POST ECONOMETRIC POLICY EVALUATION: A CRITIQUE

Beth Ingram and Eric M. Leeper

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ABSTRACT

An increasingly popular approach to policy evaluation involves applying the parameters calibrated for a real business cycle model that does not include policy to a different model, where policy does affect private decisions. This technique, in effect, estimates a model that misspecifies how private behavior depends on policy. The calibrated parameters depend on policy behavior, but calibrators overlook this dependence when projecting policy effects. This procedure repeats the "Keynesian" errors that Lucas (1976) noted in his influential critique of (then) standard methods of econometric policy evaluation and produces predictions of policy consequences that may be no more useful than ones from traditional econometric models.
Post Econometric Policy Evaluation: A Critique

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Robert Lucas (1976) uses a few simple examples to expose a fault in the foundations of econometric policy analysis. He argues that traditional Keynesian econometric models misspecify how private behavior depends on policy. This misspecification implies that the objects the modelers interpret as describing private behavior depend on both private and policy behavior. Therefore, analyses of policy changes that hold these objects fixed "are of no value in guiding policy" [Lucas and Sargent (1978, p. 50) emphasis in original]. To address Lucas's criticisms, modern policy analysis is conducted in general equilibrium models with maximizing private agents. This paper shows that, ironically, these maximizing models invite and often entail committing precisely the same "Keynesian" sins.

One constructive response to Lucas's critique has been the development of the rational expectations econometrics program. At the general level described by Lucas and Sargent (1981), the program requires estimating the dependence of private agents' optimal decision rules on tastes, technology, and the decision makers' environment. Alternative policies correspond to different environments, and policy analysis consists of tracing out the decisions

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of individual agents for various policy environments.

In practice, after adopting parameterizations of preferences, technology, and the environment, rational expectations econometricians proceed in several steps. First, they specify how private behavior depends on policy. Second, they identify and estimate the aspects of private behavior that can reasonably be held fixed when the policy environment changes (i.e., parameters of tastes and technology), separating private parameters from policy parameters. Finally, they evaluate the economic effects of alternative policies that are characterized by changes in policy parameters. Note that failure to execute the first two steps correctly prevents the last step from being useful.

An increasingly popular approach to the first two steps is the calibration technique pioneered by Kydland and Prescott (1982). In the original real business cycle (RBC) model, Kydland and Prescott use a simple structure without policy (and in which policy is irrelevant) to match data. To calibrate the model, they find values for parameters of private behavior that allow their equilibrium to mimic certain comovements of variables over the business cycle.¹ Recent work adds monetary and fiscal policies to versions of Kydland and Prescott’s setup (or Hansen’s (1985) modification) with two objectives in mind: (a) to explore whether adding policy helps the model to match observed correlations more closely;² and (b) to

¹Long and Plosser (1983) also use a simple model without policy to reproduce certain features of business cycles. Their calibration procedures differ substantially from Kydland and Prescott’s and do not seem to have been followed as widely.

obtain welfare comparisons of various policies.\textsuperscript{3}

In the recent work, the authors choose values for the parameters of individual behavior, and then solve and simulate the model. A finding that the empirical patterns emerging from the model are "close" to those observed in actual data is interpreted as \textit{prima facie} evidence that the model's welfare implications for policy should be taken seriously.

Frequently, RBC modelers transport the parameter values Kydland and Prescott used in their model without policy to the new models with policy.\textsuperscript{4} This procedure leads to the following paradox. Kydland and Prescott's model assumes that policy doesn't affect private decision rules. There is no policy evaluation to perform.\textsuperscript{5} Alternatively, if policy does affect private behavior, then the parameters Kydland and Prescott calibrate are \textit{reduced-form}

\textsuperscript{3}Papers that evaluate the welfare consequences of various policies include Danthine, Donaldson, and Smith (1987), Cooley and Hansen (1989,1990), Cassou (1990), Greenwood and Huffman (1990), Hansen and Imrohoroglu (1990), Imrohoroglu (1990), King and Rebelo (1990), and McGrattan (1990).


\textsuperscript{5}Sargent (1984) points out a logical problem inherent in rational expectations econometrics. If policy behavior has been purposeful historically, then there again is no policy evaluation to conduct. He argues modelers must sidestep the problem and should instead treat policy historically as being "arbitrary," that is, sub-optimal. Also see Sims's (1986, 1987) remarks.
parameters for some underlying model embedding monetary and fiscal policies. Thus, if there is any policy evaluation left to perform, Kydland and Prescott's calibrated parameters must be functions of policy behavior and should change systematically with policy. When RBC modelers evaluate alternative policies, however, the calibrated parameters are held fixed.

This is precisely the sort of procedure that Lucas criticized. But the modern manifestation of Lucas's critique has a twist. Even if RBC modelers correctly specify the decision rules of the private sector, they forecast the effects of a contemplated policy change using incorrect values for private parameters. Thus, applying Kydland and Prescott's parameter values to models in which policy matters effectively continues the traditional misspecification of the dependence of private behavior on policy. Although the modern error is more subtle, there is no guarantee it is less severe.

This leads us to update Lucas's critique: We "shall argue that the features which lead to success in [matching business cycle correlations] are unrelated to quantitative policy evaluation, that the [calibrated RBC] models are (well) designed to perform the former task only, and that simulations using these models can, in principle, provide no useful information as to the actual consequences of alternative economic policies. These contentions will be based not on deviations between estimated and 'true' structure prior to a policy change but on the deviations between the prior 'true' structure and the 'true' structure prevailing afterwards" [Lucas (1976, p. 20), emphasis in original].

We illustrate the problem by constructing a hypothetical monetary growth economy,

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6By misspecifying how private decisions depend on the policy functions, RBC modelers incorrectly execute the first two steps of the rational expectations econometric program when they estimate private parameters.
which is a simplified version of models being used to evaluate policy. In this economy, agents hold money to satisfy a transactions constraint. Since the optimal savings rate depends on the expectation of money growth tomorrow relative to all future money growth, the parameters of the money process enter the decision rules of private agents.

Maintaining the hypothetical economy as "truth," we retrace the steps taken by modelers who transport parameters from one model to another model. RBC calibrators who insert Kydland and Prescott's parameters into this monetary model have (implicitly) estimated a different model that ignores money and monetary policy. Consequently, policy parameters do not enter their estimated savings function. By forcing all the variation in savings to be accounted for by private behavioral parameters alone, the procedure confounds the response of savings to money growth with the response of savings to other shocks. Our example displays the resulting bias in the parameter estimates as a function of policy behavior.

Within the hypothetical economy we conduct rational expectations policy experiments. Given some historical behavior of policy, we suppose there is an unanticipated permanent change in policy today, and calculate the true path of the savings rate under the new policy. This path is then compared to the paths predicted by an RBC calibrator and a naive time series modeler. The calibrator's biased estimates of private parameters produce biased forecasts of the new level of the savings rate when policy changes.

Since the true savings rate depends on the conditional expectation of future money growth, both the actual change in savings and the RBC calibrator's predicted change vary

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7Our example characterizes calibration as a type of moments estimator. Thus, this paper is silent on the controversy over calibration versus estimation [see Singleton (1988) and Watson (1990)].
with this expectation. We illustrate both of these changes in the savings rate as a function of the current state. The two exercises show that, as popularly implemented, the RBC calibration approach may be inferior to naive time series models. This demonstrates the well-known fact that fully specified theories impose stringent restrictions, which, if false, produce false empirical predictions.

I. An Illustrative Monetary Model.

As our hypothetical "true" model, we use a variant of the cash-in-advance economies in Lucas (1980,1982), Svensson (1985), and Lucas and Stokey (1987). Two considerations guided our choice of model. First, it is a simplified version of the setup that Kydland (1989), Cooley and Hansen (1989,1990), and others use to match data and study the welfare implications of monetary policy. Our simpler model abstracts from labor/leisure choices and assumes that capital depreciates fully each period. This has the benefit that it understates the real effects of monetary policy, and, thereby, understates the biases produced by RBC calibration procedures. Second, the simpler model has analytical solutions, which are derived in Coleman (1989), enabling our example to show explicitly how the private sector's decision rules depend on policy behavior. In more complicated models, this dependence can get obscured because the equilibria must be approximated and solved numerically.

The model has a representative agent and a monetary authority. Capital held from period t-1 to period t, k_t, yields output of the single consumption good equal to \theta_t k_t^\alpha, \alpha \in (0,1), where \theta_t is an exogenous stochastic technology shock. Capital depreciates completely each period. The consumer discounts utility at rate \beta \in (0,1). At time t, the agent chooses consumption, c_t, pre-transfer nominal balances, m_{t+1}, and capital, k_{t+1}, to solve:
\[
\max \quad E_0 \sum_{i=0}^{\infty} \beta^i \ln(c_i)
\]

subject to \quad \begin{align*}
p_i(c_i + k_{t+1}) + \tilde{m}_{t+1}/M_t &= p_i\theta_i k_{t+1}^\alpha + m/M_t, \\
p_i c_i &\leq m/M_t, \\
\ln(\theta_i) &= \rho \ln(\theta_{t+1}) + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2), \quad \left| \rho \right| < 1,
\end{align*}

(1)

given \(k_0\) and \(M_0\). \(p_t\) is the price at time \(t\) of the consumption good in terms of money, \(m_t\) represents nominal balances held from \(t-1\) to \(t\), and \(M_t\) is the aggregate nominal money stock at \(t\). To make the agent’s problem stationary, we use the aggregate money stock to normalize nominal variables. The cash-in-advance constraint in equation (1) imposes the condition that nominal purchases of the consumption good cannot exceed the quantity of nominal balances carried into the period. *Ceteris paribus*, if the agent chooses higher nominal balances today, \(\tilde{m}_{t+1}\), she can consume more tomorrow.

After making decisions at time \(t\), the agent receives a lump-sum monetary transfer equal to \((1/z_{t+1}-1)M_t\) so that post-transfer money holdings are \(m_{t+1} = \tilde{m}_{t+1} + (1/z_{t+1}-1)M_t\). This ensures that any seigniorage collected by expanding the money supply is rebated to the consumer in a way that does not affect her decisions. The evolution of the nominal aggregate money stock is \(z_{t+1}M_{t+1} = M_t\). Therefore,

\[
\tilde{m}_{t+1}/M_t = m_{t+1}/M_t - 1/z_{t+1} + 1.
\]

The budget constraint can now be rewritten as:

(2) \quad \begin{align*}
p_i(c_i + k_{t+1}) + m_{t+1}/M_t + 1 - 1/z_{t+1} &= p_i\theta_i k_{t+1}^\alpha + m/M_t,
\end{align*}
Monetary policy is a specification of the stochastic process governing $z_t$, the inverse of the growth rate of the aggregate money stock. The agent knows this process, but the realization of $z_{t+1}$ is not revealed until the end of period $t$. The policy rule in effect in the hypothetical economy allows money growth to respond to technology shocks:

$$z_{t+1} = \mu + \delta \ln(\theta_t).$$

The steady state (inverse) growth rate of money is $\mu$, which is positive.

The appendix shows that this policy specification implies the equilibrium consumption and savings decisions:

$$c_t = \left[ \frac{\mu + \delta \ln(\theta_t)}{\mu + \frac{\delta \ln(\theta_t)}{1 - \alpha \beta + \frac{\delta \ln(\theta_t)}{1 - \alpha \beta}} \right] \theta_t k_t^\alpha,$$

$$k_{t+1} = \left[ 1 - \frac{\mu + \delta \ln(\theta_t)}{\mu + \frac{\delta \ln(\theta_t)}{1 - \alpha \beta + \frac{\delta \ln(\theta_t)}{1 - \alpha \beta}} \right] \theta_t k_t^\beta.$$

Note that the stochastic processes governing consumption and capital depend on the policy parameters $\mu$ and $\delta$.\(^8\)

In this economy, monetary policy affects real allocations by altering the intertemporal rate of return on cash balances. These real effects arise only if realized shocks today change

\(^8\)An equilibrium for this model requires that the return on holding money is less than $1/\beta$ for all states of the world, otherwise the optimal choice of capital is negative. In all the simulation results reported below, we check that the realized rates of inflation and money growth exceed $\beta$. 

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expected future money growth and inflation relative to their normal values. Expressions (4) and (5) show that a higher than average technology shock has both direct and indirect effects on consumption and capital decisions. If \( \delta \) is negative, these effects work in opposite directions. The direct effect raises consumption (and the capital stock) by the marginal propensity to consume (and invest) out of current income. The indirect effect arises because a higher than average technology shock leads the agent to predict a lower value for \( z \) next period, according to the policy rule (3). The lower \( z \) represents a larger lump-sum transfer at the end of the period and, therefore, a higher expected inflation tax next period. The higher tax induces the agent to hold fewer real balances today, which, via the financing constraint, drives up the current price level. This reduces consumption and increases investment. Thus, when \( \delta \) is negative, monetary policy induces a smoother consumption path by counteracting the direct effect of the technology shock on consumption. It also produces a more variable capital stock series by reinforcing the direct effect on investment.

To reproduce the RBC calibration procedures, we estimate the private agent's preference and technology parameters using a stochastic growth economy without money. When monetary policy is irrelevant and preferences and technology are the same as in the true hypothetical model, the equilibrium decision rules reduce to the well-known Levhari and Srinivasan (1969) rules:

\[
\begin{align*}
c_t &= (1 - \alpha \beta) \theta_t k_t^\alpha, \\
k_{t+1} &= \alpha \beta \theta_t k_t^\alpha.
\end{align*}
\]

A calibrator who estimates \( \alpha \) and \( \beta \) using (6) and (7), when the true decision rules are
(4) and (5), will obtain calibrated parameters that are a composite of $\alpha$, $\beta$, the policy parameters, and the technology shock. This is exactly the type of bad inference that Lucas (1976) criticized.

II. The Calibration Experiment

To proceed with our experiment, we generate a target data set using the true monetary model (decision rules (4) and (5)). The calibrator, using a model in which policy is irrelevant (decision rules (6) and (7)), chooses parameter settings for $\alpha$ and $\beta$ so that certain statistics calculated from the model without policy are equal to the values of the statistics calculated in the target data set.

More specifically, the calibrator generates simulated time series from decision rules (6) and (7) and calculates the variances of consumption and capital. He then adjusts $\alpha$ and $\beta$ until the variances in the simulated series are equal to the variances in the target data set. The experiment formalizes calibration as the "simulation estimator" discussed in Lee and Ingram (1990). We do this formal estimation to emphasize that the problem lies in transporting parameters estimated in a model without policy to a model with policy, not in the algorithm used to obtain the initial parameter estimates.

For each of the 100 replications of the experiment, we generate realizations of the technology shock according to its assumed distribution. The true values of the calibrated parameters are $\alpha = 0.36$ and $\beta = 0.96$.\(^9\) To eliminate sampling error, the calibrator is given

\(^9\)The other parameter settings are: the variance of the disturbance to technology, $\sigma^2 = .00721$ and the unconditional mean of the inverse of money growth, $\mu = .4073$. The experiments we conduct consider various settings of the serial correlation parameter for the (continued...)
the realizations of the technology shock that occurred in the actual data set. This ensures that any deviation of the estimates from the true values can be attributed directly to bias in the estimation procedure arising from misspecifying the private agent's decision rules.\textsuperscript{10}

Figure 1 contains the estimation results for $\rho = 0$ and for four values of $\delta$. When $\rho = 0$, there is no persistence in the technology shock. Figure 2 contains the results for $\rho = 0.95$, which is the value Hansen (1985) and other RBC modelers give to the autoregressive coefficient in the technology process. Each figure shows the histograms of the estimates for 100 replications of simulations of length 160, which is approximately the number of observations available in quarterly time series since World War II. Table 1 reports the mean values of the estimated parameters for both values of the serial correlation parameter, $\rho$, and for each of the four values of the policy parameter, $\delta$.

When $\rho = 0$, the estimates of $\alpha$ are decisively biased away from the true value, $\alpha = 0.36$. The bias is positive when $\delta$ is negative and negative when $\delta$ is positive. In all cases, the skewness of the empirical distribution implies that none of the replications produces an estimate equal to the true value. (The estimates for $\beta$ are close to 0.96, so we do not report the histograms for this parameter.)

The results are similar when $\rho$ is increased to 0.95, where we graph the estimates of $\beta$

\textsuperscript{9}(...continued)

\textsuperscript{10}With two free parameters, two target statistics, and no sampling error, the calibrator will choose $\alpha$ and $\beta$ to match the variances of consumption and capital exactly.
only. In that case, as illustrated in Figure 2, some of the estimates of $\beta$ exceed unity. In three cases ($\delta = -0.2$, -0.1, and 0.2), none of the replications produces an estimate equal to the true value. Since the calibrator uses the same random draws of the technology shock that generated the true data, these deviations from truth arise solely from the calibrator's misspecification of how private decisions depend on policy.

III. Some Implications of Bad Parameter Estimates

We now conduct two rational expectations policy experiments that involve changing the responsiveness of monetary policy to the technology shock, as described by $\delta$ in the policy rule. Suppose that policy $\delta_1$ has been in effect historically, and we wish to forecast the savings rate from today onward if policy is changed to $\delta_2$ today.\footnote{We follow the standard practice and assume that the announced policy changes are well-understood and perfectly credible.} It is convenient to express the outcomes of the experiments in terms of the savings rate. The true savings rate function is obtained by dividing decision rule (5) through by output:\footnote{To avoid notational clutter, we suppress the dependence of this function on the parameters $\mu$ and $\rho$, which are not central to the present exposition.}

\begin{equation}
 s(\theta; \alpha, \beta, \delta) = 1 - \frac{\mu + \delta \ln(\theta)}{\mu - \alpha \beta + \delta \ln(\theta)}.
\end{equation}

The exact procedure that Lucas criticized would evaluate the effect of a change in $\delta$ using the decision rules from the estimated model without policy. Since the savings rate from this model is $s = \alpha \beta$ (see equation (7)), it is clear that the predicted savings rate would be the
same under both policies, even though savings would actually change according to (8).

The modern error is more subtle than this traditional error. To forecast the effect of the policy change, the calibrator uses the correct savings function, (8), with the correct value of \( \delta_2 \) inserted. He also uses the calibrated private parameters, \( \hat{\alpha} \) and \( \hat{\beta} \) that were estimated from the model without policy (decision rules (6) and (7)). Thus, the calibrator predicts that the new savings rate function is \( s(\theta_i; \hat{\alpha}, \hat{\beta}, \delta_2). \)

He then forecasts future savings rates by forming forecasts of the future technology shock, the \( \theta_i \)'s.\(^{14}\) Conditional on information available at the time of the contemplated policy change, time \( T \), he forms the predicted time series:

\[
s(E_T^i[\theta_{T+k}^i]; \hat{\alpha}, \hat{\beta}, \delta_2) \quad \text{for } k \geq 1.
\]

An econometrician, who takes a naive forecasting approach and has a time series on the savings rate under the initial monetary policy, \( \{s_t(\delta_1), t=1,2,\ldots,T\} \), might model savings as a univariate autoregressive process. The econometrician estimates:

\[
s_t(\delta_1) = \gamma_0 + \gamma_1 s_{t-1}(\delta_1) + \eta_t, \quad t = 1,2,\ldots,T,
\]

where \( \eta_t \) is a mean zero i.i.d. process with finite variance. The econometrician has no knowledge of the true economic structure. To forecast savings after the change in policy, he

\(^{13}\)Using the incorrect parameter estimators in the correct savings rate function is equivalent to using the correct estimators in some incorrect decision rule. To see this, simply define the new function \( s'(\theta_i; \alpha, \beta, \delta_2) = s(\theta_i; \alpha, \beta, \delta_2) \).

\(^{14}\)We assume the calibrator knows the correct process for the technology shock. This gives the calibrator the benefit of the doubt that he has not miscalibrated this process, as well as the private parameters. Eichenbaum (1990) presents arguments, which are different from ours, for being very uncertain about the standard RBC specification of the process for technology shocks.
simply sets all future $\eta$'s to zero and uses the estimated law of motion for savings, (9), to project savings conditional on the time $T$ observed rate.

Figure 3 shows sample time paths of the actual new savings rate and the predictions produced by the calibrator and the time series econometrician.$^{15}$ The paths are 200 periods long, representing a 50-year forecast horizon for quarterly data. As seen in the figure, the biased calibrated private parameters cause the calibrator to systematically over-predict the savings rate.

Because the contemplated policy change does not alter the steady state savings rate, forecasts from the time series model are unbiased predictors of the savings rate. On average, the naive approach dominates the procedure that transports parameters estimated in the model without policy. The figure shows that having the correct decision rule does not ensure accurate forecasts of policy effects if the private parameters have been poorly estimated.

The second experiment exploits the fact that the private agent's savings decision depends on the expectation of future monetary policy, conditional on current information. Therefore, the predicted change in the savings rate, due to an unanticipated change in monetary policy today, will depend on the current realization of the technology shock. Figure 4 displays the actual and the calibrator's predicted percent changes in the savings rate induced by a hypothetical change in policy today as a function of the current technology shock.$^{16}$ We assume the forecaster knows today's realized technology shock exactly. The figure shows

$^{15}$In the experiment, the initial monetary policy was $\delta_1 = -0.2$ and the new policy was $\delta_1 = -0.1$. The serial correlation parameter is $\rho = 0.95$.

$^{16}$Policy was $\delta_1 = -0.2$ and is changed to $\delta_2 = 0.2$ today; $\rho = 0.0$. 

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that the bias, given by the vertical distance between the two lines, increases with the magnitude of the technology shock.

IV. Concluding Remarks

Kydland and Prescott's (1982) findings are remarkable because they emerge from an extraordinarily simple economic model. Precisely because of its simplicity, however, the model (including its associated parameter values) cannot be expected to be structural with respect to a broad class of changes in the environment. It's not hard to think of changes in the environment that would force the modeler to alter the parameter values used. For example, if another consumption good were introduced, it wouldn't be sensible to transport Kydland and Prescott's preference and technology parameters to the new model. Adding policy variables to Kydland and Prescott's setup is a conceptually identical exercise: Their model with policy is a different model (including its associated parameter values).

The practice of transporting parameters that were estimated in one model to another, completely different model, is widespread and growing. We have argued that the practice is also logically flawed. Many of the calibrated parameters of private behavior that RBC modelers treat as invariant are actually reduced-form parameters like those prevalent in traditional Keynesian models. Exercises that hold these parameters fixed when policy changes repeat the errors of traditional econometric analysis that Lucas (1976) so neatly pointed out. Our example shows that there is no reason to believe the modern errors are

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17This point is not new. Researchers at the Cowles Commission showed that certain aspects of economic behavior will be structural (or invariant) for only particular classes of changes in the environment [see, for example, Marschak (1953) and Hurwicz (1962)].
smaller than those that Lucas and Sargent (1978) argue contributed to the poor economic policies (and consequent performance) of the American economy in the 1970’s.

In our example, calibration procedures result in biased estimators of private behavior. We cannot tell whether the biases that show up in actual calibration exercises tend to over- or underpredict policy effects, since the direction of bias depends on what policy behavior was during the estimation period. Even though the biases appear to be "small," they can have profound consequences for forecasting policy effects. Surprisingly, large mistakes emerge even in the example in this paper where monetary policy is relatively unimportant for private decisions. Still more surprising, by imposing severe (and false) restrictions on private behavior, the practice of transporting parameters may do worse than atheoretical time series models.¹⁸

¹⁸This is one aspect of Sims’s (1982, p. 334) general contention that "the positive program of rational expectations econometrics . . . reproduces the main faults of standard policy evaluation in exaggerated form."
References


Eichenbaum, Martin, "Real Business Cycle Theory: Wisdom or Whimsy?" unpublished manuscript, Northwestern University, July 1990.


Watson, Mark W., "Measures of Fit for Calibrated Models," unpublished manuscript, Northwestern University, September 1990.
Appendix: Solving the Model

The constraints that define the solution to the optimization problem are:

(A.1) \( p_c c_t \leq m_t / M_t \)

(A.2) \( p_t (c_t + k_{t+1}) + m_{t+1} / M_t + 1 - 1 / z_{t+1} - m_t / M_t - p_t \theta_i k^\alpha_t = 0 \)

(A.3) \( 1 / p_c c_t = \lambda_{1t} + \lambda_{2t} \)

(A.4) \( \lambda_{1t} p_t = \alpha k_{t+1}^{\alpha-1} E_t \{ \lambda_{1t+1} p_{t+1} + \lambda_{2t+1} \} \)

(A.5) \( \lambda_{2t} / M_t = \beta E_t \{ (\lambda_{1t+1} + \lambda_{2t+1}) / M_{t+1} \} \).

(A.1) is the cash-in-advance constraint and (A.2) is the budget constraint. \( \lambda_{1t} \) is the Lagrange multiplier associated with the budget constraint at \( t \) and \( \lambda_{2t} \) is the Lagrange multiplier associated with the cash-in-advance constraint at \( t \). In equilibrium, (A.1) implies that \( p_c c_t = 1 \). Combining this with (A.3) and (A.5) yields an expression for \( \lambda_{1t} \):

(A.6) \( \lambda_{1t} = \beta E_t \{ z_{t+1} \} \).

Combining \( p_c c_t = 1 \) with (A.4) and (A.6) implies:

(A.7) \( p_t E_t \{ z_{t+1} \} = \alpha k_{t+1}^{\alpha-1} E_t \{ \theta_{t+1} p_{t+1} E_{t+1} \{ z_{t+2} \} \} \).

The budget constraint yields the feasibility condition:

(A.8) \( c_t + k_{t+1} = \theta_i k^\alpha_t \)

The solution to this problem is completely characterized by (A.1), (A.6), (A.7), and (A.8). Following Coleman (1989), we guess and verify that the solution to this problem is a set of functions of the form:

(A.9) \( c_t = \gamma \theta_i k^\alpha_t \)

(A.10) \( k_t = (1 - \gamma) \theta_i k^\alpha_t \)

(A.11) \( p_t = [\gamma \theta_i k^\alpha_t]^{-1} \)
(A.12) \[ m_t = M_t \]

(A.13) \[ \gamma_t = \lambda_{t,t}/\omega_t \]

(A.14) \[ \omega_t = \lambda_{t,t} + \alpha \beta E_t \omega_{t+1} \]

(A.15) \[ \lambda_{t,t} = \beta E_t \{ z_{t+1} \} \]

The policy authority chooses the stochastic process \( \{ z_{t+1} \}_{t=0}^{\infty} \) according to the rule in equation (3) in the text. (A.1) is satisfied since \( m_t = M_t \) and \( c_t p_t = 1 \). Adding together \( c_t \) and \( k_t \) shows that (A.8) is satisfied. Finally, substitution of the expression for \( p_t \) in (A.7), and using the expressions for \( \gamma_t \), \( \omega_t \) and \( \lambda_{t,t} \) shows that (A.7) holds.

Deriving the optimal decision rules in equations (4) and (5) is straightforward. Use the policy rule in (3) to obtain the expression for \( \lambda_{t,t} = \mu + \delta \ln(\theta_t) \). Since by assumption, \(|\alpha \beta| < 1\), (A.14) is a contraction. Solving this in the usual fashion, and using the expression for \( \lambda_{t,t} \), we obtain the dependence of \( \omega_t \), and hence the marginal propensity to consume out of current income, on expected future money growth:

\[ \omega_t = \beta \sum_{j=0}^{\infty} (\alpha \beta)^j E_t z_{t+j+1} \]

(A.16)

\[ = \frac{\mu}{1 - \alpha \beta} + \frac{\delta \ln(\theta_t)}{1 - \alpha \beta \rho} \]

Combining this expression for \( \omega_t \) with the solution for \( \lambda_{t,t} \), and substituting the result into (A.9) and (A.10) gives equations (4) and (5) in the text.
Table 1. Calibrated Private Parameters. True values are $\alpha = 0.36$ and $\beta = 0.96$.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\rho = 0.0$</th>
<th>$\rho = 0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\alpha}$</td>
<td>$\hat{\beta}$</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.462</td>
<td>0.940</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.410</td>
<td>0.958</td>
</tr>
<tr>
<td>0.1</td>
<td>0.342</td>
<td>0.943</td>
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<tr>
<td>0.1</td>
<td>0.317</td>
<td>0.974</td>
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Note: The reported estimates of $\hat{\alpha}$ and $\hat{\beta}$ are the mean values of the estimates from 100 replications of the data set. $\rho$ is the serial correlation parameter for the technology shock and $\delta$ is the response of (inverse) money growth to the technology shock. The calibrated parameters coming from fitting the model without money to a data set in which monetary policy affects private decisions.
Figure 1. Estimates of Alpha
\( \rho = 0.0, 100 \) replications

- **\( \delta = -0.2 \)**
  - True alpha
  - Mean = 0.462

- **\( \delta = -0.1 \)**
  - True alpha
  - Mean = 0.410

- **\( \delta = 0.1 \)**
  - True alpha
  - Mean = 0.342

- **\( \delta = 0.2 \)**
  - True alpha
  - Mean = 0.317
Figure 2. Estimates of Beta
rho = 0.95, 100 replications

delta = -2

true beta

mean = 0.978

delta = -1

true beta

mean = 0.970

delta = 0.1

true beta

mean = 0.951

delta = 0.2

true beta

mean = 0.942
Figure 3. Actual and Predicted Monetary Policy Effects
Effect of a policy change from $\delta = -0.2$ to $\delta = -0.1$ when $\rho = 0.95$
Figure 4. Predicted Effects of a Change in Monetary Policy
Effect on savings of a change from $\delta = -0.2$ to $\delta = 0.2$ when $\rho = 0.0$

Percentage Change in Savings Rate

Log of Technology Shock

Predicted change (using calibrated parameters)
Actual change (using true parameters)
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