1. A General Framework

This section describes a general equilibrium model of fiscal finance in the spirit of Chari, Jones, and Manuelli (1995). Like Sargent and Wallace (1981), we model only the fiscal financing role of monetary policy.

As in Heckman (1976) and Lucas (1988), the household supplies effective labor given by $hn$, where $h$ is human capital and $n$ is hours worked. Gross output is produced with the constant returns to scale technology $f(k, hn)$, where $k$ is physical capital. We denote gross output at $t$, $f(k_{t-1}, h_{t-1}n_t)$, by $f(t)$. Output net of undepreciated capital, $y_t$, satisfies the accounting identity:

$$c_t + x_k t + x_h t + g_t = y_t,$$

where $x_k$ and $x_h$ are investment in physical and human capital, and $g$ is government purchases. The two capital stocks evolve according to

$$k_t = x_k t + d^k(k_{t-1}, h_{t-1}n_t)$$

$$h_t = x_h t + d^h(k_{t-1}, h_{t-1}n_t),$$

where $d(\cdot, \cdot)$ represents undepreciated capital, which we denote by $d(t)$.

1.1. Firms. There are two representative firms that rent factors of production from households and sell their outputs back to households. The goods producing firm rents $k$ at rental rate $r$ and hires effective labor $hn$ at wage rate $w$ to solve

$$\max_{k_{t-1}, h_{t-1}, n_t} D_G t = f(k_{t-1}, h_{t-1}n_t) - r_t k_{t-1} - w_t h_{t-1}n_t.$$

The transactions services producing firm hires labor $l$ at wage rate $w_T$ to solve
\[
\max_{l_t} D_{T_t} = P_{T_t} T(l_t) - w_{T_t} l_t, \tag{5}
\]
with \( P_T \) the price of transactions services relative to consumption goods. Production functions are strictly concave and differentiable. Both firms behave competitively, taking all prices as given.

1.2. **Households.** The representative household owns the firms and pays taxes on capital and labor income. It has disposable income

\[
I_t = (1 - \tau^k_t)r_t k_{t-1} + (1 - \tau^n_t)w_t h_{t-1} n_t + \tau^k_t d^k_t(t) + \tau^n_t d^n_t(t) + D_{Gt} + w_{T_t} l_t + D_{T_t} + z_t, \tag{6}
\]
where \( z_t \) is transfer payments from the government. Because \( r_t \) and \( w_{T_t} \) are the returns on gross capital assets, we add back the tax levied against income derived from undepreciated capital stocks.

The household’s expenditures on consumption and new capital goods at date \( t \) must be financed with real money balances carried over from the previous period, \( M_{t-1}/P_t \), or with transactions services, \( T_t \), to satisfy the constraint

\[
\frac{M_{t-1}}{P_t} + T_t (c_t + x_t) \geq c_t + x_t, \tag{7}
\]
where \( x_t = x_{kt} + x_{ht} \) is total investment in capital. Transactions services can be thought of as a clearinghouse, money market mutual funds, or credit cards and the labor supplied to that sector reflects resources used in producing the services. Those resources should not be construed as “labor supply” in the same sense that \( n_t \) is.

The household consists of a worker-shopper pair. Each member of the household is endowed with a unit of time. The worker supplies \( n_t \) units of time to the goods producing firm and the shopper supplies \( l_t \) units of time to the transactions services producing firm; the two kinds of labor are not substitutable. The household solves

\[
\max_{\{c_t, l_t, n_t, T_t, M_t, B_t, k_t, h_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t, n_t), \quad 0 < \beta < 1, \tag{8}
\]
where \( 1 - n_t \) is leisure for the worker and \( 1 - l_t \) is leisure for the shopper, subject to (7), the budget constraint

\[
c_t + k_t + h_t + \frac{M_t + B_t}{P_t} + P_{T_t} T_t \leq I_t + \frac{M_{t-1} + (1 + i_{t-1}) B_{t-1}}{P_t}, \tag{9}
\]
the evolution of the capital stocks, (2) and (3), and \( \mathbf{0} \leq n_t, l_t \leq 1 \). Future government policy is the sole source of uncertainty; the operator \( E \) in (8) denotes equilibrium
expectations of private agents over future policy. \( B_t \) is purchases of nominal one-period government debt issued at \( t \) and \( i_t \) is the net nominal interest rate on debt issued at \( t \) and due at \( t + 1 \). The household starts with initial assets \( k_{-1} > 0 \), \( h_{-1} > 0 \), and \( M_{-1} + (1 + i_{-1})B_{-1} > 0 \).

The government finances expenditures on goods, \( g_t \), and transfer payments, \( z_t \), by levying taxes, issuing new debt, and creating new money to satisfy the restraint:

\[
\tau_t^k \left[ r_t k_{t-1} - d^k(t) \right] + \tau_t^n \left[ w_t h_{t-1} n_t - d^h(t) \right] + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1 + i_{t-1})B_{t-1}}{P_t} = g_t + z_t. \tag{10}
\]

2. An Intertemporal Example

To analyze the potential for countercyclical policies to be counterproductive, we specialize the model to obtain the analytical solutions required to completely specify how the current equilibrium depends on current and expected policies. The specialization highlights the expectations channel through which countercyclical policies affect the economy. It is also convenient to consider future policies that are constant but may differ from current policies. This allows us to derive analytically the dynamic linkages between current and future policies in equilibrium.

2.1. Assumptions. Assume labor is supplied inelastically to goods production (as in Jones, Manuelli, and Rossi 1993) and set \( \tau_t^k = \tau_t^n = \tau_t \). Further assume the following functional forms:

\[
f(k_{t-1}, h_{t-1}) = k_{t-1}^{\sigma} h_{t-1}^{1-\sigma}, \quad 0 < \sigma < 1, \tag{11}
\]

\[
T(l_t) = 1 - (1 - l_t)^\alpha, \quad \alpha > 1, \tag{12}
\]

\[
U(c_t, l_t, 1) = \log(c_t) + \gamma \log(1 - l_t), \quad \gamma > 0, \tag{13}
\]

with \( \gamma/\alpha < 1 \),

\[
d^k(k_{t-1}, h_{t-1}) = \sigma(1 - \delta_k) f(k_{t-1}, h_{t-1}), \tag{14}
\]

\[
d^h(k_{t-1}, h_{t-1}) = (1 - \sigma)(1 - \delta_h) f(k_{t-1}, h_{t-1}), \tag{15}
\]

with \( 0 \leq \delta_k, \delta_h \leq 1 \).

There are no exogenous “shocks” to technology, preferences, or endowments. Realizations of current and expected policy variables are the sole source of variation.
2.2. Solution\textsuperscript{2}. With these restrictions the equilibrium can be defined in terms of policy expectations, initial assets, and current government policy.

Our results focus on two aspects of portfolio choice: investment and money demand. We express the choices in terms of their stationary counterparts—investment as a share of expenditures and velocity.

Combining the Euler equations for physical and human capital yields a solution for the total capital stock:

\[
k_t + h_t = \left(1 - \frac{1}{\eta_t}\right) \left(1 - \delta s^g_t\right) f(k_{t-1}, h_{t-1}),
\]

where \( s^g_t \equiv g_t / y_t\), \( \delta = 1 - \sigma (1 - \delta_k) - (1 - \sigma)(1 - \delta_h) \), and

\[
\eta_t \equiv E_t \sum_{i=0}^{\infty} \beta^i \gamma \left[1 - \frac{1}{\alpha} \left(1 - \frac{1}{s^g_t + i + 1}\right)\right], \quad d_i^\mu \equiv \prod_{j=0}^{i-1} \frac{1 - \delta \tau_{t+j+1}}{1 - \delta s^g_t}, \quad d_0^\mu \equiv 1.
\]

Symmetry between the two capital stocks implies that in equilibrium \( h_t / k_t = (1 - \sigma) / \sigma \). Using this proportionality together with the solution (16) and the laws of motion for capital, we derive the investment ratio:

\[
\frac{x_{kt}}{c_t + x_{kt}} = \frac{\sigma \left[\left(1 - \frac{1}{\eta_t}\right) \left(1 - \delta \sigma s^g_t\right) - (1 - \delta_k)\right]}{(1 - \sigma) \left(1 - \delta \sigma s^g_t\right) + \sigma \left[(1 - \delta \sigma s^g_t) - (1 - \delta_k)\right]}.
\]

Solving the Euler equation for money yields

\[
(1 - T_t) \left[\frac{c_t + x_t}{c_t} - \frac{\gamma}{\alpha}\right] = \frac{\mu_t}{\rho_t},
\]

where \( \rho_t \equiv M_t / M_{t-1} \) and

\[
\mu_t \equiv \beta^\gamma E_t \sum_{i=0}^{\infty} \beta^i d_i^\mu, \quad d_i^\mu \equiv \prod_{j=0}^{i-1} \frac{1}{\rho_{t+j+1}}, \quad d_0^\mu \equiv 1.
\]

An expression for velocity comes from using the solution (19) together with (7) and other expressions:

\textsuperscript{1}This specification of depreciation is non-standard. We employ it to obtain an analytical solution, which allows clearer interpretation. In future work it will be important to check the robustness of results to the standard treatment of depreciation as proportional to the level of the capital stock.

\textsuperscript{2}Appendix A derives the model solution.
\[ v_{kt} \equiv \frac{c_t + x_{kt}}{M_t/P_t} = \frac{(1 - \sigma)\frac{\mu_t}{\eta_t} (1 - \delta_s s_t^g) + \sigma [(1 - \delta_s s_t^g) - (1 - \delta_k)]}{(1 - \sigma)(1 - s_t^g)}, \tag{21} \]

where

\[ \Delta_t \equiv \frac{\mu_t}{\left(\frac{\delta_s (1-s_t^g)}{1-\delta_s s_t^g}\right) \eta_t - \frac{\gamma}{\alpha}}. \tag{22} \]

2.3. Interpretation. We characterize the equilibrium at date \( t \) in terms of policy expectations functions \((\mu_t, \eta_t)\), current government claims, \( s_t^g \), and initial assets, \((k_{t-1}, h_{t-1}, M_{t-1}, (1 + i_{t-1})B_{t-1})\). \((\mu_t, \eta_t)\), which summarize the information agents need to form rational expectations about the equilibrium of the economy, capture the portfolio balance effects of expected policies.

\( \mu \) is the marginal value of real money balances and is ubiquitous in dynamic monetary models. All else equal, changes in \( \mu \) imply changes in expected inflation and the rate of return on money holdings. Expectation of a higher rate of money growth—and therefore of seigniorage revenues—depreciates the value of money, lowers, \( \mu \), and induces substitution away from money.

\( \eta \) captures two interdependent impacts of expected policies. One impact is a direct tax distortion, which alters the private return on real assets. To isolate this effect, consider the impact of higher expected future taxes, holding future money growth and government spending shares fixed. Further suppose that debt is identically zero and, in order to focus on substitution effects, that the revenues collected through higher distorting taxes are rebated lump sum to households. Higher future taxes reduce the expected return on investment and induce agents to substitute from capital to consumption. A lower expected return on capital also induces substitutions into nominal assets, including money.

A second impact comes from \( \eta \)'s summary of the composition of expected fiscal financing in terms of the relative sizes of the real and inflation tax bases. This trade-off can be seen heuristically from an alternative expression for the terms in \((1 - \tau)/(1 - s^g)\) that appear in the definition of \( \eta \) in (62). A transformation of the government budget constraint yields:

\[ \frac{1 - \tau_t}{1 - s_t^g} = 1 + \frac{(M_t - M_{t-1} + B_t - (1 + i_{t-1})B_{t-1})/P_t}{(1 - s_t^g) f(t)}, \quad t \geq 0. \tag{23} \]

\(^3\)For simplicity we show this for the case of complete depreciation of capital and no government transfers.
Terms in \((1 - \tau)/(1 - s^g)\) reflect the fraction of private resources absorbed by the acquisition of new nominal liabilities issued by the government. Higher \(\eta\) indicates an expected shift in future financing that expands the inflation tax base and contracts the real tax base. By reflecting the relative sizes of the two tax bases, changes in \(\eta\) generate an expected inflation effect that is not embedded in the nominal interest rate.

Expression (18) shows that the total impact of countercyclical fiscal policies consists of a direct effect, associated with fixed \(\eta\), and an amplification effect, due to induced changes in \(\eta\). Countercyclical policy raises \(s^g_t\) when the economy contracts. Hold expected policies fixed initially. Expansionary fiscal policy has a direct (negative) effect on investment, and the elasticity of that direct effect depends on \(\eta_t\), the index of expected policies being held fixed. Direct effects arise because an increase in \(s^g_t\) reduces current disposable income; how that reduction gets apportioned between consumption and investment depends on \(\eta_t\). A lower value of \(\eta\) reflects an expectation of either higher taxes or lower government spending. Lower \(\eta\) raises the elasticity of equilibrium investment with respect to current government spending.

A second effect of countercyclical fiscal policies may arise. If higher \(s^g_t\) induces agents to change their expectations of future tax rates or government spending shares, then the direct effects may get amplified. Suppose that debt-financed cyclicals increases in spending create a recognition that future taxes must rise. This reduces \(\eta_t\), amplifying the reduction in investment that higher \(s^g_t\) entails.

Expected policies may increase velocity in two ways. First, higher expected money growth reduces the return on real balances (lower \(\mu\)) and induces substitution out of money into transactions services. Second, lower expected taxes or higher expected government spending (higher \(\eta\)) induces substitution out of nominal assets into real assets.

2.4. Equilibrium Expectations. Equilibrium requires that current and future policies satisfy the government’s budget constraint and that agents’ expectations of policy are consistent with equilibrium. This creates interactions among current and future policies, whose characterization is a novel feature of this paper.

We focus on circumstances in which the economy is in a stationary equilibrium in the future (dates \(s > t\)), but starts from some other position at date \(t\). Assume future policies are constant:

\[
\rho_{t+j} = \rho_F, \quad \tau_{t+j} = \tau_F, \quad s^g_{t+j} = s^g, \quad s^z_{t+j} = s^z, \quad j > 0,
\]

(24)

where \(s^z_t = z_t/y_t\).
The government budget constraint can be expressed entirely in terms of current and expected policies. In period $t$ the constraint is

$$\left[ \frac{\rho_t - 1}{\rho_t} + \frac{B_t}{M_t} - \frac{(1 + i_{t-1})}{\rho_t} \cdot \frac{B_{t-1}}{M_{t-1}} \right] \Delta_t = \frac{s_{t}^g + s_{t}^z - \tau_t}{1 - s_{t}^g}. \quad (25)$$

Given expectations of policy embedded in $\Delta_t$ and initial government indebtedness as summarized by $(1 + i_{t-1})B_{t-1}/M_{t-1}$, (25) reports equilibrium trade-offs among current policies.

We now derive equilibrium trade-offs among future policies given the state of government indebtedness. Shift the timing of (25) forward one period and assume future interest liabilities are correctly anticipated at $t$ by substituting the expression for equilibrium $i_t$. For simplicity, assume the bond-money ratio is constant at $(B/M)_F$ in the stationary equilibrium. Re-labeling variables dated $t + 1$ with an “$F$” subscript and imposing equilibrium yields

$$\Delta_t = \left[ \frac{s_{F}^g + s_{F}^z - \tau_F}{1 - s_{F}^g} \right] \cdot \frac{1}{\left[ \frac{B_F}{M_F} - \frac{1}{\beta} \left( \frac{B_F}{M_F} \right)_t + \left( \frac{B_{F-1}}{M_{F-1}} \right) \right]}. \quad (26)$$

Given government indebtedness carried into the future, as summarized by $(B/M)_t$, (26) describes the trade-offs among future policies that are consistent with fixed $\Delta_t$ being an equilibrium.

Trade-offs between (25) and (26) determine the interactions between current policies and expectations of future policies. Any change in policy at $t$ that requires a change in $\Delta_t$ must be accompanied by a change in policy at date $F$ that is consistent with the new values of $\Delta_t$ and the new level of government liabilities, $(B/M)_t$.

In addition to expressions (18) and (21), we report simulated paths for two other variables that define investment and velocity more broadly. The additional variables are the investment-output ratio, where investment includes physical and human capital investment,

$$\frac{x_t}{y_t} = \left[ \frac{\delta_{\sigma} (1 - s_t^y) - \frac{1}{\eta_t} (1 - \delta_{\sigma} s_t^y)}{\delta_{\sigma}} \right], \quad (27)$$

and the income velocity of money

$$v_{yt} \equiv \frac{y_t}{M_t/P_t} = \frac{1}{\Delta_t (1 - s_t^y)}. \quad (28)$$
A. First-Order Conditions. The first-order conditions for the two kinds of firms yield

\[ r_t = f_k(t), \]  
\[ w_t &= f_h(t), \]  
\[ w_t h_{t-1} = f_n(t), \]  
\[ w_{Tt} = P_{Tt} T'(l_t). \]

Let \( \varphi \) be the lagrange multiplier on the household’s budget constraint and \( \lambda \) be the multiplier on the finance constraint. The household’s first-order conditions are:

\[ c_t : \quad U_c(t) - \varphi_t - \lambda_t (1 - T_t) = 0 \]  
\[ l_t : \quad U_l(t) + \varphi_t w_{Tt} = 0 \]  
\[ n_t : \quad U_n(t) + \varphi_t \left[ (1 - \tau^n_t) w_t h_{t-1} + \tau^n_t d^n_k(t) + \tau^n_t d^n_h(t) \right] + \lambda_t (1 - T_t) \left[ d^n_k(t) + d^n_h(t) \right] = 0 \]  
\[ T_t : \quad -\varphi_t P_{Tt} + \lambda_t (c_t + x_t) = 0 \]  
\[ M_t : \quad -\frac{\varphi_t}{P_t} + \beta E_t \left[ \frac{\varphi_{t+1} + \lambda_{t+1}}{P_{t+1}} \right] = 0 \]  
\[ B_t : \quad -\frac{\varphi_t}{P_t} + \beta (1 + i_t) E_t \frac{\varphi_{t+1}}{P_{t+1}} = 0 \]  
\[ k_t : \quad -\varphi_t - \lambda_t (1 - T_t) + \beta E_t \left\{ \varphi_{t+1} \left[ (1 - \tau^n_{t+1}) w_{t+1} h_{t+1} + \tau^n_{t+1} d^n_k(t + 1) + \tau^n_{t+1} d^n_h(t + 1) \right] + \lambda_{t+1} (1 - T_{t+1}) \left[ d^n_k(t + 1) + d^n_h(t + 1) \right] \right\} = 0 \]  
\[ h_t : \quad -\varphi_t - \lambda_t (1 - T_t) + \beta E_t \left\{ \varphi_{t+1} \left[ (1 - \tau^n_{t+1}) w_{t+1} h_{t+1} + \tau^n_{t+1} d^n_k(t + 1) + \tau^n_{t+1} d^n_h(t + 1) \right] + \lambda_{t+1} (1 - T_{t+1}) \left[ d^n_k(t + 1) + d^n_h(t + 1) \right] \right\} = 0 \]
A.2. **Some Accounting.** The following accounting identities are useful in solving the model. Total goods must equal gross assets:

\[ c_t + k_t + h_t + g_t = f(k_{t-1}, h_{t-1} n_t), \]

\[ y_t = f(k_{t-1}, h_{t-1} n_t) - d^k(k_{t-1}, h_{t-1} n_t) - d^h(k_{t-1}, h_{t-1} n_t), \]

\[ c_t + k_t + h_t = (1 - \delta_s s_t^q) f(t), \]

and define the share-weighted average depreciation rate as

\[ \delta_s = 1 - \sigma(1 - \delta_k) - (1 - \sigma)(1 - \delta_h). \]

From (42) and (60),

\[ y_t = \delta_s f(t), \]

\[ c_t + x_t = (1 - s_t^q) y_t = \delta_s (1 - s_t^q) f(t). \]

Define

\[ s_t = \frac{x_t}{c_t + x_t}, \]

and

\[ \tilde{s}_t = \frac{k_t + h_t}{c_t + k_t + h_t} = \frac{k_t + h_t}{(1 - \delta_s s_t^q)f(t)}, \]

so the relationship between \( s_t \) and \( \tilde{s}_t \) is

\[ \frac{1}{1 - s_t} = \frac{\delta_s (1 - s_t^q)}{1 - \delta_s s_t^q} \cdot \frac{1}{1 - \tilde{s}_t}. \]

\[ \frac{f(t)}{c_t} = \frac{1}{1 - \delta_s s_t^q} \cdot \frac{1}{1 - \tilde{s}_t} = \frac{1}{\delta_s (1 - s_t^q)} \cdot \frac{1}{1 - s_t}. \]

\[ \frac{f(t)}{c_t + x_t} = \frac{1}{\delta_s (1 - s_t^q)}. \]
A.3. **Solving the Model.** The lagrange multipliers are

\[ \varphi_t = \frac{1}{c_t} - \frac{\gamma/\alpha}{c_t + x_t}, \]  

\[ \lambda_t = \frac{\gamma/\alpha}{(1 - T_t)(c_t + x_t)}. \]  

The Euler equation for physical capital, (39) can be reduced to

\[ \frac{k_t}{c_t} = \sigma \beta E_t \left\{ \left[ \frac{1 - \delta_s T_{t+1}}{1 - \delta_s S_{t+1}} \right] \frac{1}{1 - s_{t+1}} - \frac{\gamma}{\alpha} \left( 1 - \frac{T_{t+1}}{s_{t+1}} \right) \right\} \]  

\[ \frac{h_t}{c_t} = \sigma \beta E_t \left\{ \left[ \frac{1 - \delta_s T_{t+1}}{1 - \delta_s S_{t+1}} \right] \frac{1}{1 - s_{t+1}} - \frac{\gamma}{\alpha} \left( 1 - \frac{T_{t+1}}{s_{t+1}} \right) \right\}. \]  

In the special case on which we focus, where \( \tau^k = \tau^n = \tau \), (54) and (55) further simplify to

\[ \frac{k_t}{c_t} = \sigma \beta E_t \left[ \frac{1 - \delta_s T_{t+1}}{1 - \delta_s S_{t+1}} \right] \frac{1}{1 - s_{t+1}} - \frac{\gamma}{\alpha} \left( 1 - \frac{T_{t+1}}{s_{t+1}} \right) \]  

\[ \frac{h_t}{c_t} = (1 - \sigma) \beta E_t \left[ \frac{1 - \delta_s T_{t+1}}{1 - \delta_s S_{t+1}} \right] \frac{1}{1 - s_{t+1}} - \frac{\gamma}{\alpha} \left( 1 - \frac{T_{t+1}}{s_{t+1}} \right). \]  

Note that along an equilibrium growth path, (56) and (57) imply

\[ \frac{k_t}{h_t} = \frac{\sigma}{1 - \sigma}. \]  

Adding (56) and (57) yields a difference equation in \( \tilde{s} \):

\[ \frac{1}{1 - \tilde{s}_t} = \beta E_t \left[ \frac{1 - \delta_s T_{t+1}}{1 - \delta_s S_{t+1}} \right] \frac{1}{1 - \tilde{s}_{t+1}} + E_t \left[ 1 - \frac{\beta \gamma}{\alpha} \left( 1 - \frac{T_{t+1}}{s_{t+1}} \right) \right], \]  

where

\[ \delta_s = 1 - \sigma(1 - \delta_k) - (1 - \sigma)(1 - \delta_h). \]  

The solution to this difference equation is

\[ \frac{1}{1 - \tilde{s}_t} = \eta_t, \]
where
\[
\eta_t = E_t \sum_{i=0}^{\infty} \beta^i d^n_i \left[ 1 - \frac{\gamma}{\alpha} \left( 1 - \frac{\tau_{t+i+1}}{1 - s_{t+i+1}} \right) \right], \quad d^n_i = \prod_{j=0}^{i-1} \frac{1 - \delta \sigma \tau_{t+j+1}}{1 - \delta \sigma s_{t+j+1}}, \quad d^n_0 = 1.
\] (62)

Convergence of (59) imposes restrictions on the policy processes, \( \{\tau_t, s_t^g\} \), such that
\[
\lim_{k \to \infty} \beta^k E_t \prod_{j=1}^{k} \frac{1 - \delta \sigma \tau_{t+j}}{1 - \delta \sigma s_{t+j}^g} \frac{1}{1 - \delta t+k} = 0.
\] (63)

The Euler equation for money yields the difference equation
\[
(1 - T_t) \left[ \frac{1}{1 - s_t} - \frac{\gamma}{\alpha} \right] = \beta E_t \left\{ (1 - T_{t+1}) \left[ \frac{1}{1 - s_{t+1}} - \frac{\gamma}{\alpha} \right] + \frac{\gamma}{\alpha} \right\},
\] (64)
whose solution is
\[
(1 - T_t) \left[ \frac{1}{1 - s_t} - \frac{\gamma}{\alpha} \right] = \frac{\mu_t}{\rho_t},
\] (65)
where \( \rho_t \equiv M_t/M_{t-1} \) and
\[
\mu_t \equiv \beta E_t \sum_{i=0}^{\infty} \beta^i d^\mu_i, \quad d^\mu_i \equiv \prod_{j=0}^{i-1} \frac{1}{\rho_{t+j+1}}, \quad d^\mu_0 \equiv 1.
\] (66)

Convergence of (64) imposes restrictions on the policy process \( \{\rho_t\} \) such that
\[
\lim_{k \to \infty} \beta^k E_t \prod_{j=1}^{k} \frac{1}{\rho_{t+j-1}} (1 - T_{t+k}) \left[ \frac{1}{1 - s_{t+k}} - \frac{\gamma}{\alpha} \right] = 0.
\] (67)

A.4. Recursive Representations. Note that \( \mu_t \) and \( \eta_t \) have the following recursive representations:
\[
\mu_t = \beta \left( \frac{\gamma}{\alpha} + E_t \frac{\mu_{t+1}}{\rho_{t+1}} \right),
\] (68)
\[
\eta_t = 1 - \beta \frac{\gamma}{\alpha} E_t \left( \frac{1 - \tau_{t+1}}{1 - s_{t+1}^g} \right) + \beta E_t \left( \frac{1 - \delta \sigma \tau_{t+1}}{1 - \delta \sigma s_{t+1}^g} \right) \eta_{t+1}.
\] (69)
References


