A NEW KEYNESIAN MODEL

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1. The Model

This is a standard New Keynesian model with monopolistic competition and sticky prices in goods markets.\(^1\) We extend the model to include lump-sum taxes and a potentially non-trivial fiscal financing decision.

1.1. Households. The representative household chooses \(\{C_t, N_t, M_t, B_t\}\) to maximize

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ C_{t+i}^{1-\sigma} \frac{1-\eta}{1+\eta} + \delta \frac{(M_{t+i}/P_{t+i})^{1-\kappa}}{1-\kappa} \right]
\]

with \(0 < \beta < 1, \sigma > 0, \eta > 0, \kappa > 0, \chi > 0\) and \(\delta > 0\). \(C_t\) is a composite consumption good that combines the demand for the differentiated goods, \(c_{jt}\), using a Dixit and Stiglitz (1977) aggregator:

\[
C_t = \left[ \int_0^1 c_{jt}^{\frac{\sigma-1}{\theta}} dj \right]^\frac{\sigma}{\sigma-1}, \theta > 1.
\]

The household chooses \(c_{jt}\) to minimize expenditure on the continuum of goods indexed by the unit interval, leading to the demand functions for each good \(j\)

\[
c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t,
\]

where

\[
P_t \equiv \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}
\]

is the aggregate price level at \(t\).

Using (4), the household’s budget constraint is

\[ C_t + \frac{M_t}{P_t} + E_t \left( Q_{t,t+1} \frac{B_t}{P_t} \right) + \tau_t \leq \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \Pi_t, \tag{5} \]

where \( \tau_t \) is lump-sum taxes/transfers from the government to the household, \( B_t \) is one-period nominal bonds, \( Q_{t,t+1} \) is the stochastic discount factor for the price at \( t \) of one unit of composite consumption goods at \( t+1 \), and \( \Pi_t \) is profits from the firm, which the household owns. The household maximizes (1) subject to (5) to yield the first-order conditions

\[ \chi \frac{N^\sigma_t}{C_t} = \frac{W_t}{P_t}, \tag{6} \]

\[ Q_{t,t+1} = \beta \left( \frac{C_t}{C_{t+1}} \right)^\sigma. \tag{7} \]

If \( R_t \) denotes the risk-free gross nominal interest rate between \( t \) and \( t+1 \), then absence of arbitrage implies the equilibrium condition

\[ E_t \left[ \frac{Q_{t,t+1} P_t}{P_{t+1}} \right] = \frac{1}{R_t}, \tag{8} \]

so the first-order conditions imply that real money balances may be written as

\[ \frac{M_t}{P_t} = \delta^\kappa \left( \frac{R_t}{R_t - 1} \right)^{-1/\kappa} C_t^{\sigma/\kappa}. \tag{9} \]

We assume the government demands goods in the same proportion that households do, so the government’s demand is

\[ g_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} G_t, \tag{10} \]

where \( G_t = \left[ \int_0^1 g_{jt}^{\theta+1} \, dj \right]^{\theta \sigma - 1} \).

1.2. **Firms.** A continuum of monopolistically competitive firms produce goods using labor. Production of good \( j \) is given by

\[ y_{jt} = A_t N_{jt}, \tag{11} \]

where \( A_t \) is an aggregate technology shock, common across firms.

From (3) and (10), the demand curve firm \( j \) faces is given by

\[ y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} Y_t, \tag{12} \]
where \( Y_t \) is defined by

\[
C_t + G_t = Y_t. \tag{13}
\]

Equating supply and demand for individual goods,

\[
A_t N_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} Y_t. \tag{14}
\]

The real profit flow of firm \( j \) at period \( t \) is

\[
\Pi_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{1-\theta} Y_t - \frac{W_t}{P_t} N_{jt}. \tag{15}
\]

Following Calvo (1983), a fraction \( 1 - \varphi \) firms are permitted to adjust their prices each period, while the fraction \( \varphi \) are not permitted to adjust. If firms are permitted to adjust at \( t \), they choose a new optimal price, \( p^*_t \), to maximize the expected discounted sum of profits given by

\[
E_t \sum_{i=0}^{\infty} \varphi^i Q_{t,t+i} \left[ \left( \frac{p^*_t}{P_{t+i}} \right)^{1-\theta} - \Psi_{t+i} \left( \frac{p^*_t}{P_{t+i}} \right)^{-\theta} \right] Y_{t+i}. \tag{16}
\]

where \( Q_{t,t+i} = \beta^k (C_t/C_{t+i})^\sigma \) and profits have been rewritten using (14). \( \Psi_t \) is real marginal cost, defined as

\[
\Psi_t = \frac{W_t}{A_t P_t}. \tag{17}
\]

The first-order condition that determines \( p^*_t \) is\(^2\)

\[
0 = E_t \sum_{i=0}^{\infty} \varphi^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{p^*_t}{P_{t+i}} \right)^{-\theta} + \Psi_{t+i} \left( \frac{p^*_t}{P_{t+i}} \right)^{-1-\theta} \right] \frac{C_{t+i} + G_{t+i}}{P_{t+i}}.
\]

Rearranging,

\[
0 = E_t \sum_{i=0}^{\infty} \varphi^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{p^*_t}{P_{t+i}} \right) + \theta \Psi_{t+i} \right] \frac{C_{t+i} + G_{t+i}}{P_{t+i}} \left( \frac{p^*_t}{P_{t+i}} \right)^{-1-\theta},
\]

\[
0 = E_t \sum_{i=0}^{\infty} \varphi^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{p^*_t}{P_{t+i}} \right) + \theta \Psi_{t+i} \right] (C_{t+i} + G_{t+i}) \left( \frac{1}{p^*_t} \right) \left( \frac{p^*_t}{P_{t+i}} \right)^{-\theta},
\]

which is (18).
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\[ E_t \sum_{i=0}^{\infty} \varphi^i Q_{t,t+i} \left[ (1 - \theta) \left( \frac{p^*_{t+i}}{P_{t+i}} \right) + \theta \Psi_{t+i} \right] \left( \frac{1}{P^*_t} \right) \left( \frac{p^*_{t+i}}{P_{t+i}} \right)^{-\theta} Y_{t+i} = 0 \]  \hspace{1cm} (18)

Using the definition of \( Q_{t,t+i} \) and rearranging, (18) is

\[ \frac{p^*_t}{P_t} = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} (\varphi \beta)^i (Y_{t+i} - G_{t+i})^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} \Psi_{t+i} Y_{t+i}}{E_t \sum_{i=0}^{\infty} (\varphi \beta)^i (Y_{t+i} - G_{t+i})^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} Y_{t+i}}. \]  \hspace{1cm} (19)

Denote (19) as

\[ \frac{p^*_t}{P_t} = \left( \frac{\theta}{\theta - 1} \right) \frac{K_{1t}}{K_{2t}}, \]  \hspace{1cm} (20)

where \( K_{1t} \) denotes the numerator and \( K_{2t} \) denotes the denominator. Note that these two expressions have the following recursive representations:

\[ K_{1t} = (Y_t - G_t)^{-\sigma} \Psi_t Y_t + \varphi \beta E_t K_{1t+1} \left( \frac{P_{t+1}}{P_t} \right)^{\theta} \]  \hspace{1cm} (21)

and

\[ K_{2t} = (Y_t - G_t)^{-\sigma} Y_t + \varphi \beta E_t K_{2t+1} \left( \frac{P_{t+1}}{P_t} \right)^{\theta-1}. \]  \hspace{1cm} (22)

Solving (20) for \( p^*_t \) and using the result in the price index

\[ P_{t-1}^{1-\theta} = (1 - \varphi) \left( p^*_t \right)^{1-\theta} + \varphi P_{t-1}^{1-\theta}, \]  \hspace{1cm} (23)

yields

\[ \pi_t^{\theta-1} = \frac{1}{\varphi} - \frac{1 - \varphi}{\varphi} \left( \mu \frac{K_{1t}}{K_{2t}} \right)^{1-\theta}, \]  \hspace{1cm} (24)

where \( \mu \equiv \theta/(\theta - 1) \).

Note that (20) determines \( p^*_t \) relative to aggregate \( P_t \), implying that we are free to choose a normalization for \( P_t \). Let \( P_t = 1 \) and define

\[ \tilde{K}_{1t} = (Y_t - G_t)^{-\sigma} \Psi_t Y_t P_t^\theta + \varphi \beta E_t \tilde{K}_{1t+1} \]  \hspace{1cm} (25)

and

\[ \tilde{K}_{2t} = (Y_t - G_t)^{-\sigma} Y_t P_t^{\theta-1} + \varphi \beta E_t \tilde{K}_{2t+1}. \]  \hspace{1cm} (26)

Then (20) becomes
\[ p_t^* = \mu \frac{\tilde{K}_{1t}}{K_{2t}}, \]  
and the expression for inflation becomes
\[ \pi_t^{\theta - 1} = \frac{1}{\varphi} - \frac{1 - \varphi}{\varphi} \left( \mu \frac{\tilde{K}_{1t}}{K_{2t}} \right)^{1 - \theta}. \]  

1.3. **Aggregation.** We assume that individual labor services may be aggregated linearly to produce aggregate labor:
\[ N_t = \int_0^1 N_{jt} dj. \]  
Linear aggregation of individual market clearing conditions implies
\[ A_t N_t = \Delta_t Y_t, \]  
where \( \Delta_t \) is a measure of relative price dispersion defined by
\[ \Delta_t = \int_0^1 \left( \frac{p_{jt}}{P_t} \right)^{-\theta} dj. \]  
Now the aggregate production function is given by
\[ Y_t = \frac{A_t}{\Delta_t} N_t. \]  
It is natural to define aggregate profits as the sum of individual firm profits,
\[ \Pi_t = \int_0^1 \Pi_{jt} dj. \]  
(15) and (33) imply that the aggregate profit flow can be expressed as
\[ \Pi_t = Y_t - \frac{W_t}{P_t} N_t. \]  
Substituting (34) into the household’s budget constraint, (5), and combining the result with the government’s budget constraint, yields the aggregate resource constraint
\[ \frac{A_t}{\Delta_t} N_t = C_t + G_t. \]  
We now derive the law of motion of relative price dispersion. From the definition of price dispersion, (31) and (23), relative price dispersion evolves according to
\[ \Delta_t = (1 - \varphi) \left( \frac{p_t^*}{P_t} \right)^{-\theta} + \varphi \pi_t^\theta \Delta_{t-1}, \] (36)

where \( \pi_t = P_t/P_{t-1} \).

1.4. **Policy Specification.** Monetary policy is assumed to follow a standard Taylor-type rule

\[ R_t = \alpha_0 + \alpha_1 \pi_t + \alpha_2 (Y_t - Y_t^*) + \varepsilon_t^{MP} \] (37)

and taxes are permitted to respond to the state of government indebtedness

\[ \tau_t = \gamma_0 + \gamma_1 \frac{B_{t-1}}{P_{t-1}} + \varepsilon_t^{\tau}, \] (38)

where the \( \varepsilon \)'s are i.i.d. random variables.

The processes for \( \{G_t, \tau_t, M_t, B_t\} \) must satisfy the government budget identity

\[ G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - R_{t-1}B_{t-1}}{P_t}. \] (39)

1.5. **Stochastic Specification.** The remaining exogenous processes are assumed to obey univariate AR(1) processes with i.i.d. errors. Government purchases obey

\[ G_t = (1 - \rho_G) \bar{G} + \rho_G G_{t-1} + \varepsilon_t^G, \] (40)

where \( \varepsilon_t^G \) has bounded support \([\underline{\varepsilon}^G, \overline{\varepsilon}^G]\) to ensure \( C_t > 0 \) for all \( t \) in all states. Technology obeys

\[ \log(A_t) = (1 - \rho_A) \bar{A} + \rho_A \log(A_{t-1}) + \varepsilon_t^A, \] (41)

where \( \varepsilon_t^A \sim N(0, \sigma_A^2) \).

**References**


