J. Statist. Phys.* **26**, 613–628.

The proof of Lemma 7.4(iii) is incorrect. One way to fix it is as follows: Assume that $K(x)$ has at least k ends with positive probability. Choose any finite tree T containing x so that with positive probability, $T \subset K(x)$ and $K(x) \setminus V(T)$ has at least k infinite components. Then with positive probability, $T \subset K(x)$, $K(x) \setminus V(T)$ has at least k infinite components, all edges in T are assigned values less than $1/2$, and all edges incident to $V(T)$ but not in T are assigned values greater than $1/2$. On this event, ω' contains T and T is part of a spanning tree in ω' with at least k ends.

The proof of Theorem 2.12 is incorrect. Moreover, the statements of Theorem 2.12 and Remark 2.13 can be strengthened. This also affects the first part of Theorem 1.2, of course. Here is a replacement:

Theorem 2.12 (Site-percolation threshold)**.** *If $\mathbf{P}$ is a G-invariant site percolation on a nonamenable Cayley graph $X = X(G)$ with $\mathbf{P}[o \in \omega] \ge d/(d + \iota_E(X))$, then there is an infinite component with positive probability. Moreover, if $\mathbf{P}[o \in \omega] > d/(d + \iota_E(X))$, then with positive probability, there is a component with $p_c < 1$.*

Proof. Write $n(x, y) := |\{z \in K(x)\,;\ z \sim y\}|$ for the number of neighbors of y in the component of x. Note that for each x, we have $\sum_{y \in \partial_V K(x)} n(x, y) = |\partial_E K(x)|$. We use the G-invariant mass transport corresponding to the expected mass transfer from x to y of

$$f(x, y) := \mathbf{E}\left[\frac{\mathbf{1}_{\{|K(x)|<\infty,\, y \in \partial_V K(x)\}} n(x, y)}{|\partial_E K(x)|}\right].$$

The mass that leaves o has mean $\mathbf{P}[o \in \omega, |K(o)| < \infty]$, which must equal the mean mass that enters o, namely

$$\mathbf{E}\Bigg[\sum_{x \sim o} \frac{\mathbf{1}_{\{o \notin \omega,\, |K(x)| < \infty\}} |K(x)|}{|\partial_E K(x)|}\Bigg] < \frac{d(1 - \mathbf{P}[o \in \omega])}{\iota_E(X)}$$

(provided $\mathbf{P}[K(o) = X] < 1$). Therefore, we get the estimate

$$\mathbf{P}[|K(o)| = \infty] > \mathbf{P}[o \in \omega]\left(1 + \frac{d}{\iota_E(X)}\right) - \frac{d}{\iota_E(X)}\,.$$

This proves the first assertion; the second follows as in the proof of Theorem 2.4. ∎

Remark 2.13. If there are triangles in X, then the threshold $d/(d + \iota_E(X))$ may be replaced by $d'/(d' + \iota_E(X))$, where d' is the maximum number of neighbors of o that are pairwise nonadjacent.

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