S212 Exam 2 Review Sheet

Questions will be drawn from the topics arising in sections 6.1 (area between curves), 6.2 (computing volumes by integrating cross-sectional area), 7.8 (improper integrals), 8.1 arclength, 8.2 surface area of surfaces of revolution, 9.1 (modeling with differential equations), 9.3 (solving separable differential equations), 9.4 (population models such as exponential & logistic model).

Although there will not be questions relating solely to earlier material, i.e. pre-Exam 1 material, it will be necessary for you to review techniques of integration, properties of inverse trig functions, etc. in order to answer the questions on the newer material.

From the unit on ODE's (ordinary differential equations):

Know how to verify that a given function solves an ODE and an initial condition.

Be familiar with the 3 models for population growth we studied, namely:

$\frac{dP}{dt} = kP$ (exponential model)

$\frac{dP}{dt} = kP \pm I$ (exponential model with immigration)

$\frac{dP}{dt} = kP(1 - \frac{P}{K})$ (logistic model)

Note: exponential model with immigration was discussed in class–remember the pythons–and also appears in exercise 13, pg. 599. Also, if you need the explicit solution to the logistic equation on the exam then the solution (formula 7 on page 595) will be given to you.

For an ODE of the form $\frac{dy}{dt} = f(y)$, be able to determine (a) the equilibria and (b) the intervals of y-values for which any solution $y(t)$ is increasing/decreasing.

Be able solve separable ODE's and to satisfy a given initial condition using the method of section 9.3.

Regarding improper integrals:

Know definition of convergence/divergence of integral with either upper limit integration ∞, lower limit of integration $-\infty$ or both in terms of a limit.

Know definition of convergence/divergence of integral over a finite interval $[a, b]$ when the integrand has a discontinuity (vertical asymptote in its graph).

Know how to *recognize when an integral is improper* in this second sense by looking for "trouble" in the form of an integrand with a vertical asymptote.

Know the situation vis-a-vis convergence/divergence of the "key example" $f(x) = \frac{1}{x^p}$ for both types of improper integrals.

Be able to apply the comparison test to either type of improper integral by comparing a complicated integral to an integral of a bigger or smaller function that is easier to handle. In particular, be able to deal with "distracters" that get in the way of understanding convergence/divergence of an improper integral.

Regarding arclength, area, surface area, volume:

Know how to set up integral to compute the arclength of a graph given in the form $y = f(x)$ or $x = g(y)$.

Know how to set up integral to compute the surface area of a surface of revolution when the surface is formed by (i) rotating a graph $y = f(x)$ about the x-axis, (ii) rotating a graph $y = f(x)$ about the y-axis, (iii) rotating a graph $x = g(y)$ about the x-axis or (iv) rotating a graph $x = g(y)$ about the y-axis. Remember that they all fit the formula:

$$\text{Surface area } = 2\pi \int \text{radius}\, ds$$

where ds is the "arclength element" given by either $\sqrt{1 + f'(x)^2}\, dx$ or $\sqrt{1 + g'(y)^2}\, dy$.

Know that if a solid region D in 3-dimensional space is sliced by parallel planes (say perpendicular to the x-axis, for instance, for x ranging from a to b) and the area of each cross-section is $A(x)$ then

$$\text{Volume of } D = \int_a^b A(x)\, dx.$$

In particular, for a solid obtained by rotating a planar region about the x-axis or y-axis (or some other line parallel to the x or y-axis, know how to use disks or washers to compute the volume.

$$\text{If cross-sections are disks: Volume } = \pi \int (\text{radius})^2.$$

$$\text{If cross-sections are washers (annuli): Volume } = \pi \int \left\{(\text{big radius})^2 - (\text{small radius})^2\right\}.$$

Look over volume examples other than disks/washers like tetrahedron, pyramid, etc.