Analyzing Fiscal Sustainability*

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Abstract

We study the implications of fiscal policy behavior for sovereign risk in a framework that develops a country’s fiscal limit, the point at which for economic or political reasons taxes and spending can no longer adjust to stabilize debt. A real business cycle model maps the economic environment—expected fiscal policy, the distribution of exogenous disturbances, and private agents’ behavior—into a distribution for the maximum sustainable debt-GDP ratio. Default is possible at any point on this fiscal limit distribution. Calibrations of the model to Greek and Swedish data illustrate how the framework can be used to study actual fiscal reforms undertaken by developed economies facing sovereign risk pressures.

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1 Introduction

In the recent recession and financial crisis, several advanced economies have run into serious sovereign risk problems. What distinguishes countries with and without sovereign risk troubles today? At the top of the list are the past and prospective fiscal policies they pursue. This is why troubled countries, and even some less-troubled nations, are adopting drastic fiscal austerity measures intended to deliver long-lasting fiscal consolidation.

Understanding how fiscal policies determine a country’s sovereign risk requires explicit modeling of fiscal behavior. Every economy faces a fiscal limit, the point at which for economic or political reasons taxes and spending can no longer adjust to stabilize debt. In the absence of a shift in monetary policy to a regime that stabilizes debt, at the fiscal limit a government has no choice but to default on its outstanding debt obligations. That limit depends on the entire economic and political environment: expected fiscal policy behavior, the distribution of exogenous disturbances, and private agents’ behavior. In most cases a country’s fiscal limit will not be revealed by historical policy choices. Rather, the fiscal limit answers the counterfactual question, “after accounting for country-specific economic and political constraints, what is the maximum expected present value of primary surpluses?”

This paper answers that question with a simple real business cycle model that maps economic environments, especially fiscal policy regimes, into conditional and unconditional distributions of the fiscal limit. A conditional distribution reflects the notion that bondholders’ expectations of repayment depend on the current state of the economy, including shock realizations and policy regime. For some analyses, particularly of long-run fiscal reforms, the unconditional fiscal limit distribution is more appropriate.

By mapping policy behavior into fiscal limit distributions, the paper provides a tool to examine the efficacy of fiscal reforms pursued by countries that are under sovereign risk pressures. Both the nature and the credibility of proposed reforms matter for their likely success in reducing sovereign risk. Credible shifts to a stabilizing regime can insure against risk premia even when fundamentals are poor. An identical shift that is less-than credible does little to bring down debt-service costs. If investors do not have full information about fiscal policy, however, even reforms that are intended to be credible may not produce an immediate drop in sovereign risk premia, as it takes time for investors to grow convinced that the reforms will last.

Our framework builds on Bi (2012), a closed-economy RBC model with fixed capital and a proportional tax levied against labor income. For given government purchase and

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1Our treatment of the fiscal limit as country-specific contrasts with the more typical analysis, which International Monetary Fund (2012, chapter 3) exemplifies, that extrapolates from past policy behavior to draw inferences about sovereign debt sustainability.
transfers processes, an economic fiscal limit arises from the peak of the dynamic Laffer curve. Because that peak depends on the state of the economy, the joint distribution of the fundamental disturbances and private agents’ optimal decision rules induce a distribution for the maximum sustainable government debt that equals the expected discounted present value of maximum primary surpluses. In a stochastic environment, “maximum debt” is a distribution, not a point.

Government transfers in the model may be stationary or grow as a share of GDP, depending on the prevailing regime. Growing transfers reflect either the rapid growth in the government’s share of the economy that has occurred in some countries, or demographic shifts and the old-age benefits that governments in advanced economies have promised their citizens. Stationary transfers represent a reform regime. Transition probabilities between the two regimes reflect the likelihood and the persistence of reforms. Non-stationary transfers have several effects. First, they provide a rationale for high and rising debt and tax rates, which push the economy close to its fiscal limit. Second, the distribution of the fiscal limit, conditional on residing in the non-stationary transfers regime, has a fat tail, which carries interesting asset-pricing implications. Finally, even if the prevailing transfers regime is stationary, if agents do not regard the regime as permanent, then a higher probability of returning to the non-stationary regime shifts the fiscal limit and makes debt risky.

We do not model default as a strategic decision taken by an optimizing sovereign. Instead, we appeal to political frictions that make default decisions intrinsically uncertain. The distribution of the fiscal limit reports the probability that a particular debt level can be supported by taxing income at the peak of the Laffer curve. The maximum sustainable debt levels in the upper tail of the distribution are obtained only if the economy receives a run of good shocks. Given the lower probability of receiving such good shocks, default becomes more likely if current debt is in the upper tail. Analogously, even if the economy receives a run of bad shocks debt levels in the lower tail can be supported, so default is less likely. Default is possible at any point on the distribution. Randomness inherent in the politically-determined default decision is modeled as a random draw of the “effective fiscal limit” from its model-based distribution. Default occurs when the current level of debt exceeds the effective limit; otherwise, debt obligations are fully honored.

Households base their expectations of default on the model-determined fiscal limit distribution. If the transfers regime is a latent state, so households are uncertain about the conditional fiscal limit distribution, they must make a probabilistic inference about the prevailing regime. Households update their beliefs about the transfers regime only gradually. This lack of complete information can generate risk premium paths that are similar to those observed in Euro Area countries: even after a reform that moves policy to the stationary
transfers regime, risk premia can continue to rise until agents are convinced the reform is credible.

In section 4, we calibrate the model to Greek data to examine a variety of policy scenarios motivated by fiscal developments in Greece. Those developments include an increase in transfers as a share of GDP since 1970 of about 13 percentage points, along with a sevenfold increase in the government debt-GDP ratio. On the heels of steady growth in transfers, low productivity can generate large risk premia. Policy experiments exploit the conditional fiscal limit distributions to focus on short-run matters.

Sweden in the 1990s is a case study of largely credible long-run fiscal reforms that dramatically shifted its unconditional fiscal limit distribution and reduced the riskiness of its sovereign debt. When calibrated to Swedish data, the model predicts pre-crisis risk premia arising at debt levels like those that Sweden experienced in the 1990s. After the fiscal reforms, Sweden’s fiscal limit shifted out substantially, allowing debt to be risk free even at debt-GDP ratios higher than observed during Sweden’s crisis.

2 Contacts with the Literature

We selectively survey the existing work to place our proposed framework in context. An important branch of the literature builds on Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008) to model sovereign defaults as strategic decisions made by a welfare-maximizing government in response to bad productivity shocks. Sovereign default risk helps standard real business cycle models reproduce key business cycle facts in emerging economies, particularly volatile consumption and countercyclical interest rates and net exports. By modeling default as an optimal response to exogenous shocks, however, the strategic default literature is largely silent about the policy behavior that got the country into a sovereign debt crisis in the first place and also about the policy reforms that might resolve the crisis. These are the issues at the heart of our paper.

Another line of work follows Bohn’s (1998) reduced-form regressions of surpluses on debt to infer fiscal policy behavior. Ghosh, Kim, Mendoza, Ostry, and Qureshi (2011) estimate the responses of primary surpluses to debt levels for 23 advanced economies to argue that the responses are weaker at higher levels of debt, a phenomenon the authors dub “fiscal fatigue.” Under the assumption that the government always follows its historically estimated surplus rule, there is a level of debt beyond which the government can no longer service its

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3 Cuadra and Sapriza (2008) is an exception to the silence about fiscal policy. That paper allows two types of governments, each embracing different preferences over public goods, that alternate in power to show that more polarized economies may face higher default rates.
debt because fatigue sets in. The authors compute a debt limit for each country that is
fully determined by the risk-free interest rate, the recovery rate, and the support of the
shock to primary balances. “Fiscal space” is defined as the difference between the long-run
average debt ratio and the debt limit. These calculations, however, are backward-looking,
grounded in past policies that are assumed to be immutable. Changes in policy rules—
like those European countries are now implementing—would alter the country’s debt limit,
destabilizing this backward measure of fiscal space.

Juessen, Linnemann, and Schabert (2011) compute a government’s debt repayment ca-
pacity using a Laffer curve argument, but then impose that the actual tax rate is always
constant. Whenever current debt exceeds the debt limit, default occurs in an amount neces-
sary for equilibrium. In this setting, the risk premium is determined solely by the stochastic
level of productivity, rather than by policy choices, making their setup inappropriate for the
questions we seek to address.

We seek to model how a country’s fiscal limit distribution varies systematically with the
economic environment, including the specification of policy behavior. To do that, we turn
to describe the framework that we propose.

3 Our Approach

Following Bi (2012), we lay out a closed economy model in which the fiscal limit, a measure-
ment of the government’s ability to service its debt, arises endogenously from the economy’s
dynamic Laffer curve.

3.1 Model With linear production technology, output is determined by productivity, $A_t$,
and labor supply, $1 - L_t$. Household consumption, $c_t$, and government purchases, $g_t$, satisfy
the aggregate resource constraint

$$c_t + g_t = A_t(1 - L_t)$$

where the level of productivity follows an $AR(1)$ process with $A$ being the steady-state level
of technology

$$A_t - A = \rho^A(A_{t-1} - A) + \varepsilon^A_t \quad \varepsilon^A_t \sim \mathcal{N}(0, \sigma^2_A).$$

The government finances exogenous unproductive purchases and lump-sum transfers to
households, $z_t$, by collecting tax revenue and issuing one-period bonds, $b_t$. Government
purchases obey
\[ g_t - g = \rho^g (g_{t-1} - g) + \varepsilon^g_t \quad \varepsilon^g_t \sim \mathcal{N}(0, \sigma^2_g) \] (3)

where \( g \) represents steady-state purchases. Government transfers to households have risen as a share of output in many developed economies since 1970. We allow transfers to follow one of two regimes: in one regime transfers are stationary and in the other they grow exponentially. The transfers regime is indexed by \( rs^z_t \):

\[
z_t = \begin{cases} 
(1 - \rho^z)z + \rho^z z_{t-1} + \varepsilon^z_t & \text{if } rs^z_t = 1 \\
\mu^z z_{t-1} + \varepsilon^z_t & \text{if } rs^z_t = 2 
\end{cases}
\]

with \( \mu^z > 1 \) and \( \varepsilon^z_t \sim \mathcal{N}(0, \sigma^2_z) \). The transfer regime, \( rs^z_t \), evolves according to the transition matrix

\[
P^z \equiv \begin{pmatrix} p^z_1 & 1 - p^z_1 \\ 1 - p^z_2 & p^z_2 \end{pmatrix}. \quad (4)
\]

The government adjusts the tax rate, \( \tau_t \), in response to deviations of debt from steady state according to the rule

\[
\tau_t - \tau = \gamma (b^d_t - b)
\] (5)

where \( b^d_t \) is the post-default level of debt, defined below. The tax adjustment parameter, \( \gamma \), is positive. This rule captures the idea that fiscal authorities tend to increase tax rates when government debt rises. With lump-sum taxes, any \( \gamma > 0 \) guarantees an equilibrium exists, while \( \gamma \) must be sufficiently large to ensure debt is bounded in equilibrium. Distorting labor taxes, though, are subject to a Laffer curve that imposes an upper bound on tax revenues. When transfers can grow explosively, the feedback rule in (5) is not sufficient to ensure that government debt is default-free, as section 3.3 explains.

The default scheme at each period depends on an effective fiscal limit, \( b^*_t \), which is drawn from a distribution, \( B_t^* \). The distribution arises endogenously from the distorting taxes, as discussed below. If the government’s obligations at the beginning of period \( t \) are less than the effective fiscal limit, then it repays its debt in full and no default occurs; otherwise, the government partially defaults by a fraction \( \delta_t \). This rule determines the default rate \( \Delta_t \)

\[
\Delta_t = \begin{cases} 
0 & \text{if } b_{t-1} < b^*_t \\
\delta_t & \text{if } b_{t-1} \geq b^*_t 
\end{cases}
\]

where \( b^*_t \sim B_t^* \). In the baseline calibration, we consider a fixed default rate of 0.2: whether or not the government defaults depends on the existing debt level and the effective fiscal limit; once default occurs, a fixed fraction of debt is written off. In the alternative scenario, we
also consider stochastic default rates that follows an empirical distribution, $\delta_t \sim \Omega$, where $\Omega$ is derived from sovereign debt defaults and restructures observed in emerging market economies.

Let $q_t$ be the price of a sovereign bond in units of consumption goods at time $t$. For each unit of bonds, the government promises to pay the household one unit of consumption in the next period. This bond contract is not enforceable: at time $t$, the government may partially default on its outstanding liabilities, $b_{t-1}$, by the fraction $\Delta_t$, with post-default government liabilities defined as $b^d_t = (1 - \Delta_t)b_{t-1}$. These considerations yield the government’s flow budget constraint

$$\tau_t A_t (1 - L_t) + b_t q_t = (1 - \Delta_t) b_{t-1} + g_t + z_t. \tag{6}$$

### 3.2 Households

With access to the sovereign bond market, a representative household chooses consumption, leisure, and bond purchases to solve

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \tag{7}$$

$$s.t. \quad A_t (1 - \tau_t)(1 - L_t) + z_t - c_t = b_t q_t - (1 - \Delta_t)b_{t-1} \tag{8}$$

taking prices, $q_t$, and policies, $(\tau_t, z_t, \Delta_t)$, as given. $E_t$ is the mathematical expectation conditional on the information available at time $t$, including sovereign default information. The utility function is strictly increasing and strictly concave in consumption and leisure. $\beta \in (0,1)$ is the discount factor.

The household’s first-order conditions are

$$\frac{u_L(t)}{u_c(t)} = A_t (1 - \tau_t) \tag{9}$$

$$q_t = \beta E_t \left[ (1 - \Delta_{t+1}) \frac{u_c(t+1)}{u_c(t)} \right]. \tag{10}$$

The marginal rate of substitution between consumption and leisure equals the after-tax wage. Bond prices in equation (10) reflect the household’s expectation about the probability and magnitude of sovereign default in the next period. The optimal solution to the household’s maximization problem also implies the transversality condition

$$\lim_{j \to \infty} E_t \beta^{j+1} \frac{u_c(t + j + 1)}{u_c(t)} (1 - \Delta_{t+j+1}) b_{t+j} = 0. \tag{11}$$

### 3.3 Distribution of Fiscal Limit

Distorting taxes have important implications for how much revenue the government can collect. Consider an increase in the tax on labor
income. If the household’s work effort remained unchanged, the tax base would also remain fixed and tax revenues would rise unambiguously. A higher income tax, however, reduces the after-tax return to working and induces households to work less. The resulting impact on revenue collections is ambiguous, but generally at low tax rates, an increase in tax rates raises revenues, while at high tax rates, tax hikes can actually reduce revenues. This phenomenon is the basis of the “Laffer curve.” In this model, the Laffer curve is dynamic, as the shape of the Laffer curve depends on the state of the economy. For given levels of productivity and government purchases, \((A_t, g_t)\), a tax rate exists that maximizes revenues. At the tax rate at the peak of the Laffer curve, denoted by \(\tau^{\text{max}}(A_t, g_t)\), government collects the maximum level of tax revenue for the given state, denoted by \(T^{\text{max}}(A_t, g_t)\).

The government’s ability to service its debt also depends on the size of government purchases and lump-sum transfers, which are political decisions that grow out of conflicts and compromises among parties with different ideologies [Persson and Svensson (1989) and Alesina and Tabellini (1990)]. To avoid developing a structural political economy model, we specify the processes for government purchases and transfers to capture the trends and fluctuations of government expenditures observed in the data.

The fiscal limit is the maximum level of debt that the government is able to pay back, defined as the sum of discounted expected maximum primary surpluses in all future periods. The dynamic and stochastic nature of the Laffer curve and shock processes imply that the fiscal limit is stochastic with a probability distribution that depends on all the features of the economy, including private sector behaviors, the nature of policy behavior, and the properties of the random disturbances in the economy.

### 3.3.1 Conditional Distribution

We first consider the conditional, or state-dependent, distribution of fiscal limits, defined as

\[
\mathcal{B}^*(A_t, g_t, r s_t^z) \sim \sum_{j=0}^{\infty} \beta^j \frac{u_c^{\text{max}}(A_{t+j}, g_{t+j})}{u_c^{\text{max}}(A_t, g_t)} \left( T^{\text{max}}(A_{t+j}, g_{t+j}) - g_{t+j} - z(r s_{t+j}^z, A_{t+j}) \right) \tag{12}
\]

\(u_c^{\text{max}}\) is the marginal utility of consumption when the tax rate is at the peak of the Laffer curve, \(\tau^{\text{max}}\). Given the parameters of the model and the specifications of the shock processes, a unique mapping between the peak of dynamic Laffer curve and the exogenous state of the economy determines the state-dependent distribution of the fiscal limit. The conditional distribution implies that households’ expectations about the government’s ability to pay back its debt depend on the current state of the economy, including the transfers regime, as the notation \(\mathcal{B}^*(A_t, g_t, r s_t^z)\) makes explicit.
3.3.2 Unconditional Distribution  In the long run, the state of the economy today plays a less significant role in determining the government’s ability to service its debt. The unconditional distribution $B^*$ is no longer time-varying and is defined by

$$B^* \sim E \left( \sum_{h=1}^{\infty} \beta^h \frac{u_{\max}(t+h)}{u_{\max}(t)} (T_{t+h}^{\max} - g_{t+h} - z_{t+h}) \right)$$  (13)

3.3.3 Discussion  In linearized models, where transfers are stationary, positive feedback from government debt to taxes, like the tax rule specified in equation (5), can keep sovereign debt from exploding unless the tax adjustment parameter is too small [Leeper (1991), Bohn (1998)]. This is not guaranteed, however, if the tax rate is approaching the peak of Laffer curve or if transfers follow the Markov regime-switching process specified in section 3.1. Trabandt and Uhlig (2011) use a neoclassical growth model to show that Denmark and Sweden are already on the “slippery side” of their curves, where lower tax rates will raise revenues. Even if the average tax rate is far from the peak of Laffer curve, which is arguably more relevant for most countries, rising transfers raise debt and by the specified tax rule, the tax rate. With regime-switching transfers, there can be prolonged periods during which rising transfers steadily raise government debt. Forward-looking agents may still be willing to purchase sovereign debt if they expect the explosive transfers regime to end. If transfers stay in that regime for too long, however, debt may rise to such a level that the government will be unable to repay its debt in full, even if it consistently follows a tax rule designed to stabilize debt. A positive probability of eventually hitting the peak of Laffer curve in the future can spur sovereign default fears today even if the current tax rate is well below the peak of Laffer curve.

We assume that the effective fiscal limit, $b^*_t$, is a draw in each period $t$ from the fiscal limit distribution. As shown in the numerical analysis below, the distribution can be quite dispersed, especially when transfers currently reside in the explosive regime. The distribution reports the probability that a particular debt level can be supported by taxing income at the peak of the Laffer curve, given the stochastic processes for transfers, government purchases and productivity. The maximum sustainable debt levels in the upper tail of the distribution are obtained only if the economy receives a run of good shocks. Given the lower probability of receiving such good shocks, default becomes more likely if current debt is in the upper tail. Similarly, the debt levels in the lower tail can be supported even if the economy receives a run of bad shocks, and therefore default is less likely. Default is possible at any point on the distribution. If a debt level of $b^{**}$ is associated with a probability of $p^{**}$ in the distribution, then it implies that with the probability $p^{**}$ a run of shocks may occur that makes a debt
level that is equal to or higher than $b^{**}$ unsustainable. Full details of how the fiscal limit distributions are computed appear in appendix A.

We use the case of Greece to explore conditional distributions of fiscal limits in section 4, and the case of Sweden to understand unconditional distributions in section 5.

4 DEBT CRISIS IN GREECE

4.1 Timeline Through the early and mid 2000s, the view seemed to be that the discipline instilled by membership in the euro area would force the Greek government to conform to the euro zone’s fiscal standards. Despite robust economic growth during this period, persistent deficits in Greece maintained debt at about 100 percent of GDP. A series of U.S.-based financial events in 2008—the subprime mortgage crisis and the failures of Bear Stearns and Lehman Brothers—induced investors to more carefully assess the riskiness of Greek sovereign debt and drove Greek-German interest rate spreads to a couple of hundred basis points.

The impact on Greek rates of these global shocks may have been contained had it not been for the “data revisions” the Greek Ministry of Finance began to announce in 2009. The 2009 budget deficit, initially forecasted to be 2 percent of GDP, was revised upward to 3.7 percent in January, to 5.1 percent in a mid-year review, and to 12.7 percent by late November. Eurostat eventually announced a final value of 15.8 percent of GDP. This fiscal news alerted markets to the true state of fiscal policy in Greece and triggered a steady rise in risk premia throughout 2010. Figure 1 reports daily spreads between 10-year yields on Greek sovereign bonds and German Bunds.

In May 2010, Greece’s socialist PASOK-led government narrowly approved sweeping fiscal changes that cut public sector wages, reformed pensions, and raised taxes. These austerity measures were part of a bailout agreement between Greece and the troika (the International Monetary Fund, the European Commission, and the European Central Bank). Because the changes did not grow out of a clear political consensus among Greeks on the need for fiscal consolidation, they triggered violent public protests and widespread criticism that raised doubts about the ruling government’s ability to complete its term, which ended in 2011, much less see the reforms through. Greek risk premia confirm these doubts, as they continued their relentless rise through 2010, reaching nearly 10 percentage points by year end.

Since the first troika agreement, each quarter Greece missed its fiscal targets and the Greek government has announced additional austerity actions. None of these actions tempered the rise in risk premia. An October 2011 EU summit sought to calm financial markets by reasserting member countries’ commitments to sustainable fiscal policies. Summit leaders
called on Private Sector Involvement (PSI) to help Greece reach a 120 percent debt-GDP ratio. Under PSI, private investors would agree to a 50 percent haircut on Greek bond holdings, while Eurozone countries would provide 30 billion euro to the PSI package and contribute to recapitalizing Greek banks [Council of the European Union (2011)]. Within a month of the summit, premia had risen by more than 500 basis points.

Figure 1 makes clear that risk premia behaved quite different beginning in the second half of 2011. Over the 18 months from September 2009 to June 2011, the premium rose 1164 basis points—65 basis points per month on average. But in just the six months from July through December 2011, the increase was 1655 basis points, averaging 276 basis points per month. Very little news about the current state of Greek finances arrived in the latter period, but plenty of news arrived about the stability of the Greek government and prospective future fiscal states.

With Prime Minister Papandreou’s resignation in early November 2011, Lucas Papademos, former governor of the Bank of Greece and former Vice President of the ECB, became prime minister of a caretaking coalition government. On February 13, 2012 the new government approved the terms of the second troika bailout. Conditions included a 22 percent cut in the minimum wage, large reductions in public employment, and substantial cuts in pension, health, and defense spending. The agreement also included an increase to 53.5 percent for the haircut taken by private bond holders, a schedule for coupon payments on Greek bonds through 2042, a reduction of interest rates of the Greek Loan Facility, and a promise by national central bank holders of Greek bonds to pass earnings from those bonds back to Greece [Council of the European Union (2012)].

4.2 Model Calibration We calibrate the quarterly model to Greek data from 1971–2007 to illustrate uses of the conditional (or state-dependent) fiscal limit. Steady-state government purchases are 17 percent of GDP, lump-sum transfers are 14 percent of GDP, and government debt-GDP ratio is close to 60 percent, which produces a tax rate of 0.34 in steady state. The productivity shock has persistence of 0.9721 and a standard deviation of 0.47 percent of the steady-state level; and the transfers shock has persistence of 0.9634 and a standard deviation of 0.8 percent of the steady-state level. The parameter $\gamma$ is calibrated to 0.1 to match the Greek data, implying the government raises the tax rate by 1 percentage point.

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4In September 2012, the ECB announced its Outright Monetary Transactions program, which consists of potentially unlimited purchases by the ECB of short-term sovereign debt issued by qualifying member nations. Our analysis does not examine this or similar actions taken by the European Union to assist Greece.

5Due to lack of quarterly data for fiscal measures, we calibrate the model using interpolated data. Appendix B describes the data sources.

6For simplicity, we keep government spending at its steady state to compute the fiscal limit distribution. Bi (2012) shows that in a similar setup, time-varying spending has limited impact on the fiscal limit distribution.
point in response to an increase of government debt by 10 percent of GDP. Transfers follow a Markov regime-switching process. In the explosive regime, transfers growth, \( \mu_z \), is 1.0037 to match the observation that transfers from the Greek government to the private sector rose by 13 percentage points of GDP over the 40-year sample. In the baseline case, regime persistence, parameterized by \( p_{z1} \) and \( p_{z2} \), is set at 0.9937 so that the regimes are symmetric with expected duration of 40 years. Table 1 summarizes the parameter settings.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate (( \beta ))</td>
<td>0.99</td>
</tr>
<tr>
<td>Steady state leisure (( L ))</td>
<td>0.75</td>
</tr>
<tr>
<td>Persistence of productivity (( \rho_A ))</td>
<td>0.9721</td>
</tr>
<tr>
<td>Standard deviation of productivity (( \sigma_A ))</td>
<td>0.0047A</td>
</tr>
<tr>
<td>Persistence of transfers (( \rho_z ))</td>
<td>0.9634</td>
</tr>
<tr>
<td>Standard deviation of transfers (( \sigma_z ))</td>
<td>0.008z</td>
</tr>
<tr>
<td>Response of taxes to debt (( \gamma ))</td>
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</tr>
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<td>Spending-GDP ratio (( g/y ))</td>
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</tr>
<tr>
<td>Transfers-GDP ratio (( z/y ))</td>
<td>0.14</td>
</tr>
<tr>
<td>Tax rate (( \tau ))</td>
<td>0.34</td>
</tr>
<tr>
<td>Transfers growth (( \mu_z ))</td>
<td>1.0037</td>
</tr>
<tr>
<td>Regime-switching parameters (( p_{z1}/p_{z2} ))</td>
<td>0.9937/0.9937</td>
</tr>
</tbody>
</table>

Table 1: Baseline Calibration for the Greek Economy

As the results reveal, even this very simple model generates non-linearities that play a critical role in pricing sovereign debt. We solve the full non-linear model, coupled with the fiscal limit described in section 3.3, using the monotone mapping method. The solution method, based on Coleman (1991) and Davig (2004), discretizes the state space and conjectures candidate decision rules that reduce the system to a set of first-order expectational difference equations. Decision rules map the state at period \( t \) into the end-of-period government debt, denoted as \( b_t = f^b(\psi_t) \) with \( \psi_t = (b_t^d, A_t, z_t, r s_t^z) \). After finding the decision rules, we solve for the bond-pricing rule, \( q_t = f^q(\psi_t) \), using the government budget constraint, and the interest rate rule, \( R_t = f^R(\psi_t) \). Details appear in appendix C.

4.3 Decision Rules

In the benchmark calibration, the default rate is fixed at 20 percent. The top panels of figure 2 show the cumulative conditional distributions (CDFs) for fiscal limits, and the bottom panels are the corresponding risk premia. The top left panel reports the CDFs, \( B^*(A_t = A^i, r s_t^z = 1) \), when current productivity is at different levels (\( i = low, ss, high \)), while current transfers reside in the stationary regime. All future states (\( A_{t+i}, r s_{t+i}^z \)) evolve according to their stochastic specifications. Since the effective fiscal limit
at each period is a random draw from the conditional distribution, the CDF illustrates the
default probability at each debt level scaled by the steady-state output: if the amount of
debt the government issues at period $t$ is $b_t$, then the CDF illustrates the probability that the
government will default in the following period, $\Phi(b_t \geq b_{t+1}^*)$. The solid line is the probability
when current productivity is at steady state, while the dashed line and the dash-dotted
lines are CDFs when current productivity is 8 percent below and above the steady state.
Current productivity changes contemporaneous tax revenues directly and future tax revenues
indirectly depending on the shock persistence, so productivity has a significant impact on the
distributions. At the debt ratio of 200 percent, for example, the default probability is 0.25
when productivity is at steady state, and less than 0.1 when productivity is 8 percent above
steady state, but rises to 0.9 when productivity is 8 percent below steady state. Symmetric
changes in productivity produce asymmetric changes in default probabilities because possibly
explosive transfers in the future generate fat tails in the fiscal limit distribution.

The risky interest rate on government bond $R_t$ can be computed in terms of the current
state $\psi_t = (b^d_t, A_t, z_t, rs_t^z)$, as appendix C describes. The risk-free rate $R_f$ is computed from
an identical specification, but conditional on the assumption that the government never
defaults. The risk premium, $r_t$, is defined

$$r_t \equiv R_t - R_f^\Delta$$

where $q_t^{\Delta=0}$ is the bond-pricing function when government debt is perfectly safe.

The bottom left panel of figure 2 shows the risk premia that the government has to pay
at different debt levels. Sovereign risk premia follow the fiscal limit distribution closely:
they are flat when the debt level is far from the fiscal limit and begin to rise, sometimes
rapidly, when default becomes possible. The maximum risk premium can actually go above
20 percent points. This happens because a higher $\Delta_{t+1}$ reduces the debt burden and tax
rate next period, reducing the marginal utility of consumption next period and the discount
factor. The risky discount rate, $\frac{u_c(t+1)}{u_c(t)} |(\Delta_{t+1} > 0)$, is lower than the risk-free discount rate,
$\frac{u_c(t+1)}{u_c(t)} |(\Delta_{t+1} \equiv 0)$.

The topright panel of figure 2 reports the conditional distributions when current transfers
are either in the stationary or the explosive regime, while current productivity is at steady
state. All else being equal, the default probability can be significantly higher in the explosive
regime: when debt is 150 percent of output, default occurs with 30 percent probability
when the current transfers grow exponentially, but less than 5 percent when transfers are
stationary. Importantly, the distribution has a fat tail even when current transfers are
stationary—the possibility that future transfers may switch to the explosive regime implies that future fiscal surpluses could be significantly lower, constraining the government’s ability to service its debt today even if current transfers are stationary.

4.3.1 Alternative Default Rate Defaulting countries and creditors typically engage in prolonged debt negotiations whose outcomes are uncertain. In the benchmark calibration, the default rate is fixed at 20 percent. As an alternative, we assume the default rate is stochastic and follows an empirical distribution.

We derived the distribution from empirical evidence on sovereign debt defaults and restructurings observed in emerging market economies from 1983 to 2005. Moody’s (2009) reports the total amounts of defaulted debt during rated sovereign bond defaults since 1983, Panizza (2008) provides a thorough dataset on public debt in developing countries, and Sturzenegger and Zettelmeyer (2008) estimate the haircuts associated with sovereign debt restructurings. Based on these three sources, we compute the default rate, defined as the share of actual defaulted debt over total public debt. The default rate falls between 0 and 0.1 with 70 percent probability, between 0 and 0.3 with 90 percent probability, and between 0 and 0.5 with 100 percent probability [see Bi (2012) for more details]. Figure 3 shows the decision rules for risk premia when default rates are stochastic. They follow the baseline case closely, but with different magnitudes of risk premia. Since the distribution has a mean of 0.1, risk premia peak a little above 10 percent points.

4.4 Fiscal Reform Greece has experienced persistent transfers growth during the past three decades which, in combination with rampant tax evasion, has led to soaring government debt. Mounting pressure from financial markets has forced the Greek government to adopt a variety of fiscal austerity measures. On February 10, 2012, the Greek cabinet approved a new austerity plan, which is estimated to improve the 2012 budget deficit by 3.3 billion euro. It remains an open question whether the fiscal austerity measures are credible.

We consider two extreme scenarios against the baseline case—a less-credible versus a more-credible reform. The smaller is the regime-switching probability $p^*_1$, the more likely transfers will switch from the stationary regime to the explosive regime, and the less credible is the fiscal reform. We consider an incredible reform with $p^*_1 = 0.9306, p^*_2 = 0.9937$—even if transfers are stationary today, with 7 percent probability the government will renounce on the fiscal reform and revert to the explosive regime next period. This yields an expected duration for the stationary regime of only 4 years. We contrast this to a credible reform with $p^*_1 = 0.9987, p^*_2 = 0.9937$—once the transfers are stationary, the probability of leaving that regime is less than 0.2 percent, giving the reform an expected duration of 200 years.

Figure 4 compares the fiscal limit distributions for these two reform scenarios to the
baseline calibration with an expected duration of 40 years. The top panel illustrates the comparison when current transfers are stationary while current productivity is at the steady state. For a stark contrast, the dotted line is the fiscal limit when transfers are always zero. The solid line is the baseline case, the dashed line shows the incredible reform, and the dash-dotted line illustrates the credible reform. Everything is identical across the four scenarios except the expectation of future transfers. The area between the dashed line and the solid line measures how much the fiscal limit shrinks due to the incredible reform, and equals the expected present value of future transfers increases due to a higher probability of switching to the explosive regime. Similarly, the area between solid line and the dash-dotted line is the expansion in the fiscal limit due to the credible reform. The bottom panel repeats the same comparisons except that current transfers are in the explosive regime.

Figure 4 makes clear that if fiscal reform is credible, the current transfers regime matters a great deal in determining the default probability, as the dash-dotted line is much less dispersed in the top panel than in the bottom. On the other hand, if fiscal reform is not credible, containing transfers growth temporarily does little to reduce the default probability and risk premia, as shown by the dashed lines in both panels. Speculation that the general election in Greece in 2013 may overturn many fiscal austerity measures suggests that markets may not be confident in the credibility of Greek fiscal reforms.

An alternative way to model the fiscal reforms is through changes in the persistence of explosive regime. The higher the parameter $p_i^z$, the more likely transfers will stay in the explosive regime, and the less credible is the fiscal reform. We consider an incredible reform with $p_1^z = 0.9937, p_2^z = 0.9987$, which yields an expected duration for the explosive regime of 200 years, and also a credible reform with $p_1^z = 0.9937, p_2^z = 0.9306$, giving the explosive-transfer regime an expected duration of only 4 years. Figure 5 compares the fiscal limit distributions for these two reform scenarios to the baseline calibration. An incredible fiscal reform reduces the fiscal limit, captured by the area between the dashed line and the solid line. More interesting, if the government can commit to reducing the duration of the explosive regime, such a fiscal reform can raise the fiscal limit regardless of whether current transfers are stationary or explosive, because the explosive regime is expected to be short-lived. In contrast, the credible reform in figure 4, modeled as a more persistent stationary regime, can raise the fiscal limit if the government can switch transfers to the stationary regime, but has limited impact if current transfers come from the explosive regime.

4.5 Signal Extraction We have so far assumed that households can observe the transfers regime so there is no uncertainty about the distribution of the fiscal limit at any point in time.
Now we extend the model to explore the implications of confronting agents with a signal-extraction problem: they cannot observe the transfer regime and instead make a probabilistic inference regarding the regime in place. We continue to assume that agents know the transfers process and its transition probabilities. They behave as Bayesian updaters and form their inference by combining the current realization of transfers, \( z_t \), with their prior beliefs. Denote the updated probability that the current transfers regime is stationary by \( \omega_t = P[rs^*_t = 1|z_t] \), which can be calculated recursively from

\[
\omega_t = \frac{\omega_{t-1} p_{11}^z \eta(1) + (1 - \omega_{t-1}) p_{21}^z \eta(1)}{\sum_{i=1,2} \omega_{t-1} p_{1i}^z \eta(i) + (1 - \omega_{t-1}) p_{2i}^z \eta(i)}
\]

(16)

with \( \eta(1) = \eta(z_t - (1 - \rho^z) z - \rho^z z_{t-1}) \) and \( \eta(2) = \eta(z_t - \mu^z z_{t-1}) \). \( \eta(\cdot) \) represents the normal density with variance \( \sigma^2_z \).

Transfers regime, \( rs^*_t \), is a latent state variable to households. They observe current and past realizations of transfers, but not the regime that generated the realizations. Agents update their beliefs about how likely the transfers realizations come from the stationary regime this period, \( \omega_t \), based on observations of \( (z_t, z_{t-1}, \omega_{t-1}) \). They then compute the likelihood of transiting to each of the transfers regime next period, based on the transition probabilities and their updated belief, \( \omega_t \). Since the conditional distribution of the fiscal limit depends on the transfers regime, households decide the quantity of bonds to purchase in each period conditional on the updated belief, so \( \omega_t \) becomes a relevant state for the decision rules.

A comparison of decision rules illustrates how uncertainty about the transfers regime affects risk premia. In the baseline case without a signal-extraction problem, figure 6 compares risk premia when current transfers are at different levels: the left panel is conditional on the current transfer regime being stationary and the right panel on the regime being explosive. Regardless of the transfers regime, the current level of transfers has negligible impact on risk premia. This is because the lump-sum transfers do not change households’ decisions directly, except that higher (lower) transfers may raise (reduce) the level of government debt at the end of the period.

But if households need to infer the transfers regime, past transfers and the shocks to transfers can play an important role in shaping their inferences about the distribution of the fiscal limit, which feeds into risk premia. Now the state space becomes \( (b_t^d, A_t, z^\text{shock}_t, z_{t-1}, \omega_{t-1}) \), where \( z^\text{shock}_t = z_t - z_{t-1} \). The decision rules in figure 7 illustrate that transfers shocks of the same size can change risk premia by different amounts, depending on households’ prior beliefs. The left panel reports the case when agents currently have a strong belief that the stationary regime (regime 1) prevails; the right panel reports the outcome when agents place
a low prior probability on being in the stationary regime. For instance, when the debt level is 200 percent of GDP, a negative transfer shock of 3.6 percent of steady-state level reduces the risk premium by 3 percentage points if the prior belief of being in the stationary regime, $\omega_{t-1}$, is high (0.75), but by 8 percentage points if $\omega_{t-1}$ is low (0.25). Similarly, a positive shock of 3.6 percent of steady-state transfers raises the risk premium by 7 percentage points if $\omega_{t-1}$ is high, but by only 2 percentage points if $\omega_{t-1}$ is low. This asymmetry illustrates that a given shock to transfers can lead to very different assessments of the riskiness of sovereign debt, depending on agents’ prior beliefs about the nature of the prevailing transfers regime.

Figures 8–10 show three sets of simulations to further explore the impact of the signal-extraction problem. Figure 8 reports the baseline calibration with low-volatility in transfers shocks—only 0.8% of the steady-state level. Transfers stay in the explosive regime until period 135 and then switch to the stationary regime. After the regime switch, the true default probability drops from 0.04 to almost zero immediately, but households do not reduce the risk premium to zero immediately because they are uncertain that a regime change has occurred. Instead, risk premia decrease in the following three quarters as households gradually learn about the regime change.

Figure 9 shows the same experiment but with a larger standard deviation of transfers shocks—1.6% of the steady-state level. Again, transfers switch in period 135, but it takes another 10 quarters for households to learn the regime switch as the inference index slowly rises from 0 to 1. After the transfers regime switch, the risk premium actually increases for another 2 quarters before coming down gradually. Figure 10 illustrates a more extreme case: other than transfers switching from explosive regime to stationary regime at period of 135, the economy also receives a run of negative productivity shocks for 8 quarters from period 133 to 140. Lower productivity shrinks the tax base and further deteriorates the government budget, raising risk premia.

5 Swedish Fiscal Reform in 1990s

Large and seemingly permanent changes in fiscal behavior in Sweden following the recession and banking crisis in the early 1990s illustrate uses of the unconditional distribution of fiscal limits.

5.1 Timeline In the early 1990s, Sweden experienced a boom-bust cycle that severely tested the prevailing policy regime. After deregulating the financial system, the economy boomed in the late 1980s, with rapid growth in GDP and employment. By 1989–1990 the

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7This section draws liberally from Swedish Ministry of Finance (2001), Jonung and Hagberg (2005), Jonung (2009), Reinhart and Rogoff (2008), and Wetterberg (2009).
boom had ended and the bust began. The resulting recession was comparable to Sweden’s experience in the Great Depression, with GDP falling for three consecutive years and unemployment rising from 1.5 percent in 1989 to over 8 percent in 1993. Large automatic stabilizers built into Swedish fiscal rules swung the general government balance from a 5 percent surplus in 1989 to nearly a 12 percent deficit in 1993. The Swedish government responded with a thorough policy reform.

The fiscal framework that was introduced in 1993 consists of three components covering both central and local governments. First, a ceiling on total expenditures, excluding interest payments, was introduced at the central government level. Sweden’s Ministry of Finance prepares the budget and presents it to Riksdag, which votes on the expenditure ceiling and how to divide the budget into 27 expenditure areas. Second, a budget surplus target of 1 percent of GDP over the business cycle was adopted at the general government level to partially pre-fund benefits to Sweden’s aging population. Third, a balanced budget at the local government level was introduced in 2000.

Under this fiscal framework, the Swedish government was able to reduce public expenditures from 60 percent of GDP in 1993 to 45 percent of GDP in 2007 by cutting social benefits, public subsidies, capital expenditures and public consumption. The successful fiscal reform has earned applause from sovereign debt rating agencies. After the 1993 downgrade of Swedish debt, Standard & Poor’s (1997) revised its long-term foreign currency rating outlook for Sweden from negative to stable, largely due to “expected fiscal strengthening” arising from the reforms. In the context of the 2007-2009 economic downturn, Standard & Poor’s (2009) wrote, “the established fiscal rules have served Sweden well” and, “the Kingdom’s substantial fiscal buffers support its creditworthiness in the current adverse economic environment.” Despite the decline in fiscal performance as a result of rising government spending and declining tax revenue, rating agencies believe that the deterioration in public finances will be temporary as the Swedish government has a solid history of fiscal discipline and credible rules in place.

Figure 11 suggests that a shift in the level of transfers and government spending occurred between 1992 and 1997. Sweden’s financial crisis started in 1992, while the expenditure ceiling on central government spending was introduced in 1997. Claeyss (2008) identifies the breakpoint for government spending as the third quarter of 1995 and for transfers as the second quarter of 1996. We set the breakpoint to be 1997 in order to highlight the comparison before and after the fiscal reform, but different breakpoints do not affect our results qualitatively.

Countercyclical fiscal policy is an important feature of Swedish data, and we allow government spending and transfers to respond to the countercyclical changes of output, measured
by the parameters $\alpha^g$ and $\alpha^z$ that is estimated for the period of 1980 to 2007.\footnote{We assume the transfers always follow a stationary process as they have been stable over time in Sweden.}

\begin{align*}
\ln \frac{g_t}{g} &= \alpha^g \ln \frac{A_t}{A} \quad (17) \\
\ln \frac{z_t}{z} &= \alpha^z \ln \frac{A_t}{A} \quad (18)
\end{align*}

Table 2 shows the estimated $\alpha^g$ and $\alpha^z$, the average tax rate, and the ratios of government spending and transfers to GDP in different episodes. First, there was a sharp decline in the level of transfer payments, from 22 to about 19 percent of GDP. Second, government spending shifted from being countercyclical in the early period ($\alpha^g < 0$) to being procyclical in the latter period ($\alpha^g > 0$), which may be a consequence of the 1997 expenditure ceiling policy.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Response of spending to productivity ($\alpha^g$)</td>
<td>-0.142</td>
<td>-0.183</td>
<td>0.196</td>
</tr>
<tr>
<td>Response of transfers to productivity ($\alpha^z$)</td>
<td>-1.65</td>
<td>-1.70</td>
<td>-1.066</td>
</tr>
<tr>
<td>Average tax rate ($\tau$)</td>
<td>49.7</td>
<td>49.6</td>
<td>49.9</td>
</tr>
<tr>
<td>Spending-GDP ratio ($g/y$)</td>
<td>27.3</td>
<td>27.6</td>
<td>26.7</td>
</tr>
<tr>
<td>Transfers-GDP ratio ($z/y$)</td>
<td>21</td>
<td>22</td>
<td>19.1</td>
</tr>
</tbody>
</table>

Table 2: Swedish Fiscal Data (1980–2007).

5.2 Parameter Calibration To discuss the unconditional distributions in the long run, the model is calibrated to annual data and table 3 summarizes the calibration of the parameters. The productivity shock has a persistence of 0.661 and a standard deviation of 0.015. The degree of countercyclical government spending and lump-sum transfers, $\alpha^g$ and $\alpha^z$, the transfers-GDP ratio, $z/y$, and the government spending-GDP ratio are calibrated to pre-crisis (1980–1997) and post-crisis (1997–2007) data. The steady-state tax rate, $\tau$, is calibrated to the average level of tax ratio in the data.

5.3 Policy Experiments We treat as the baseline the calibration from the pre-crisis period when Swedish sovereign bonds were downgraded by the rating agencies, government spending and transfers are countercyclical, and the average tax rate and the share of transfers are relatively high. We simulate the distribution of the fiscal limit for this baseline scenario and then contrast it to the distributions obtained under three alternative calibrations that are designed to capture the post-crisis fiscal reforms.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate ($\beta$)</td>
<td>0.95</td>
</tr>
<tr>
<td>Steady state leisure ($L$)</td>
<td>0.75</td>
</tr>
<tr>
<td>Persistence of productivity ($\rho^A$)</td>
<td>0.661</td>
</tr>
<tr>
<td>Standard deviation of productivity ($\sigma_A$)</td>
<td>0.015</td>
</tr>
<tr>
<td>Average tax rate ($\tau$)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Pre-Crisis</th>
<th>Post-Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response of spending to productivity ($\alpha^g$)</td>
<td>−0.183</td>
<td>0.196</td>
</tr>
<tr>
<td>Response of transfers to productivity ($\alpha^z$)</td>
<td>−1.70</td>
<td>−1.066</td>
</tr>
<tr>
<td>Spending-GDP ratio ($g/y$)</td>
<td>0.276</td>
<td>0.267</td>
</tr>
<tr>
<td>Transfers-GDP ratio ($z/y$)</td>
<td>0.22</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 3: Calibration for the Swedish Economy

Table 4 summarizes the policy settings in the baseline model and in the three alternatives. The first alternative scenario, labeled “post-crisis”, is a counter-factual exercise that asks what the fiscal limit would be if the government were to reduce the tax rate and the share of transfers in GDP to their post-crisis levels, but continued to follow the pre-crisis countercyclical expenditure rules.

The second and third alternative scenarios, labeled “post-crisis-procyclical” and “post-crisis-ceiling” respectively, offer two explanations for government expenditure data from 1997 to 2007. In the “post-crisis-procyclical” case, government spending policy is assumed to have shifted from countercyclical to procyclical. In the “post-crisis-ceiling” case, on the other hand, expenditure ceilings on government spending and transfers are imposed, while countercyclical spending and transfers policies are calibrated to the pre-crisis levels. The ceiling rules are given by

$$
\log \frac{g_t}{g} = \min \left( \alpha^g \log \frac{A_t}{A} - \alpha^g \sigma_A \right)
$$

(19)

$$
\log \frac{z_t}{z} = \min \left( \alpha^z \log \frac{A_t}{A} - \alpha^z \sigma_A \right)
$$

(20)

where $\sigma_A$ is one standard deviation for the technology shock. Equations (19) and (20) operate asymmetrically. When productivity is high, expenditures tend to be low and the constraints do not bind. When productivity is low, however, expenditures automatically tend to be higher than normal. If the productivity shock is sufficiently bad, the automatic expansion in expenditures may be bounded above as the ceiling binds, implying that the government can conduct countercyclical expenditure policies only within some range.
Table 4: Alternative Swedish Fiscal Policies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pre-Crisis</th>
<th>Post-Crisis</th>
<th>Post-Crisis (procyclical)</th>
<th>Post-Crisis (ceiling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response of spending to productivity ($\alpha^g$)</td>
<td>−0.183</td>
<td>−0.183</td>
<td>0.196</td>
<td>−0.183</td>
</tr>
<tr>
<td>Response of transfers to productivity ($\alpha^z$)</td>
<td>−1.70</td>
<td>−1.70</td>
<td>−1.066</td>
<td>−1.70</td>
</tr>
<tr>
<td>Spending-GDP ratio ($g/y$)</td>
<td>0.276</td>
<td>0.267</td>
<td>0.267</td>
<td>0.267</td>
</tr>
<tr>
<td>Transfers-GDP ratio ($z/y$)</td>
<td>0.215</td>
<td>0.19</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>Expenditure ceiling</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td></td>
</tr>
</tbody>
</table>

$g_t \leq g^\text{ceil}$
$z_t \leq z^\text{ceil}$

5.3.1 Fiscal Limits Figure 12 compares the distributions of the fiscal limit under the baseline and the three alternative scenarios. The top panel compares the pre-crisis and post-crisis cases. In the pre-crisis baseline calibration, the distribution, centered at a debt-output ratio of 70 percent, is quite dispersed, implying that Swedish sovereign debt holders may have had good reasons to place probability on default in the early 1990s, even when the debt was at relatively modest levels. This, of course, was the time when Swedish sovereign debt was downgraded. On the other hand, fiscal reform that led to a smaller government in terms of the share of transfers in GDP and the average level of taxation shifted the fiscal limit markedly to the right, with the mean moving to 140 percent, as shown by the solid line labeled “post-crisis.”

The dotted-dashed line, labeled “post-crisis-pro,” uses identical policy settings as “post-crisis” except that government spending switches from countercyclical to procyclical, with $\alpha^g$ changing from −0.183 to 0.196, and transfers become somewhat less countercyclical, with $\alpha^z$ changing from −1.70 to −1.066. Altering the cyclical nature of government expenditures has little effect on the mean of the distribution, but reduces its dispersion. Expenditure ceilings have a more subtle influence on the distribution of the fiscal limit, as the dashed line shows. Asymmetry in expenditure rules induces asymmetry in the fiscal limit: the upper tail is substantially fatter than the lower tail, shifting risk away from moderate debt-output ratios. Taken together, the results for procyclical spending and expenditures ceiling policies provide some support for the argument that such policies can cushion the Swedish economy from risk premia on government debt.
6 Conclusion

By mapping policy behavior into a country’s fiscal limit distribution, this paper provides a tool to evaluate fiscal reforms that are undertaken in countries under sovereign risk pressures.

The next step is to model monetary policy. Recent work has found that interactions between monetary and fiscal policies—particularly the possibility that monetary policy may be operating at or near the lower bound on nominal interest rates—can play an important role in determining the impact of fiscal policies [Christiano, Eichenbaum, and Rebelo (2011), Davig and Leeper (2011), and others]. Moreover, in the presence of a fiscal limit, monetary policy’s ability to control inflation can be jeopardized [Sims (2004, 2011) Cochrane (2011), Davig, Leeper, and Walker (2010), and Leeper (2011)].

A more ambitious extension is to endogenize fiscal policy choices in a structural political economy framework and, therefore, to relate a country’s fiscal limit to its underlying political factors and institutions. By modeling political costs of fiscal austerity measures and sovereign defaults, this extension can help to bridge the fiscal policy literature and the strategic default literature, see Eaton and Gersovitz (1981) and Arellano (2008).
REFERENCES


A Simulation Procedure for Fiscal Limits

In this model, the choices of household consumption and labor supply only depend on the income tax rate and the exogenous state variables \((A_t, g_t)\). Assume the utility function is \(u(c, L) = \log c + \phi \log L\). The household first-order conditions can be written as,

\[
1 - L_t = \frac{A_t(1 - \tau_t) + \phi g_t}{A_t(1 + \phi - \tau_t)} \quad (21)
\]

\[
c_t = \frac{(A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t} \quad (22)
\]

The tax revenue, \(T_t\), is,

\[
T_t = \tau_t \frac{A_t(1 - \tau_t) + \phi g_t}{1 + \phi - \tau_t}
= (1 + 2\phi)A_t - \phi g_t - \left(\frac{(A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}\right) \quad (23)
\]

The tax revenue reaches to the maximum level, \(T_t^{\text{max}}\), when the tax rate reaches the peak point of the Laffer curve, \(\tau_t^{\text{max}}\).

\[
\tau_t^{\text{max}} = 1 + \phi \sqrt{(1 + \phi)\phi(A_t - g_t)} \quad (24)
\]

\[
T_t^{\text{max}} = (1 + 2\phi)A_t - \phi g_t - 2\sqrt{(1 + \phi)\phi A_t(A_t - g_t)} \quad (25)
\]

A.1 Conditional Fiscal Limit

Since there exists a unique mapping between the exogenous state space \((A_t, g_t)\) to \(\tau_t^{\text{max}}\) and \(T_t^{\text{max}}\), the conditional distribution of fiscal limit, \(B^*(A_t, g_t, rs_t^z)\), can be obtained using Markov Chain Monte Carlo simulation:

1. For each simulation \(i\), we randomly draw the shocks for productivity, \(A_{t+j}\), government purchases, \(g_{t+j}\), and the transfer regime, \(rs_{t+j}^z\) for 200 years conditional on the starting state \((A_t, g_t, rs_t^z)\). Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables using the household’s first-order conditions and the budget constraints, and the discounted sum of maximum fiscal surplus,

\[
B^*_i(t) = \sum_{j=0}^{\infty} \beta^j \frac{u_{c\max}(A_{t+j}, g_{t+j})}{u_{c\max}(A_t, g_t)} \left(T_{t+j}^{\text{max}}(A_{t+j}, g_{t+j}) - g_{t+j} - z(rs_{t+j}^z, A_{t+j})\right) \quad (26)
\]

2. Repeat the simulation for 100,000 times and obtain the conditional distribution of
\( B^*(A_t, g_t, rs_t^z) \) using the simulated \( B_i^*(t) \) for \( i = 1, ..., 100000 \).

3. Repeat the first and second steps for all possible exogenous states \((A_t, g_t, rs_t^z)\) within the discretized state space.

A.2 Unconditional Fiscal Limit  
The unconditional distribution \((B^*)\) can be obtained in a similar way:

1. For each simulation \( i \), we randomly draw the shocks for productivity \((A_j)\), government purchases \((g_j)\), and the transfer regime \((rs_j^z)\) for 400 years and drop the first 200 as burn-in period. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the discounted sum of maximum fiscal surplus \( B_i^* \).

2. Repeat the simulation for 100,000 times and obtain the unconditional distribution of \( B^* \) using the simulated \( B_i^* \) for \( i = 1, ..., 100000 \).

B Greek Data  
The data of government debt is from European Commission (2009), while the rest fiscal data is from the OECD Economic Outlook No. 84 for the period between 1971 and 2010. We interpolate the annual frequency data to obtain quarterly frequency series using the method of Chow and Lin (1971) and the seasonally adjusted quarterly real GDP series for the interpolation. The average tax rate is defined as the ratio of the total tax revenue over the GDP, including social security, indirect and direct taxes. The government purchases are government final consumption of expenditures. Lump-sum transfers are defined as the sum of social security payments, net capital transfers and subsidies. We detrend the data of the real GDP per worker from Penn World Table Version 6.2, see Heston, Summers, and Aten (2009), to estimate the shock process of productivity.

C Solving the Nonlinear Model  
C.1 Baseline Model without Signal Extraction  
Other than the end-of-period government debt, all other variables are either exogenous or can be computed in terms of the current state \( \psi_t = (b_t^d, A_t, z_t, rs_t^z) \).

\[
\begin{align*}
\tau_t &= \tau + \gamma (b_t^d - \bar{b}) \\
A_t - A &= \rho^A (A_{t-1} - A) + \varepsilon_t^A \\
c_t &= \frac{(A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}
\end{align*}
\]
The decision rule for government debt, \( b_t = f^b(\psi_t) \), is solved in the following steps:

1. Define the grid points by discretizing the state space \( \psi_t \). Make an initial guess of the decision rule \( f^b_0 \) over the state space.

2. At each grid point, solve the following core equation and obtain the updated rule \( f^b_i \) using the given rule \( f^b_{i-1} \).

\[
\frac{b^d_t + z_t + g_t - \tau_tA_t n(\psi_t)}{f^b_i(\psi_t)} = \beta E_{t+1} \frac{c(\psi_{t+1})}{c(\psi_t)} (1 - \Delta_{t+1})
\]

where \( \psi_{t+1} = ([f^b_{i-1}(\psi_t), b^*_{t+1}, \delta_{t+1}], A_{t+1}, g_{t+1}, z_t, rs^z_{t+1}) \). The integral on the right-hand side is evaluated using numerical quadrature.

3. Check the convergence of the decision rule. If \( |f^b_i - f^b_{i-1}| \) is above the desired tolerance (set to \( 1e^{-6} \)), go back to step 2; otherwise, \( f^b_i \) is the decision rule.

### C.2 Model with Signal Extraction

The state space becomes \( (b^d_t, A_t, z^{\text{shock}}_t, z_{t-1}, \omega_{t-1}) \), where \( z^{\text{shock}}_t = z_t - z_{t-1} \). Solving the model with signal extraction follows similar steps as the baseline model.

1. Define the grid points by discretizing the state space \( \psi_t \). Make an initial guess of the decision rule \( f^b_0 \) over the state space.

2. At each grid point, update the households’ inference \( \omega_t \),

\[
\omega_t = \frac{\omega_{t-1} p^z_{i1} \eta_t(1) + (1 - \omega_{t-1}) p^z_{i1} \eta_t(1)}{\sum_{i=1,2} \omega_{t-1} p^z_{i1} \eta_t(i) + (1 - \omega_{t-1}) p^z_{i2} \eta_t(i)}
\]
with \( \eta_t(1) = \eta(z_t - (1 - \rho^2)z - \rho^2 z_{t-1}) \) and \( \eta_t(2) = \eta(z_t - \mu z_{t-1}) \). \( \eta(.) \) represents the normal density with variance \( \sigma^2_z \).

3. At each grid point, solve the following core equation and obtain the updated rule \( f^b_t \) using the given rule \( f^b_{t-1} \).

\[
\frac{b^d_t + z_t + g_t + \tau_t A_t n(\psi_t)}{f^b_t(\psi_t)} = \beta E_t \frac{c(\psi_t)}{c(\psi_{t+1})} (1 - \Delta_{t+1})
\]  

(32)

The integral on the right-hand side is evaluated using numerical quadrature.

\[
E_t \frac{1 - \Delta_{t+1}}{c_{t+1}} = \int_{\epsilon_{t+1}^A} \int_{\epsilon_{t+1}^\prime} (X^1_{t+1} + X^2_{t+1}) \Xi_{t+1}
\]  

(33)

with

\[
X^1_{t+1} = \eta_{t+1}(1)(\omega_t p^z_{t+1} + (1 - \omega_t)(1 - p^z_{t+1}))
\]

(34)

\[
X^2_{t+1} = \eta_{t+1}(2)(\omega_t(1 - p^z_{t+1}) + (1 - \omega_t)p^z_{t+1})
\]

(35)

\[
\Xi_{t+1} = (1 - \Phi(b_t \geq b^*_{t+1})) \frac{1}{c_{t+1}} \mid_{\text{no default}} + \Phi(b_t \geq b^*_{t+1}) \int_{\delta_{t+1}} \frac{1 - \delta_{t+1}}{c_{t+1}} \mid_{\text{default}}
\]

(36)

4. Check the convergence of the decision rule. If \( |f^b_t - f^b_{t-1}| \) is above the desired tolerance (set to \( 1e - 6 \)), go back to step 2; otherwise, \( f^b_t \) is the decision rule.
Figure 1: Greek Risk Premia: Daily Greek Sovereign Bond Yield Spreads Over German Bund (10-year yields)
Figure 2: Risk premia and conditional (state-dependent) distributions of fiscal limits when calibrating to Greek data: baseline case. Distributions of fiscal limit are estimated from Monte Carlo Markov Chain simulations.

Figure 3: Risk premia for model calibrated to Greek data: alternative default rate case.
Figure 4: State-dependent distributions of fiscal limits for Greece calibration: fiscal reforms with different $p_z^1$. Baseline is specified as $p_z^1 = p_z^2 = 0.9937$, a less credible reform features $p_z^1 = 0.9306, p_z^2 = 0.9937$, and a more credible reform is specified as $p_z^1 = 0.9987, p_z^2 = 0.9937$. Dotted lines depict the fiscal limit when transfers are identically zero.
Figure 5: State-dependent distributions of fiscal limits for Greece calibration: fiscal reforms with different $p_z^2$. Baseline is specified as $p_1^z = p_2^z = 0.9937$, a less credible reform features $p_1^z = 0.9937, p_2^z = 0.9987$, and a more credible reform is specified as $p_1^z = 0.9937, p_2^z = 0.9306$. Dotted lines depict the fiscal limit when transfers are identically zero.
Figure 6: Risk premia when calibrating to Greek data: baseline case.

Figure 7: Risk premia when calibrating to Greek data: the case with signal extraction.
Figure 8: Simulation in the model with signal extraction: standard deviation for transfer shock is 0.8% of the steady-state level. Transfer regime switches at the period of 135.
Figure 9: Simulation in the model with signal extraction: standard deviation for transfer shock is 1.6% of the steady-state level. Transfer regime switches at the period of 135.
Figure 10: Simulation in the model with signal extraction: standard deviation for transfer shock is 1.6% of the steady-state level. Transfer regime switches at the period of 135, and negative productivity shocks of 2 standard deviations occur from period of 133 to 140.
Figure 11: Swedish fiscal data: dashed lines are measured on the left axis, and solid lines are measured on the right axis. GDP, transfers, and government spending are detrended.
Figure 12: Estimated CDF for fiscal limits under alternative fiscal policies when calibrating to Swedish data. Top panel compares the pre-crisis case to the post-crisis case with countercyclical government spending. Bottom panel compares three post-crisis cases: countercyclical government spending, expenditure ceiling, and procyclical government spending.