Erratum to

Identities and Inequalities for Tree Entropy

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The proof of Theorem 3.1 made an incorrect appeal to the monotone convergence theorem and should be replaced by the following:

Proof. The hypothesis is equivalent to $D \in \mathsf{DetAlg}$. Write $Q := (I + P)/2 \in \mathsf{Alg}$, which is the transition operator for the lazy random walk associated to P. Since $I - cQ \in \mathsf{Alg} \subseteq \mathsf{DetAlg}$ for $c \in \mathbb{R}$, it follows that $2D(I - cQ) \in \mathsf{DetAlg}$; in particular for $c = 1$, we obtain $\Delta \in \mathsf{DetAlg}$. Since $DQ \ge 0$, we have $2D \ge 2D - 2cDQ \ge 2D - 2DQ = \Delta$ for $0 \le c \le 1$. It is easy to see that $2D - 2cDQ \to \Delta$ in the measure topology as $c \uparrow 1$ (for its definition, see Fack and Kosaki (1986), §1.5), whence also $\log(2D - 2cDQ) \to \log \Delta$ in the measure topology as $c \uparrow 1$. Because of (3.1), we may apply the dominated convergence theorem to $\log^+(D - cDQ)$ (see Fack and Kosaki (1986), Theorem 3.6) and the monotone convergence theorem to $\log^-(D - cDQ)$ (see Fack and Kosaki (1986), Theorem 3.5(ii)), obtaining

$$\operatorname{Det} \Delta = \lim_{c \uparrow 1} \operatorname{Det}(2D - 2cDQ)\,. \tag{3.3}$$

Since $2D - 2cDQ = 2D(I - cQ)$, the fundamental theorem of Fuglede and Kadison (1952) yields $\operatorname{Det}(2D - 2cDQ) = 2 \operatorname{Det} D \cdot \operatorname{Det}(I - cQ)$. On the other hand, for $0 < c < 1$,

$$\log \operatorname{Det}(I - cQ) = \Re \operatorname{Tr} \log(I - cQ)$$

by Theorem 1 (2^o) of Fuglede and Kadison (1952) (or Theorem I.6.10 of Dixmier (1981)) and

$$\log(I - cQ) = -\sum_{k \ge 1} c^k Q^k / k$$

(in the norm topology). Therefore,

$$\log \operatorname{Det}(I - cQ) = -\sum_{k \ge 1} \Re \operatorname{Tr}_\rho c^k Q^k / k = -\sum_{k \ge 1} \operatorname{Tr}_\rho c^k Q^k / k\,,$$

whose limit as $c \uparrow 1$ is

$$-\sum_{k \ge 1} \operatorname{Tr}_\rho Q^k / k = \int -\sum_{k \ge 1} \frac{1}{k} q_k(o; G)\, d\rho(G, o)$$

by the monotone convergence theorem, where $q_k(o; G)$ is the k-step return probability for Q. By Lemma 3.5 of Lyons (2005),

$$\sum_{k \geq 1} \frac{1}{k} q_k(o; G) = \log 2 + \sum_{k \geq 1} \frac{1}{k} p_k(o; G) \,. \tag{3.4}$$

Comparing (2.2) with equations (3.3), (2.6), and (3.4), we deduce the equality in (3.2). ∎

The exhaustion of G by finite subnetworks G_n preceding Theorem 3.3 should specify that G_n are induced subnetworks.

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