Stabilization versus Sustainability: Macroeconomic Policy Tradeoffs

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November 19, 2010

Abstract

Worldwide monetary and fiscal policies in the past few years have put into sharp relief the fundamental tradeoff between short-run stabilization and long-run sustainability that policymakers face. The paper is organized around this question: How do the effects of routine monetary and fiscal operations designed to achieve macroeconomic stabilization objectives change when the economy moves from a debt-GDP level where the probability of default is nil to a higher level—the “fiscal limit”—where that default probability is non-negligible? Three main results emerge. First, when the economy is near its fiscal limit a transitory monetary policy contraction reduces output more, and it reduces inflation only in the very short run, before leading to a sustained rise in inflation in the medium term. Second, higher government spending may be appreciably more inflationary when the economy is staring at its fiscal limit. These effects arise even though monetary policy actively target inflation and fiscal policy passively adjusts taxes to stabilize debt. Third, specification of the central bank’s instrument—risky versus risk-free short-term nominal interest rate—matters for the link between expected default rates and inflation.

Keywords: Fiscal sustainability, Sovereign debt default, Fiscal limit

JEL Codes: H60, E30, E62, H30


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1 Introduction

Worldwide monetary and fiscal policies in the past few years have put into sharp relief the fundamental tradeoff between short-run stabilization and long-run sustainability that policymakers face. Financial markets hit Greek and Irish policymakers over the head with this tradeoff by dramatically driving up sovereign debt risk premia and forcing fiscal consolidation, even as their economies were slipping deeper into recession. Britain’s new government has announced massive fiscal austerity plans in the midst of a weak economic recovery, on the grounds of ensuring bondholders that U.K. fiscal policy is sustainable.

Figure 1 nicely encapsulates the tension facing policymakers. Risk premia have been rising in fiscally troubled European economies, but so have inflation rates. So far, only Greece has inflation rates well above the European Central Bank’s target rate, but inflation in other countries in the figure is on an upward trajectory. Outside the monetary union, the Bank of England has consistently exceeded its inflation target of 2 percent, triggering the requirement that the bank’s governor write a letter of explanation to the U.K. Chancellor of the Exchequer. It is quite likely that as worldwide inflation rates pick up, central banks will begin to raise policy interest rates and shift back to their usual inflation fighting stance, even if debt-GDP ratios remain unusually high.\(^1\)

Research has found that sometimes fiscal consolidations, which are almost always undertaken when economies are near their fiscal limits, can be expansionary, in contrast to the predictions of models calibrated to normal fiscal times. At the same time, inflation in many economies experiencing higher debt levels has also increased, despite the depth of the recession and the implied levels of excess capacity. More generally, this suggests that economies may behave significantly differently in times of crisis. In this paper we are interested in assessing how the effects of routine monetary and fiscal operations designed to achieve macroeconomic stabilization objectives change when the economy moves from a debt-GDP level where the probability of default is nil to a higher level where that default probability is non-negligible.

To explore this issue, we develop a model of the “fiscal limit” in the context of a conventional new Keynesian model, with monetary and fiscal policy interactions. Monetary policy has real effects in our sticky-price economy, which implies that, in addition to the usual impacts on intertemporal consumption decisions, it also influences the size of the tax base and real debt service costs. At the same time, changes in government spending and/or distortionary tax rates, besides having the usual fiscal consequences, also influence inflation through aggregate demand and labor supply effects. On top of this rich mix of monetary

\(^1\)Canada, Norway, and Sweden have already begun to raise policy rates, though in the absence of high net sovereign debt levels.
Figure 1: Sovereign bond risk premia over comparable German Bund (right scale) and monthly HICP inflation rates at annual rates (left scale). Sources: ECB, European Commission, and Eurostat.

and fiscal policy interactions we introduce a fiscal limit as in Bi (2009), whereby there is a partial default on outstanding government debt when the economy breaches a maximum sustainable debt-output ratio. This ratio depends upon the state of the economy, as well as stochastic fluctuations in political risk, so that bondholders demand significant risk premia on government debt prior to hitting the fiscal limit, not unlike the kind of premia observed in figure 1.

Two main results emerge when the maintained assumption is that government policy rules do not vary with the level of debt government. First, when the economy is near its fiscal limit a transitory monetary policy contraction reduces output more, and it reduces inflation only in the very short run, before leading to a sustained rise in inflation in the medium term. Second, higher government spending may be appreciably more inflationary when the economy is staring at its fiscal limit. These effects arise even though monetary policy actively target inflation and fiscal policy passively adjusts taxes to stabilize debt. They are also loosely consistent with the recent correlations between risk premia and inflation rates in figure 1.

The results hinge on the nature of the monetary policy rules—whether the central bank’s instrument is the risky, as is common in the literature, or the risk-free nominal interest rate.
Positing that the central bank sets the risky rate to satisfy a Taylor rule is equivalent to postulating a risk-free rule that includes the usual positive response to inflation and a negative response to the expected default rate. Higher expected default rates are accommodated with lower risk-free rates and, therefore, higher inflation. Fiscal policy, meanwhile, responds to the reduced value of outstanding debt by passively reducing expected primary surpluses.

The paper considers the two extreme policy specifications—the instrument is either the risky rate on government bonds or the risk-free rate on which households base their decisions—recognizing that in practice monetary policy rates probably lie between the extremes in economies facing sovereign debt risk.

The plan of the paper is as follows. The next section outlines our general model. Section 3 uses a simple endowment economy to explore how the possibility of debt default can cause the monetary authorities to lose control of inflation, even though monetary policy remains active and fiscal policy passive in the sense of Leeper (1991). Given that the loss of inflation control suggested by this section would have real effects in our sticky-price economy, we explore the local determinacy properties of our model in section 4, and find that a high rate of default could result in equilibrium determinacy, but due to the resource costs of the inflationary consequences of default rather than the issues raised by Schabert (2010). We also conclude that these determinacy issues are not a concern for our benchmark calibration. Section 5 defines and describes the computation of the fiscal limit for the model of section 2, while the model’s calibration is laid out in section 6. Equilibrium decision rules for the model are discussed in section 7. We then analyze the dynamic impacts of an exogenous monetary contraction and a fiscal expansion in section 8, illustrating the import of the level of outstanding debt for macroeconomic policy effects.

2 A General Model

As our aim is to explore how the possibility of sovereign debt default interacts with monetary and fiscal policies, we consciously use a conventional new Keynesian model of the kind typically used to explore monetary and fiscal policy interactions (see, for example, Benigno and Woodford (2004)), modified only by allowing government debt to be risky. Specifically, households in our economy supply labor to imperfectly competitive intermediate goods producing firms who do not completely adjust prices in the face of shocks since they face costly Rotemberg (1982)-style price adjustment. Moreover, rather than rendering fiscal policy redundant by balancing the budget through lump-sum taxes, we assume that households’ labor and profit income is taxed. This influences their labor supply decisions, which in turn affects firms’ marginal costs and pricing decisions. Taken together, this implies a relatively
rich set of monetary and fiscal policy interactions: monetary policy has real effects due
to the assumption of price stickiness, which in turn affects both the size of the tax base
and real debt service costs. While fiscal policy, in the form of tax or government spending
changes have the obvious fiscal consequences, but also influence inflation either through the
labor supply response to distortionary taxation or the aggregate demand effect of changes
in government spending.

We then further extend this model to allow for the possibility of sovereign default where
a fixed rate of default is applied whenever the economy hits its fiscal limit. This fiscal limit is
defined as the maximum expected present value of future primary surpluses, where the exact
position of the Laffer curve underpinning that definition depends upon both the state of the
economy and the political constraints on taxing at that maximum level. Given the stochastic
nature of the fiscal limit (due to exogenous fluctuations in productivity, government spending
and political risk) investors may demand risk premia on government debt before reaching
the fiscal limit. This, in turn, affects debt service costs and debt dynamics and may imply
quite different monetary and fiscal policy impacts in comparison to an economy operating
well away from its fiscal limit.

2.1 Households Our cashless economy is populated by a large number of identical house-
holds of size 1, who have preferences given by,

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \]

where \( \beta \in (0, 1) \) is the households’ subjective discount factor, \( c_t \) is consumption and \( n_t \) the
households’ labor supply. The household receives nominal wages \( W_t \) and monopoly profits
\( \Upsilon_t \) from the firm, both of which are taxed at the rate, \( \tau_t \), and lump-sum transfers \( z_t \) from the
government. The household chooses consumption, \( c_t \), hours worked, \( n_t \), and nominal bond
holdings, \( B_t \), to maximize utility subject to their budget constraint,

\[ P_t c_t + \frac{B_t}{R_t} = (1 - \delta_t)B_{t-1} + (1 - \tau_t) (W_t n_t + P_t \Upsilon_t) + P_t z_t \]  \hspace{1cm} (1)

Each period there is some probability that the government will choose to default on the
fraction \( \delta_t \in [0,1] \) of debt outstanding at the beginning of period \( t \). This probability of
default is endogenous to the model, but is taken as given by households. Bonds, therefore,
pay a risky yield of $R_t$. First-order conditions for this optimization problem are:

$$\frac{1}{R_t} = \beta E_t \frac{u_c(t + 1) 1 - \delta_{t+1}}{u_c(t) \pi_{t+1}}$$  \(2\)

$$-\frac{u_n(t)}{u_c(t)} = w_t (1 - \tau_t)$$  \(3\)

where $w_t \equiv W_t/P_t$ is the real wage. The first condition describes the household’s optimal allocation of consumption over time, and the second, their optimal labor supply decision. Notice in the case of the latter, labor income is taxed so that changes in the tax rate will influence households’ desire to work.

Households can also trade in a risk-free bond, which is, however, in zero net supply, so where Euler equation (2) determines the risky nominal interest rate,

$$\frac{1}{R^f_t} = \beta E_t \frac{u_c(t + 1) 1 - \delta_{t+1}}{u_c(t) \pi_{t+1}}$$  \(4\)

defines the risk-free nominal interest rate.

2.2 Final Good Production  
Final goods production is for the purposes of private and public consumption and competitive final goods firms buy the differentiated products produced by intermediate goods producers in order to construct consumption aggregates, which have the usual CES form,

$$Y_t = \left( \int_0^1 y_t(i)^{\frac{\theta - 1}{\theta}} di \right)^{\frac{1}{\theta - 1}}$$  \(5\)

where $Y_t$ is aggregate output, $y_t(i)$ the output of intermediate good firm $i$, and $\theta > 1$ is the elasticity of demand for each firm’s product. Cost minimization on the part of final goods producers results in the following demand curve for intermediate good $i$,

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t$$  \(6\)

and an associated price index for final goods,

$$P_t = \left( \int_0^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$  \(7\)

2.3 Intermediate Goods Production  
The imperfectly competitive intermediate goods firms enjoy some monopoly power in producing a differentiated product such that the face a
downward sloping demand curve, (6), but are also subject to Rotemberg (1982) quadratic-adjustment costs in changing prices of the form,  
\[ \frac{\phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} \pi^* - 1 \right)^2 P_t Y_t \] 
such that large price changes in excess of steady-state inflation rates are particularly costly, possibly as a result of customer dissatisfaction. The quadratic price adjustment costs renders the firm’s problem dynamic,

\[
\max \sum_{t=0}^{\infty} R_{0,t} \left( p_t(i)y_t(i) - mc_t P_t y_t(i) - \frac{\phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} \pi^* - 1 \right)^2 P_t Y_t \right) 
\]

satisfying

\[
y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t
\]

where \( mc_t = w_t/A_t \) is the real marginal cost implied by a linear production function, \( y_t(i) = A_t n_t(i) \). Productivity, \( A_t \) is common to all firms and follows an AR(1) process:

\[
\log A_t = \rho_A A_{t-1} + \varepsilon_t^A \varepsilon_t^A \sim i.i.d. N(0, \sigma_A^2)
\]

The first-order condition, after imposing symmetry across firms, is,

\[
(1 - \theta) + \theta mc_t - \phi \left( \frac{\pi_t}{\pi^*} - 1 \right) \frac{\pi_t}{\pi^*} + \beta \phi \frac{u_t(t+1)}{u_t(t)} \left( \frac{\pi_{t+1}}{\pi^*} - 1 \right) \frac{\pi_{t+1}}{\pi^*} Y_{t+1} = 0
\]

which represents the non-linear New Keynesian Phillips curve (NKPC) under Rotemberg pricing and which would, upon linearization, correspond to the standard NKPC under Calvo (1983) pricing.

The associated monopoly profit, which is taxed by the government when received by households, is,

\[
\Upsilon_t = Y_t - mc_t Y_t - \frac{\phi}{2} \left( \frac{\pi_t}{\pi^*} - 1 \right)^2 Y_t
\]

### 2.4 Government

Combining the households’ budget constraints and noting the equivalence between factor incomes and national output allows us to derive the government budget constraint:

\[
\frac{B_t}{R_t} + \tau_t (W_t n_t + \Upsilon_t) = (1 - \delta_t) B_{t-1} + P_t g_t + P_t z_t
\]

where we see that while fiscal policy in the form of tax, transfers and government spending changes will obviously affect debt dynamics, monetary policy, default and risk premia will also have a role to play, especially when debt stocks are large and the economy approaches
its fiscal limit. The government’s budget constraint can be rewritten as:

$$b_{t-1} \frac{1 - \delta_t}{\pi_t} = \frac{b_t}{R_t} + T_t - g_t - z_t$$

where $z_t$ is assumed to be fixed and $g_t$ follows an $AR(1)$ process:

$$\log \frac{g_t}{g^*} = \rho \frac{g_{t-1}}{g^*} + \varepsilon^g_t, \quad \varepsilon^g_t \sim i.i.d. N(0, \sigma^2_g) \quad (12)$$

We assume that fiscal and monetary policy follow simple rules of the form,

$$\tau_t - \tau^* = \gamma (b^d_t - b^*) \quad (13)$$

$$R^d_t - R^* = \alpha (\pi_t - \pi^*) + \varepsilon^R_t, \quad \varepsilon^R_t \sim i.i.d. N(0, \sigma^2_R) \quad (14)$$

where $b^d_t = (1 - \delta_t)b_{t-1}$. The interest rate rule is defined in terms of $R^d_t$, which represents either the interest rate containing default risk premia, $R_t$, or the risk-free rate of interest, $R^f_t$. Schabert (2010) and Uribe (2006) both consider the case of a rule defined in terms of the interest rate on risky government bonds, but we consider both possibilities below.

Finally, it is helpful to iterate the government’s budget constraint forward to obtain.

$$\frac{b_t}{\pi_t} = \sum_{i=0}^{\infty} \beta^i E_t \frac{c_t}{\pi_{t+i}} s_{t+i} \quad (15)$$

where the primary surplus is given by $s_t = T_t - g_t - z_t$ and we can see that in defining the intertemporal budget constraint future surpluses are discounted at the risk-free rate of interest.

### 2.5 Aggregate Resource

The aggregate resource constraint is

$$c_t + g_t = A_t n_t \left( 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi^*} - 1 \right)^2 \right) \quad (16)$$

In the aggregate, deviations of inflation from target generate real resource costs.

### 3 Simple Analytics: Default and Inflation

This section uses a simple analytical model to describe the link between default and inflation, where default is costless in the sense that the defaulting government is neither forced to reform its policies by dramatically reducing deficits nor is it locked out of credit markets for some period. Consistent with the economy described in section 2, we assume that
monetary policy is active and fiscal policy is passive. This policy combination makes the linkages between inflation and default quite different from those described in Uribe (2006), who considers a similar economy, but assumes that fiscal policy does not seek to stabilize government debt. In our setup, depending on the specification of the monetary rule, default may make it difficult for the monetary authority to hit its inflation target even if monetary policy actively targets inflation and fiscal policy passively adjusts surpluses to stabilize debt.

Consider a constant endowment, cashless economy in which the equilibrium real interest rate, $1/\beta$, is also constant. Government default is the sole source of uncertainty and for the current purposes, the decision to default by the fraction $\delta_t \in [0, 1]$ on outstanding debt carried into period $t$ is exogenous and follows a known stochastic process. Let $R_t$ be the gross risky rate of return on nominal government debt and $\pi_t = P_t/P_{t-1}$ be the inflation rate. Household optimization yields the Fisher relation

$$\frac{1}{R_t} = \beta E_t \left[ \frac{1 - \delta_{t+1}}{\pi_{t+1}} \right]$$

while trade in risk-free bonds (assumed to be in zero net supply) gives an analogous relation for the risk-free interest rate, $R_f^t$,

$$\frac{1}{R_f^t} = \beta E_t \left[ \frac{1}{\pi_{t+1}} \right]$$

The government’s budget constraint is

$$\frac{B_t}{P_t} + s_t = \left( 1 - \delta_t \right) R_{t-1} \frac{B_{t-1}}{P_{t-1}} \pi_t$$

where $s_t$ is the primary surplus. Write this constraint at $t + 1$, take expectations conditional on information at $t$, impose the Euler equation $\beta^{-1} = E_t(1 - \delta_{t+1}) R_t/\pi_{t+1}$, and solve for $B_t/P_t$ to yield

$$\frac{B_t}{P_t} = \beta E_t \frac{B_{t+1}}{P_{t+1}} + \beta E_t s_{t+1}$$

When the real interest rate is fixed, both the nominal rate and the inflation rate reflect default, so that the expected default rate drops out once expectations are taken. This implies that only surprises in default directly affect the evolution of real government debt in this flexible-price endowment economy. In light of this, we obtain, by iterating on (20) and imposing the household’s transversality condition

$$\frac{B_t}{P_t} = \sum_{j=1}^{\infty} \beta^j E_t s_{t+j}$$
Expression (21) is the usual intertemporal equilibrium condition that equates the value of government debt to the expected present value of “cash flows,” which are primary surpluses.

Fiscal policy sets the surplus in order to stabilize the post-default value of government debt
\[ s_t - s^* = \gamma \left[ (1 - \delta_t) \frac{B_{t-1}}{P_{t-1}} - b^* \right] \] (22)
where \( s^* \) and \( b^* \) are target and steady state values for the surplus and real debt and \( b_{t-1} = B_{t-1}/P_{t-1} \).

Substituting (22) into (19) and taking expectations at time \( t \) yields the evolution of expected debt
\[ E_t b_{t+1} + (s^* - \gamma b^*) = [\beta^{-1} - \gamma(1 - E_t \delta_{t+1})] b_t \] (23)
One result that emerges immediately from (23) is that stability of the debt process in the face of debt default requires that
\[ \gamma > \frac{\beta^{-1} - 1}{1 - E_t \delta_{t+1}} \] (24)
a condition that potentially is far more demanding than the usual one that \( \gamma > \beta^{-1} - 1 \), particularly when substantial default rates are possible. Provided this condition is fulfilled, however, fiscal policy remains passive and capable of stabilizing the real value of government debt.

When specifying monetary policy behavior, we must choose which interest rate to adopt as the policy instrument. Following Uribe (2006) and Schabert (2010), our benchmark case assumes that monetary policy sets the risky nominal interest rate, \( R_t \), according to a simple Taylor rule
\[ \frac{1}{R_t} = \frac{1}{R^*} + \alpha \left( \frac{1}{\pi_t} - \frac{1}{\pi^*} \right) \] (25)
Monetary policy targets inflation by setting \( \alpha/\beta > 1 \). Aside from being the dominant rule in the literature, in the context of our cashless model it is natural for monetary policy to be implemented by varying the contractual interest rate on government debt, rather than the risk-free interest rate on private debt, over which the government has no direct control and which is in zero net supply in equilibrium. More generally, in the transmission from the very short-term rates targeted through open market operations to the wider economy and, ultimately inflation, the central bank would expect to see a significant degree of pass through to the contractual interest rates employed throughout the economy.\(^2\) Indeed, since government bonds typically form the collateral for the repo contracts undertaken by central

\(^2\)Empirical evidence suggests that the rate at which policy interest rates pass through to bank interest rates is quite high—about 90 percent within a quarter [Gambacorta (2008)]. We are implicitly assuming similarly high rates of pass through to government bond yields.
banks, it is inevitable that without an offsetting policy adjustment, the policy rates pick up some of the default risk.$^3$

It is instructive nonetheless to contrast this rule with one specified in terms of the risk-free rate

$$\frac{1}{R_t} = \frac{1}{R^*} + \alpha \left( \frac{1}{\pi_t} - \frac{1}{\pi^*} \right)$$

(26)

Combining the policy rule defined in terms of the risk-free interest rate with the Fisher relation, (18), yields the dynamic equation for inflation

$$\frac{1}{\pi_t} - \frac{1}{\pi^*} = \frac{\beta}{\alpha} \mathbb{E}_t \left( \frac{1}{\pi_{t+1}} - \frac{1}{\pi^*} \right)$$

(27)

which implies monetary policy hits its target inflation rate, provided the policy behavior is sufficiently active, $\beta/\alpha < 1$. $^4$ Although default can weaken the passivity of a fiscal rule defined in terms of the post-default level of debt, provided it satisfies (24), fiscal policy remains passive, and an active monetary policy can successfully target inflation when the central bank’s instrument is the risk-free nominal rate.

But if monetary policy controls the risky interest rate, $R_t$, default influences the ability of the monetary authority to target inflation, even if fiscal policy remains passive and monetary policy is active. To see this, combine the monetary policy rule in (25) with the Fisher relation to yield the dynamic equation for inflation

$$\frac{1}{\pi_t} - \frac{1}{\pi^*} = \frac{\beta}{\alpha} \mathbb{E}_t \left( \frac{1 - \delta_{t+1}}{\pi_{t+1}} - \frac{1}{\pi^*} \right)$$

(28)

which now depends on the expected default rate.

Active monetary policy implies that the unique locally bounded solution for inflation is

$$\frac{1}{\pi_t} = \frac{1}{\pi^*} \left( 1 - \frac{\beta}{\alpha} \right) \left\{ 1 + \mathbb{E}_t \sum_{i=1}^{\infty} \left( \frac{\beta}{\alpha} \right)^i \prod_{j=1}^{i} (1 - \delta_{t+j}) \right\}$$

(29)

In the absence of default, $\delta_t \equiv 0$, monetary policy achieves its inflation target exactly, $\pi_t = \pi^*$. Higher expected default rates in the future raise current inflation. The farther into

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$^3$Sims (2008) emphasizes that the unconventional operations of many central banks—particularly the Fed and the ECB—in recent years have made the central banks’ balance sheets riskier. If foreign reserves are an important component of the bank’s assets, as for the ECB, then surprise appreciation of the euro devalues its assets relative to its liabilities. The Fed’s increased holdings of long-term Treasuries expose its balance sheet to more interest-rate risk than normal. Riskiness is exacerbated if the central bank is not assured that the fiscal authority will back it in times of large declines in asset values.

$^4$Throughout this paper, we restrict attention to locally bounded solutions, recognizing the validity of Cochrane’s (2010) argument that there are a continuum of explosive solutions to expressions like (27).
the future default is expected, the more it is discounted by \( \beta/\alpha < 1 \), and the smaller is its impact on inflation at time \( t \). Notice also that if the default rate is constant, \( \delta_t \equiv \delta \in [0, 1] \), then more aggressive monetary policy enhances the central bank’s control of inflation. A constant default rate yields the solution for inflation

\[
\pi_t = \pi^* \left[ 1 - \frac{(1 - \delta) \beta}{\alpha} \right]
\]

so that \( \pi_t \to \pi^* \) as \( \alpha \to \infty \). A more aggressive monetary policy response to inflation reduces the inflationary consequences of default.

Finally, consider a stylized experiment. At time \( t \) news arrives that raises the expected default rate at \( t+1 \), \( E_t \delta_{t+1} > 0 \), but all subsequent expected default rates are zero, \( E_t \delta_{t+j} = 0 \) for \( j > 1 \). Then (29) reduces to

\[
\bar{\pi}_t = \pi^* \left[ 1 - \frac{1}{1 - \frac{\beta}{\alpha} E_t (\delta_{t+1})} \right] > \pi^*
\]

and again we see that higher expected default raises inflation, but the extent to which it does so is mitigated by a more aggressive monetary response to inflation in the form of a higher \( \alpha \).

The source of this inflationary response to default can be seen in contrasting the interest rate rules when defined in terms of risky and risk-free interest rates. A risk-free rule, coupled with a passive fiscal policy, can successfully target inflation. To see why the rule defined in terms of the risky-rate cannot, it is helpful to return to the simple case where the default rate is constant, \( \delta_t \equiv \delta \in [0, 1] \), so that \( \frac{1}{R_t} = \frac{1-\delta}{R^*_f} \). Rewrite (25) in terms of the risk-free rate as

\[
\frac{1}{R^*_f} = \frac{1}{R^*} + \frac{\alpha}{1-\delta} \left[ \frac{1}{\pi_t} - \left( \frac{1}{\pi^*} - \frac{\delta}{\alpha R^*} \right) \right]
\]

The monetary policy rule defined in terms of the risky rate of interest can be transformed into a rule of the same form as that defined in terms of the risk-free rate, but with two important differences. First, default does not make monetary policy less active; in fact, it raises the coefficient on excess inflation, \( \frac{\alpha}{1-\delta} > \alpha \). Second, default raises the effective inflation target from \( \pi^* \) to \( \pi^* \frac{1-\delta/\alpha}{1-\delta} \). Intuitively, a higher rate of default creates partial monetary policy accommodation: in the presence of default, the monetary authority must allow the risky rate of interest to rise to induce bondholders to continue holding the stock of government bonds. Given the monetary policy rule, the monetary authority will not raise interest rates without a rise in inflation. Bondholders attempt to sell bonds, increasing aggregate demand as they try to increase their consumption paths. This behavior pushes up the price level until
bondholders are being compensated for their default risk and inflation and interest rates are consistent with the monetary rule. Stronger responsiveness of policy to inflation, higher $\alpha$, reduces the effective rise in the inflation target needed to achieve the rise in interest rates desired by bondholders.

As a general proposition, the possibility of default can undermine the central bank’s control of inflation: there is a tight connection between expected default rates and inflation, as in Uribe (2006), but the mechanism differs from Uribe’s. Uribe obtains his result through a standard fiscal theory of the price level mechanism by coupling an active monetary policy rule like (25) with an active fiscal rule akin to setting $\gamma = 0$ in (22), just as in Loyo (1999) and, more recently, Sims (2010). Such analyses echo the logic of Sargent and Wallace’s (1981) unpleasant arithmetic, where the fiscal consequences of a tight monetary policy can ultimately generate a worsening inflation situation because fiscal policy does not adjust to stabilize government debt. In contrast, our results stem from the monetary policy response to default, but where the policy rule remains active and fiscal policy passive. Although we also find a positive link between default and inflation, that link differs in crucial aspects. For example, in Uribe (2006) delaying default supports unstable inflation dynamics for longer, making it more difficult for the monetary authorities to hit their inflation target. In our active monetary/passive fiscal regime, though, the impact of future default on prices is discounted so that delaying default reduces the immediate inflationary consequences of default. Furthermore, in Uribe (2006) raising $\alpha$ and making monetary policy more active further destabilizes inflation dynamics and moves the economy farther from its inflation target. More active monetary policy in our environment reduces deviations from the inflation target due to default.\textsuperscript{5}

This section illustrates that the specification of the monetary policy instrument—risky versus risk-free nominal rate—matters for the inflation consequences of sovereign default risk and the ability of active monetary policy to target inflation in the face of such risk. Our simple model sharply dichotomizes between risky and risk-free interest rates, a dichotomy that is difficult to achieve in practical settings. Actual central bank instruments in countries facing sovereign default risk probably fall somewhere along the continuum between the two types of interest rates in the model.

4 Simple Analytics: Default and Determinacy

This section examines the implications of default for the determinacy of equilibrium in the model that section 2 describes. There are several reasons for doing so. First, the simple

\textsuperscript{5}In the full new Keynesian model, where real interest rates and default rates are endogenous, higher $\alpha$ raises real debt service more, which raises default probabilities and inflation, as shown in figure 10.
endowment economy in section 3 implies that default prevents inflation from achieving its target. In our sticky-price economy, movements in inflation have real effects, and these can be illustrated by considering the determinacy properties of a linearized version of our benchmark model. Second, it is useful to obtain local determinacy conditions for a log-linearized version of our model before turning to solving the full non-linear version. Local results help to confirm when we can expect the non-linear code to converge. Third, the determinacy analysis combines the results on determinacy under trend inflation due to Ascari and Ropele (2009), with the analysis of determinacy in an environment with distortionary taxation, due to Linnemann (2006). As such, the analysis is interesting in its own right.

Assume that government spending and transfers are zero and begin by considering the case of lump-sum taxation. These assumptions simplify the analytics, before using numerics to study the interactions between distortionary taxation and default. We assume there is an exogenous and constant rate of default every period, \( \delta_t \equiv \delta \in [0, 1] \). Varying this fixed default rate only affects the dynamics of our system if it affects the steady state of the system, and it only does that through the monetary policy rule. As noted above, in the presence of default, it is as if the inflation target has risen from \( \pi^* \) to \( \bar{\pi}^* \equiv \frac{\pi^*}{1 - \delta \beta / \alpha} \) (33)

This implies that as we approach the fiscal limit, default risk premia emerge and, given the standard specification of the interest rate rule, this leads to monetary accommodation of default risk that is effectively the same as raising the inflation target. We now explore the implications for determinacy of increasing the fixed default rate, which raises risk premia.

Appendix A details the log-linearization of our dynamic system. The appendix shows that the dynamic system can be written as

\[
\begin{bmatrix}
E_t \hat{\pi}_{t+1} \\
E_t \hat{y}_{t+1} \\
\hat{b}_t
\end{bmatrix} = \begin{bmatrix}
\frac{\phi_1}{\beta \phi_2} & -\frac{\phi_3}{\beta \phi_2} & 0 \\
\gamma_2 - \gamma_1 \frac{\phi_1}{\beta \phi_2} & 1 + \frac{\gamma \phi_1}{\beta \phi_2} & 0 \\
\beta^{-1}(\alpha \beta - 1) & 0 & \beta^{-1}(1 - \gamma \bar{\pi}^*)
\end{bmatrix} \begin{bmatrix}
\hat{\pi}_t \\
\hat{y}_t \\
\hat{b}_{t-1}
\end{bmatrix}
\]

where a hat denotes the percentage deviation of that variable from its steady-state value, \( \phi_i > 0 \) for \( i = 1, 2, 3 \) are bundles of parameters contained in the new Keynesian Phillips curve defined in the appendix. The other parameter bundles are found in the consumption-Euler
equation and are defined as

\[
\begin{align*}
\gamma_1 &= 1 - \phi(\bar{\pi}^* - 1) \frac{y^*}{c^*} \\
\gamma_2 &= \alpha - \phi(\bar{\pi}^* - 1) \frac{y^*}{c^*}
\end{align*}
\]

where \( \phi \geq 0 \) determines the costs of adjustment in Rotemberg (1982) pricing. Parameters \( \gamma_1 \) and \( \gamma_2 \) are positive until default raises inflation to such an extent that the second term dominates the first in either definition.

Since taxes are lump-sum, the debt dynamics are decoupled from the dynamics of the rest of the system. Debt will be dynamically stable if

\[
\beta^{-1}(1 - \gamma \bar{\pi}^*) < 1
\]

which requires

\[
\gamma > \frac{(1 - \beta)}{\bar{\pi}^*}
\]

Fiscal policy must passively raise lump-sum taxes rise by more than debt service costs in order to stabilize the debt.

We can now set aside debt dynamics to analyze the determinacy of the rest of the dynamic system. This part of the system can be represented as

\[
\begin{bmatrix}
E_t \hat{\pi}_{t+1} \\
E_t \hat{y}_{t+1}
\end{bmatrix} = \begin{bmatrix}
\tilde{\beta}^{-1} & -\tilde{\beta}^{-1} \kappa \\
\gamma_1 (\bar{\gamma}_2 - \tilde{\beta}^{-1}) & 1 + \gamma_1 \tilde{\beta}^{-1} \kappa
\end{bmatrix} \begin{bmatrix}
\hat{\pi}_t \\
\hat{y}_t
\end{bmatrix}
\]

where

\[
\tilde{\beta}^{-1} = \frac{\phi_1}{\beta \phi_2}, \quad \kappa = \frac{\phi_3}{\phi_1}, \quad \bar{\gamma}_2 = \frac{\gamma_2}{\gamma_1}
\]

which is in exactly the same form as the model considered in Woodford (2003, p. 677).

Consider the trace of the transition matrix

\[
trA = 1 + \tilde{\beta}^{-1}(1 + \kappa \gamma_1)
\]

and determinant

\[
det A = \tilde{\beta}^{-1}(1 + \gamma_2 \kappa)
\]
which imply
\[ \det A - trA = \tilde{\beta}^{-1}(1 + \gamma_2 \kappa) - 1 - \tilde{\beta}^{-1}(1 + \kappa \gamma_1) = \tilde{\beta}^{-1}\kappa(\alpha - 1) - 1 \]

and
\[ \det A + trA = \tilde{\beta}^{-1}(1 + \gamma_2 \kappa) + 1 + \tilde{\beta}^{-1}(1 + \kappa \gamma_1) = \tilde{\beta}^{-1}(2 + (\gamma_1 + \gamma_2)\kappa) + 1 \]

The set of determinacy conditions in Woodford (2003, p. 677) that are relevant for our model are given by

\[ \det A > 1 \]
\[ \det A - trA > -1 \]
\[ \det A + trA > -1 \]

and provided all parameter bundles are positive, it is only the second condition that bites, requiring
\[ \tilde{\gamma}_2 = \frac{\alpha - \phi(\bar{\pi}^* - 1)\frac{\kappa}{1 - \phi(\bar{\pi}^* - 1)^2}}{1 - \phi(\bar{\pi}^* - 1)^2} > 1 \]

This reduces to the usual Taylor principle: $\alpha > 1$. Therefore, provided default rates are not too high, the usual combination of active monetary policy and passive fiscal policy will ensure determinacy of our sticky-price economy.

As steady-state inflation rises—as a result of steady-state increases in the default rate, $\delta$—the parameter combinations $\gamma_1$ and $\gamma_2$ can turn negative, and may overturn the necessary conditions for stability and imply indeterminacy. Notice that $\det A - trA > -1$ irrespective of the steady-state rates of default and inflation, so the conditions outlined above remain the relevant case for determinacy. The other conditions for determinacy may be breached when
\[ \gamma_2 = \alpha - \frac{\phi(\bar{\pi}^* - 1)}{1 - \frac{\phi}{2}(\bar{\pi}^* - 1)^2} < \frac{\tilde{\beta} - 1}{\kappa} \]
or when
\[ \gamma_1 + \gamma_2 = 1 + \alpha - \frac{2\phi(\bar{\pi}^* - 1)}{1 - \frac{\phi}{2}(\bar{\pi}^* - 1)^2} < -\frac{2}{\kappa} \]
either of which may occur for an active interest rate rule and a high enough default rate. In other words, when we move to our sticky price economy, at high default rates the ac-
commodation of risk premia through rising inflation can result in the backward bending
of the Phillips curve detailed in Ascarì and Ropele (2009), which can render a standard
active/passive policy mix indeterminate.

Finally, we note that this indeterminacy arising from default is different from that re-
ported in Schabert (2010). By assuming that the government exogenously imposes a default
rate (which we assumed applied in every period in this section, but which applies only upon
hitting the fiscal limit in the numerical analysis below) we circumvent the indeterminacy
that arises when the rate of default, \( \delta_t \), is endogenously determined by the need to satisfy
the government’s intertemporal budget constraint. Uribe (2006) similarly avoids indeter-
minacy by imposing a default rate which ensures the inflation target holds. Instead, the
potential indeterminacy we have identified, comes from the resource costs of rising inflation
in a sticky-price economy, where default results in a monetary accommodation that raises
inflation.

To assess whether or not this is an important feature of our own model, we now numeri-
cally assess the determinacy properties of the full model used in the numerical analysis below,
which features government spending, transfers and fiscal adjustment through distortionary,
rather than lump-sum, taxation. We fix the level of government spending and transfers at
their steady-state values and assume a constant default rate, \( \delta_t \equiv \delta \in [0, 1] \). Figures 2 and 3
plot the combinations of \( \alpha \) and \( \gamma \) (the monetary and fiscal rule parameters) necessary to en-
sure determinacy conditional on different default rates. When there is no default risk—figure
2—we find that determinacy requires the usual mixture of active/passive policies found in
Leeper (1991), despite the presence of distortionary taxation.\(^6\)

When we raise the default rate from zero to 0.01 (as used in the numerical results below)
there is no significant change in the nature of the stability conditions [figure 3]. However,
there are regions of the policy-rule parameter space where the steady state is not well defined,
typically because the interest rate rule implies that the steady-state rate of inflation is
sufficiently negative that the policy rule implies the nominal interest rate breaches its zero
lower bound.

This section has shown that a standard monetary policy rule, like that used by Uribe
(2006) and Schabert (2010), which implicitly raises the inflation target in the presence of
default risk, can induce indeterminacy if the default rate is sufficiently high. However, for
the rates of default considered upon hitting the fiscal limit in the main body of the paper,
the model remains locally determinate for the policy rule parameters adopted.

\(^6\)In other words, in the region of the steady-state, our model does not feature the distortionary tax effects
which can render an active monetary policy indeterminate as in Linnemann (2006).
Figure 2: Determinacy and Stability with no Default ($\delta = 0$). + denotes a determinate equilibrium; ◊ denotes an indeterminate equilibrium; ⋆ denotes no stable equilibrium.

Figure 3: Determinacy and Stability with Default ($\delta = 0.01$). + denotes a determinate equilibrium; ◊ denotes an indeterminate equilibrium; ⋆ denotes no stable equilibrium.
5 Computing the Fiscal Limit

Laffer curves provide a natural starting point for quantifying the fiscal limit from the tax revenue side of the government’s budget constraint. At the peak of the Laffer curve tax revenues reach their maximum and, given some minimum level of total government expenditures, the expected present value of primary surpluses and, therefore, the value of government debt, are maximized. Revenues, expenditures, and discount rates, of course, vary with the shocks hitting the economy, generating a distribution for the maximum debt-GDP level that can be supported. We refer to this as the distribution of the fiscal limit. This section describes more precisely how we derive that distribution.

5.1 Laffer Curve  

Assume the utility function is

\[ u(c_t, n_t) = \log c_t + \chi_n \log(1 - n_t) \]

Labor supply can be solved analytically as a function of \((\tau_t, \pi_t, A_t, g_t)\) using the first-order conditions. Work effort is given by

\[ n_t = \frac{w_t X_{1,t} + \chi_n g_t}{w_t X_{1,t} + \chi_n X_{2,t}} \]  

with

\[ X_{1,t} = 1 - \tau_t \]

\[ X_{2,t} = A_t \left( 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi^s} - 1 \right)^2 \right) \]

Total tax revenue is

\[ T_t = (w_t n_t + \Upsilon_t) \tau_t \]

\[ = A_t n_t \tau_t \left( 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi^s} - 1 \right)^2 \right). \]

When the monetary authority keeps the inflation rate at its target \((\pi_t = \pi^*)\) and transfers are at their steady-state level \((z_t = z^*)\), the peak of the Laffer curve is a function only of the exogenous state of the economy \((A_t, g_t)\).

\[ \tau_t^{\max} = \tau^{\max}(A_t, g_t) \]

\[ T_t^{\max} = T^{\max}(A_t, g_t) \]

Evidently, the stochastic processes governing the exogenous states induce stochastic processes for both the tax rate that maximizes revenues and the level of revenues.

5.2 Distribution of the Fiscal Limit  

The fiscal limit is defined, following Bi (2009), as the maximum expected present value of future primary surpluses. Importantly, the notion
of a fiscal limit that we develop is the private sector’s perception of the limit.

\[
\mathcal{B}^* = E \sum_{t=0}^{\infty} \beta^t \sum_{\text{political factor}} \frac{u_c^{\max}(A_t, g_t)}{u_c^{\max}(A_0, g_0)} (T_t^{\max}(A_t, g_t) - g_t - z)
\]

Calculation of the fiscal limit uses the stochastic discount factor that obtains when tax rates are at the peak of the Laffer curve, $\beta^t u_c^{\max}(A_t, g_t)/u_c^{\max}(A_0, g_0)$, but modified to allow for a regime-switching political risk parameter $\beta_p^t \in \{\beta_p^L, \beta_p^H\}$ with transition matrix of

\[
\begin{bmatrix}
    p_{LL} & 1 - p_{LL} \\
    1 - p_{HH} & p_{HH}
\end{bmatrix}
\]

Higher political risk—lower $\beta_p^t$—lends itself to multiple interpretations that reflect the private sector’s beliefs about policy. Most straightforward is the idea that policymakers are believed to have effectively shorter planning horizons than the private sector [see, for example, Acemoglu, Golosov, and Tsyvinski (2008)]. To see this, rewrite the discount factor in (40) as

\[
(\beta_p^t \beta_p^t)^{-1} - 1
\]

so that a lower value of $\beta_p^t$ reduces the present value of maximum surpluses. An alternative interpretation is that a lower $\beta_p^t$ implies that private agents place probability mass on both the maximum surpluses, $s^{\max}$ reflected in (40), and on surpluses being zero. Rewrite the surpluses as

\[
\beta_p^t s^{\max} + (1 - \beta_p^t) \cdot 0
\]

Nothing we do hinges on the precise interpretation attached to $\beta_p^t$. As a practical matter, setting $\beta_p^t < 1$ serves to shift down the distribution of the fiscal limit, which generates risk premia at lower levels of debt like those observed in data. Uncertainty about $\beta_p^t$ generated by treating it as a Markov process increases the dispersion of the fiscal limit, which also seems important for generating plausible movements in risk premia.

Since there exists a unique mapping between the exogenous state space, $(A_t, g_t)$, to $\tau_t^{\max}$ and $T_t^{\max}$, the unconditional distribution of the fiscal limit, $f(\mathcal{B}^*)$, can be derived from a Markov Chain Monte Carlo simulation following the steps that appendix B describes.

Government adopts a fixed-rate default rule. At each date, an effective fiscal limit, $\mathcal{B}_t$, is drawn from the normal density depicted in figure 4, given by $\mathcal{N}(\bar{\mathcal{B}}^*, \sigma_{\mathcal{B}}^2)$. Government defaults on a fraction, $\delta$, of outstanding debt according to the rule

\[
\delta_t = \begin{cases} 
\delta & \text{if } b_{t-1} - 1 > \mathcal{B}_t \text{ (Above Effective Fiscal Limit)} \\
0 & \text{if } b_{t-1} \leq \mathcal{B}_t \text{ (Below Effective Fiscal Limit)}
\end{cases}
\]

If the real value of debt at the beginning of period $t$, $b_{t-1}$, exceeds the effective fiscal limit, then the government partially defaults and debt outstanding at the beginning of period $t$ becomes $b_t = (1 - \delta_t)b_{t-1}$.

The choice of $\mathcal{B}_t$, which we treat as random, is determined by political considerations that are driven by the policymakers’ assessments of the costs associated with fully meeting
1.25 1.3 1.35 1.4 1.45 1.5 1.55 1.6 1.65 1.7 1.75
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5

Debt to Steady-State GDP Ratio

5.3 Interpretation
We interpret the fiscal limit depicted in figure 4 as a necessary condition for equilibrium. It describes the distribution of the upper bound on how much government debt the economy can support, yielding the feasible set of debt-output ratios. Policy rules that make fiscal instruments react strongly enough to the state of government indebtedness serve to anchor fiscal expectations on policies that are consistent with the transversality condition, which is one of the necessary and sufficient conditions for existence of equilibrium.

We restrict attention to policies consistent with the existence of equilibrium because we take seriously Eaton and Gersovitz’s (1981) argument that sovereign debt default is about a government’s willingness, not its ability to meet its debt obligations. Rather than making the default decision a strategic choice of an optimizing government, we opt to treat the intrinsically political decision as a random draw from the feasible set of debt-output ratios. In the literature on strategic default in emerging market economies—see Eaton and Gersovitz (1981), Arellano (2008), and Pouzo (2009), among others—sovereign default is modeled as the outcome of an optimal and strategic decision by the government. Predicted levels of government debt at which sovereign default occurs, however, are much lower than levels at which sovereign risk premia are observed in developed countries. This discrepancy makes it difficult to apply those models to policymaking in advanced countries. Instead of modeling sovereign default as a strategic decision, we treat it as an exogenous political choice that is
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guided by the economy’s endogenous fiscal limit.

Although the theoretical analysis of fiscal policy in developed countries has largely abstracted from sovereign default risk, there are some exceptions. Uribe (2006) analyzes a flexible price model in which sovereign default is inevitable, as the central bank targets the price level and the fiscal authority maintains a constant tax rate. By setting an ad-hoc and fixed default threshold, he shows how the default scheme affects equilibrium dynamics. Similarly, Daniel and Shiamptanis (2010) assume government debt is constrained by an ad-hoc fiscal limit to study a small open economy in a monetary union under alternative fiscal policy responses to a fiscal crisis.

Schabert (2010) extends Uribe (2006) by assuming that sovereign default occurs when the transversality condition is violated. He shows that a monetary policy that sets the nominal risky interest rate fails to determine the equilibrium: the risky rate can affect allocations only through the risk-free rate and, therefore, the expected default rate. Since the latter, in turn, depends on the equilibrium allocation, there are many risky rate–expected default rate combinations consistent with equilibrium.

Instead of modeling sovereign default explicitly, Corsetti, Kuester, Meier, and Muller (2010) assume that the risk premium on government debt depends on the expected level of government debt, as in Garcia-Cicco, Pancrazi, and Uribe (2010). Corsetti, Kuester, Meier, and Muller (2010) show how the timing of fiscal retrenchment and the size of the risk premium affect economic outcomes at the zero lower bound for nominal interest rates.

6 Calibration

The model is calibrated at a quarterly frequency. The household discount rate is 0.99 and the net real interest rate is 4.04 percent at annual rate. The utility function is assumed to be $u(c, L) = \log c + \chi_n \log L$, where leisure, $L$, equals $1 - n$. The leisure preference parameter, $\chi_n$, is calibrated in such a way that the household spends 25 percent of its time working and the Frisch elasticity of labor supply is 3. Time endowment and the productivity level at the steady state are normalized to 1.

Parameterizations of the shock processes for $A_t$ and $g_t$ follow the literature. For instance, Schmitt-Grohé and Uribe (2007) assume $\rho_A$ to be 0.8556, $\sigma_A$ to be 0.0064, $\rho_g$ to be 0.87, and $\sigma_g$ to be 0.016. The price elasticity of demand, $\theta$, is assumed to be 11 and the Rotemberg adjustment parameter, $\phi$, is 100, which implies that 26.7 percent of the firms reoptimize each quarter [see Keen and Wang (2007)]. The gross inflation rate is calibrated to 1.03 at annual rate and the Taylor rule parameter is assumed to be 1.5.

The fiscal parameters are roughly calibrated to match Greek data from 1971 to 2007. In steady state, government purchases are 16.7 percent of GDP, lump-sum transfers are 13.34
percent of GDP, and government debt is 35.26 percent of GDP at an annual rate. The resulting tax rate is 0.315 at the steady state, which is slightly higher than the average tax rate of 0.28 in the data. The tax adjustment parameter, $\gamma$, is calibrated to 0.5 at annual rate, which is roughly consistent with estimates. The default rate, $\delta$, is assumed to 0.01, implying 4 percent annual default rate. We use a very small default rate to underscore that even small rates can generate quantitatively important effects.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Rotemberg adjustment parameter</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>$\pi^*$</td>
</tr>
<tr>
<td>Labor supply</td>
<td>$n^*$</td>
</tr>
<tr>
<td>Government spending-GDP</td>
<td>$g^<em>/y^</em>$</td>
</tr>
<tr>
<td>Government transfer-GDP</td>
<td>$z^<em>/y^</em>$</td>
</tr>
<tr>
<td>Government debt-GDP</td>
<td>$b^<em>/y^</em>$</td>
</tr>
<tr>
<td>Tax rate</td>
<td>$\tau^*$</td>
</tr>
<tr>
<td>Fiscal rule parameter</td>
<td>$\gamma_{\tau}$</td>
</tr>
<tr>
<td>Default rate</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Taylor rule parameter</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Technology</td>
<td>$A^*$</td>
</tr>
<tr>
<td>Technology shock persistence</td>
<td>$\rho_A$</td>
</tr>
<tr>
<td>Technology shock variance</td>
<td>$\sigma_A^2$</td>
</tr>
<tr>
<td>Monetary shock variance</td>
<td>$\sigma_R^2$</td>
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<tr>
<td>Fiscal shock persistence</td>
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<td>Fiscal shock variance</td>
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<td>Political factor (Low)</td>
<td>$\beta^L$</td>
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<tr>
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<tr>
<td>Political factor transition matrix</td>
<td>$\rho_{LL}$</td>
</tr>
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<td>Fiscal limit</td>
<td>$\bar{B}^*$</td>
</tr>
<tr>
<td>Fiscal limit variance</td>
<td>$\sigma_b^2$</td>
</tr>
</tbody>
</table>

Table 1: Model Calibration

The International Country Risk Guide’s (ICRG) index of political risk offers one way to...
calibrate the political factor, $\beta^p$ [see Arteta and Galina (2008)]. The ICRG index of political risk for Greece appears to follow a regime-switching process. It stayed low and stable during the period between 1984 and 1993, then rose from 60 to 80 between 1994 and 1996, and stayed at the high level until the financial crisis erupted in 2008. In this model, we calibrate $\beta^p_t$ to be a two-state symmetric Markov regime-switching process. The low state, $\beta^p_L$, is calibrated to 0.4 and the high state, $\beta^p_H$, is 0.6. We assume that the probability of switching between the two states is 0.1.

Under the calibration in table 1, the distribution of the fiscal limit has $\bar{B}^* = 1.5$ (150 percent of GDP annually) and $\sigma_b = 0.0739$, shown in figure 4. If the political risk factor, $\beta^p$ were constant and equal to unity, the distribution of the fiscal limit would be far less disperse and centered at about 300 percent of GDP.

We solve the full non-linear model laid out in section 2, coupled with the fiscal limit described in section 5, using the monotone map method, which discretizes the state space and finds fixed points in the space of decision rules. Details appear in appendix C.

7 Equilibrium Decision Rules

Underlying the analysis that follows are the equilibrium decision rules that map the state of the economy into endogenous variables. These are high-dimensional objects, but for our purposes, it is sufficient to trace out how endogenous variables vary with the debt-GDP ratio under alternative values of the exogenous states—technology, the monetary policy rule, and the level of government spending. We plot the rules as a function of the post-default debt to steady-state output ratio; that is, as a function of $b^d_t/y^* = (1 - \delta_t)b_{t-1}/y^*$.

Figure 5 reports decision rules for steady-state, high, and low levels of technology. Rules become highly non-linear at debt-GDP ratios in the range of the fiscal limit in figure 4—between 130 and 170 percent. As debt rises, the economy moves farther into the range of the fiscal limit and the probability increases that debt will exceed the effective fiscal limit that the government chooses, $b_{t-1} > B_t$, raising the probability of default. Bond holders must be compensated for the increased riskiness of government debt, raising the real interest rate and risk premium on government bonds.\textsuperscript{10} Higher real rates increase the value of goods today, inducing households to work more, raising output. Over this range of debt levels the default probability rises rapidly and, with it, the risk premium. Even though the annual default rate is only 4 percent, the risk premium rises by 14 percentage points as the default probability rises from 0 to 1. A little bit of default goes a long way in this setup.

\textsuperscript{10}Empirical work routinely finds non-linearity between sovereign risk premia and government indebtedness [see, for example, Alesina, De Broeck, Prati, and Tabellini (1992) and Bernoth, von Hagen, and Schuknecht (2006)].
Decision rules associated with exogenous variation in nominal interest rates due to monetary policy look very much like those in figure 5, with the obvious adjustments from technology to interest rates. Variations in the level of government spending, however, do little to shift decision rules for interest rates, inflation, and default probabilities.

8 Policy Effects: Without Fiscal Reforms

We now turn to examine how policy effects—exogenous changes in monetary and government spending policies—change when the economy moves from being far from its fiscal limit to within striking distance of the limit. Several European countries are now operating at or near their fiscal limits and concerns about the possibility of sovereign debt default extend beyond Greece and Ireland to include Portugal and Spain [Thomas and Kanter (2010)]. At the same time, inflation rates may soon pick up in Europe and the European Central Bank will wish to return to its usual inflation-fighting stance.

The first set of experiments in this section addresses precisely this situation by tracing out the impacts of an exogenous monetary contraction. We then explore whether the impacts of changes in fiscal spending are different when debt default is a live option.
8.1 A Monetary Contraction

Monetary and fiscal policy always must interact in specific ways to ensure that a unique equilibrium exists. When monetary policy targets inflation, even when there is no possibility of default, a monetary contraction engineered through an open-market sale of bonds increases the public’s bond holdings, raises nominal and real interest rates, and requires expected surpluses to rise. Higher outstanding debt and higher interest rates raise debt service. To prevent debt from exploding, fiscal policy must raise future taxes or reduce future non-interest spending. These interactions permit the monetary contraction to reduce inflation and economic activity.

The possibility of debt default adds further complications to the interactions—complications that can dramatically alter the effects of monetary policy. A monetary policy contraction triggers three distinct sources of dynamics: those produced by the initial increase in the policy interest rate itself; the intrinsic dynamics that arise when debt is well above its steady state level; the additional dynamics that stem from changes in the probability of default and the risk premium on sovereign debt. The dynamics following a monetary contraction may be different at high levels of debt than at low levels, even in the absence of default.

This sub-section assumes the central bank’s instrument is the risky nominal interest rate. Consider a large i.i.d. contractionary monetary policy disturbance that raises the nominal interest rate by 1.5 percentage points. When the economy is at its steady state and the probability of default is nil, the contraction has the usual effects: nominal and real interest rates rise only in the period of the shock; output, inflation and wages fall initially; higher debt brings forth higher tax rates, which keep output persistently below steady state while debt is stabilized [see figure 6].

If we consider the same economy without a monetary contraction but in a high-debt state, intrinsic dynamics kick in that are analogous to the decision rules in figure 5 and shown in figure 7. To return the high level of debt to steady state, tax rates rise, depressing labor supply, output and consumption, but raising wages. Higher wages raise marginal costs and induce firms to increase prices. Monetary policy reacts to higher inflation by sharply raising nominal rates, which raises real interest rates. In the absence of any further disturbances, the economy would converge back to steady state.

If we combine the paths in figures 6 and 7, we learn how the impacts of a monetary contraction vary with the size of the initial level of debt, even when there is no possibility of default. Figure 8 shows the differences in time paths with and without a monetary contraction when debt is at steady state and when it is near the fiscal limit. At a high level of debt, monetary contraction raises debt and taxes more, producing a persistently lower path for output.

The third layer of dynamics is triggered by the possibility of debt default. To isolate the
Figure 6: Dynamic paths with an i.i.d. contractionary monetary policy shock and without a shock: initial government debt is at steady state. Time periods are quarters.

Figure 7: Dynamic paths without a monetary policy shock: no default and initial government debt close to fiscal limit. Time periods are quarters.
Figure 8: Difference in dynamic paths with an i.i.d. contractionary monetary policy shock and without the shock in model that rules out default: initial government debt at steady state and close to fiscal limit. Time periods are quarters.

Effects of default, figure 9 reports the difference that allowing for default makes to the time paths of variables when debt is near the fiscal limit and there is no monetary policy shock. This marginal effect of default is computed by solving the model first with the default rule in expression (41), then with $\delta_t \equiv 0$, and calculating the difference in the time paths from these two solutions, conditional on debt being near the fiscal limit.

Government debt near the fiscal limit creates a probability of default, which produces a risk premium in real bond yields. Higher real rates raise debt service, which further increases debt and actual tax rates, as dictated by the tax rule in (13). Higher realized tax rates reduce hours worked and consumption. Inflation rises, as do nominal interest rates through the active monetary policy rule. This is the same phenomenon highlighted in the analytics of section 3. The higher default risk leads to bondholders demanding risk premia to induce them to hold government bonds. In the absence of such an increase in the contractual returns on government debt, bondholders would seek to sell government bonds and increase consumption. This will fuel inflation and, given the monetary authority’s commitment to the monetary policy rule, the contractual rate of return on government bonds will rise until bondholders are content to hold the outstanding stock of government debt. In other words, we observe the implicit rise in the effective inflation target discussed in section 3 as the risk of default increases and is accommodated by the monetary authorities given the form of their
Figure 9: Marginal effect of possibility of default. Difference in time paths from solving with default rule in (41) and with $\delta_t \equiv 0$, conditional on debt being near the fiscal limit.

Pulling all these dynamics together we obtain the overall effect of a monetary contraction when debt default is permitted. Figure 10 reports the differences in time paths with and with a serially uncorrelated monetary contraction, contrasting those when the economy is far from the fiscal limit—dashed lines—to those when the economy is staring at the limit—solid lines. Away from the limit, tighter monetary policy has the usual effects because the probability of debt default is essentially zero.

In the very short run, monetary contraction lowers inflation. But soon the impact of expected default dominates and inflation rises dramatically and persistently. The effects of higher debt service manifest in sharply higher debt, which brings with it higher tax rates and persistently lower output.

8.2 A Monetary Contraction: Risk-Free Rate Instrument

Analytics in section 3 made clear the import of the instrument the central bank is assumed to control. Here we repeat the monetary contraction experiment of section 8.1, but now assume that the central bank follows a rule for setting the risk-free interest rate, $R^f_t$, according to a Taylor rule.

Analytical results suggest that the increase in inflation observed in figure 10 when monetary policy control the risky interest rate, may no longer occur once it is the risk-free interest rate the monetary authority determines. Figure 11 confirms that the analytical result holds.
Figure 10: Difference in dynamic paths with an i.i.d. contractionary monetary policy shock and without the shock in model where default is possible: initial government debt at steady state and close to fiscal limit. Time periods are quarters.

in the new Keynesian model. The figure overlays results when the risk-free interest rate enters the Taylor rule with those from figure 10 for the risky interest rate.

When government debt is far from the fiscal limit and the probability of default is nil, there is no distinction between the risk-free and the risky rates, and the impacts of a monetary contraction are identical. Near the fiscal limit, however, when monetary policy adjusts the risk-free rate it combats inflation without accommodating increases in default probabilities. As a consequence, policy ends up raising the real interest rate more (dotted-dashed lines) and output contracts further. There is no tendency for inflation to rise, however.

Figure 11 confirms that the specification of monetary policy instrument is a critical step in determining the inflationary consequences of sovereign debt risk.

8.3 A Fiscal Expansion  Much of the current fiscal policy debate centers on whether fiscal consolidation can be expansionary, or at least not contractionary, for the macro economy. This debate draws on an extensive literature that finds that under some circumstances fiscal consolidations have had beneficial economic effects, or at least have not produced declines in economic activity [Giavazzi and Pagano (1990) and Alesina and Ardagna (1998) are prominent examples]. Without exception, instances where fiscal retrenchments have been expansionary arise in economies that are operating near their fiscal limits.
Figure 11: Monetary policy sets the risk-free interest rate (dotted-dashed lines) overlaid with figure 10. Difference in dynamic paths with an i.i.d. contractionary monetary policy shock and without the shock in model where default is possible: initial government debt at steady state and close to fiscal limit. Time periods are quarters.
We consider a serially correlated increase in unproductive government spending that is initially financed through debt, but in the long run brings forth higher taxes rates that stabilize debt. Policy rules for this experiment are

\[ \tau_t - \tau^* = \gamma \tau (b_t^{d} - b^*) \]  
(42)

\[ \log \frac{g_t}{g^*} = \rho g \log \frac{g_{t-1}}{g^*} + \varepsilon_t^g \]  
(43)

\[ z_t = z^* \]  
(44)

\[ R_t - R^* = \alpha (\pi_t - \pi^*) \]  
(45)

And the default rule is as described by (41): when debt breeches the effective fiscal limit chosen by the government, the government defaults by the fraction \( \delta \) on its outstanding debt. Table 1 reports the calibration of the government spending process.

As in the case of a monetary contraction, it is important to separate three sources of dynamics: those triggered by the fiscal expansion; those that arise because the economy is operating near the fiscal limit; those induced by changes in the probability of default and risk premia. Figure 12 reports the effects of a sequence of three positive government spending shocks when the economy is at steady state, far from its fiscal limit. These are the usual impacts of higher government spending in a new Keynesian model with active monetary policy: the negative wealth effect from higher anticipated taxes initially raises work effort but, as distorting tax rates rise to stabilize debt, eventually labor declines; higher demand for current and future goods raises wages, inflation, and real and nominal interest rates, which drive down private consumption.

While the direct fiscal implications of an increase in government spending are the same whether or not we are close to the fiscal limit, in our model with numerous monetary and fiscal policy interactions, there are significantly different indirect implications depending on the size of the outstanding stock of government debt. The output expansion, caused by the government spending shock, affects the size of the tax base; the jump in inflation deflates the real value of debt; and the monetary policy response to inflation affects debt service costs. Even without default risk, these effects vary significantly at high debt levels relative to low debt levels. In particular, the fiscal impacts of surprise inflation and interest rate changes are far greater when the outstanding stock of debt is large. Moreover the elasticity of tax revenues with respect to tax rates will vary as we move towards the peak of the Laffer curve.

Taken together, these effects imply that a smaller jump in inflation applied to a significantly larger debt stock, can slow the initial rise in debt when we are near the fiscal limit, which results in a relatively lower increase in taxes to stabilize debt. Since taxes are distortionary, this in turn ensures that marginal costs do not rise by as much and the inflation
increase is more modest. Given the active monetary policy stance, this is consistent with a smaller increase in real interest rates, which also helps avoid destabilizing debt-interest dynamics.

These effects are borne out by the simulations. First consider the case when default is ruled out. Figure 13 reports the difference in the dynamic paths with the fiscal disturbance and without the disturbance, contrasting when initial debt is at steady state (dashed lines) and close to the fiscal limit (solid lines). Inflation and risky real interest rate impacts from the fiscal expansion are larger at low levels of debt than at high levels.

It turns out that once default is permitted, the patterns in figure 13 are reversed. Figure 14 reports the effects of a fiscal expansion when default is possible, contrasting the impacts when debt is at steady state and when debt is near the the fiscal limit. Qualitative patterns are similar, but now debt, the risky real rate, and inflation rise substantially more when initial debt is high than when it is low.

The reason for this different response mirrors the medium-term response to a monetary contraction discussed in section 8.1. There is an implicit rise in the effective inflation target under the interest rate rule specified in terms of the risky interest rate, as the monetary authorities allow inflation to rise in partial accommodation of the default risk. This leads to an equilibrium where debt dynamics deteriorate due to the higher real interest rates,
Figure 13: Difference in dynamic paths with a serially correlated government spending shock and without the shock in model that rules out default: initial government debt at steady state and close to fiscal limit. Time periods are quarters.

but bondholders are receiving the compensation they require to hold the higher stock of government debt, despite the default risk, and the monetary authorities have maintained their interest rate rule.

There are two reasons for the muted effect of the possibility of default in the case of a government spending increase, compared to a monetary contraction. First, even when staring at the fiscal limit, higher government spending increases output in the short run, which raises revenues and tempers the impacts of the spending on debt accumulation. Second, when the economy is near the fiscal limit, there is substantial default probability even in the absence of a fiscal expansion. Higher spending nudges that probability up, but only a bit, so the risky real interest rate is not appreciably higher when default is possible than when it is ruled out.

9 Policy Effects: With Fiscal Reforms
[to be completed]

10 Concluding Remarks
[to be completed]
Figure 14: Difference in dynamic paths with a serially correlated government spending shock and without the shock in model where default is possible: initial government debt at steady state and close to fiscal limit. Time periods are quarters.

### A Log-Linearized System

The log-linearized system includes:

\[
\begin{align*}
\dot{c}_t &= \frac{c^* + g^*}{c^*} \dot{y}_t - \phi (\pi^* - 1) \frac{y^*}{c^*} \tilde{\pi}_t \\
\dot{w}_t &= \frac{n^*}{1 - n^*} (\dot{y}_t - \dot{\bar{A}}_t) + \frac{c^* + g^*}{c^*} \dot{y}_t - \phi (\pi^* - 1) \frac{y^*}{c^*} \tilde{\pi}_t + \frac{\tau^*}{1 - \tau^*} \dot{\bar{R}}_t \\
\dot{\bar{R}}_t &= E_t (\dot{c}_{t+1} + \dot{\pi}_{t+1}) - \dot{c}_t + E_t \frac{\delta}{1 - \delta} \delta_{t+1} \\
-\frac{\delta}{1 - \delta} \delta_t + \dot{b}_{t-1} - \tilde{\pi}_t &= \beta \dot{b}_t + (1 - \beta) \dot{T}_t - \beta \dot{\bar{R}}_t \\
\theta \frac{w^*}{A^*} (\dot{w}_t - \dot{\bar{A}}_t) &= \phi (\pi^* - 1) \tilde{\pi}_t - \phi \pi^* \tilde{\pi}_t + \beta \phi (\pi^* - 1) \pi E_t (\dot{c}_t - \dot{c}_{t+1} + \dot{y}_{t+1} - \dot{y}_t) + \beta \phi (2 \pi^* - 1) E_t \tilde{\pi}_{t+1} = 0
\end{align*}
\]

If taxes are distortionary and applied to labour income and monopoly profits, then

\[
T_t = \tau_t y_t (1 - \frac{\phi}{2} (\pi_t - 1)^2)
\]

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which log-linearizes as
\[ \hat{T}_t = \hat{\tau}_t + \hat{y}_t - \phi(\pi^* - 1)\frac{y^T}{T} \hat{n}_t \]

In this case our log-linearized fiscal rule is given by
\[ \hat{\tau}_t = \gamma \hat{b}_{t-1} \]

while if taxes are lump-sum, \( \hat{\tau}_t = 0 \), and the fiscal rule is
\[ T_t - T^* = \gamma((1 - \delta_t)b_{t-1} - b^*) \]

and total tax revenues log-linearize as
\[ \hat{T}_t = \frac{\gamma \pi^*}{(1 - \beta)} \hat{b}_{t-1} \]

In conjunction with the log-linearized monetary policy rule
\[ \hat{R}_t = \alpha \hat{n}_t \]

these conditions can be combined to yield the dynamic systems described in the text.

B Simulating the Fiscal Limit

The fiscal limit \( B^* \) can be obtained using Markov Chain Monte Carlo simulation:

- First, for each simulation, we randomly draw the shocks of political factor, productivity and government purchases for 1000 periods. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables using the household first-order conditions and the budget constraints. According to equation 40, we compute the discounted sum of maximum fiscal surplus by discarding the first 200 draws as a burn-in period.

- Second, we repeat the simulation for 100,000 times and obtain the distribution of the fiscal limit, which is then approximated to a normal distribution \( N(b^*, \sigma_b^2) \).

- At each period of time, the effective fiscal limit \( b_t^* \) is a random draw from the distribution.
C Solving the Non-Linear Model

Following the household first-order conditions, the labor supply and consumption can be solved in terms of \( (w_t, \tau_t, \pi_t, A_t, g_t) \):

\[
n_t = \frac{w_t X_{1,t} + \chi n g_t}{w_t X_{1,t} + \chi n X_{2,t}} \quad (C.1)
\]

\[
c_t = X_{2,t} n_t - g_t \quad (C.2)
\]

with \( X_{1,t} = 1 - \tau_t \quad (C.3) \)

\[
X_{2,t} = A_t \left( 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi^*} - 1 \right) \right) \quad (C.4)
\]

The complete model also consists of the following non-linear equations:

\[
(1 - \theta) + \theta \frac{w_t}{A_t} = \phi \left( \frac{\pi_t}{\pi^*} - 1 \right) \frac{\pi_t}{\pi^*} - \beta \phi E_t \frac{u_c(t+1)}{u_c(t)} \left( \frac{\pi_{t+1}}{\pi^*} - 1 \right) \frac{\pi_{t+1}}{\pi^*} \frac{Y_{t+1}}{Y_t} \quad (C.5)
\]

\[
Y_t = A_t n_t \quad (C.6)
\]

\[
\Upsilon_t = Y_t - w_t Y_t - \frac{\phi}{2} \left( \frac{\pi_t}{\pi^*} - 1 \right)^2 Y_t \quad (C.7)
\]

\[
b_t - 1 \frac{\delta_t}{\pi_t} = \frac{b_t}{R_t} + \tau_t (w_t n_t + \Upsilon_t) - g_t - z^* \quad (C.8)
\]

\[
\tau_t - \tau^* = \gamma (b^d_t - b^*) \quad (C.9)
\]

\[
R_t - R^* = \alpha (\pi_t - \pi^*) + \varepsilon^R_t \quad (C.10)
\]

\[
\log \frac{g_t}{g^*} = \rho_g \frac{g_{t-1}}{g^*} + \varepsilon_g \quad (C.11)
\]

\[
\log \frac{A_t}{A^*} = \rho_A \frac{A_{t-1}}{A^*} + \varepsilon_A \quad (C.12)
\]

The solution method, based on Coleman (1991) and Davig (2004), conjectures candidate decision rules that reduce the system to a set of expectation first-order difference equations. In this model, the decision rule maps the state at period \( t \) into the stock of government debt, the real wage, and the inflation rate in the same period. The state is denoted as \( \psi_t = \{b^d_t, A_t, g_t\} \) or \( \psi_t = \{b^d_t, A_t, \varepsilon^R_t\} \) depending on whether the monetary and fiscal policy shock is turned on, while the mapping is denoted as \( b_t = f^b(\psi_t), w_t = f^w(\psi_t), \pi_t = f^\pi(\psi_t) \). The state variable of the post-default government liability \( (b^d_t) \) incorporates the information of the effective fiscal limit at time \( t \) \( (b^d_t) \) and the pre-default government liability \( (b_{t-1}) \).

The conjectured rules can be substituted into the non-linear system, in which the expectation terms are evaluated using a numerical quadrature. The model is solved for each set of state variables defined over a discrete partition of the state space. The decision rules are updated at every node of the state space. The procedure is repeated until the iterations
update the current decision rules by less than some $\epsilon > 0$ (set to $1e-6$).

After finding the decision rules, we can solve the pricing rule ($q_t = f^q(\psi_t)$) using the government budget constraint. The interest rate on government bonds can also be solved using $R_t = 1/q_t$, denoted as $f^R(\psi_t)$. 
REFERENCES


