REGIME SWITCHING MONETARY AND FISCAL POLICIES

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1. An Analytical Example with Regime Switching

This note presents a stylized model of monetary and fiscal policies in which policy regime evolves randomly according to a Markov chain. This simple model is designed to illustrate that well-defined and unique equilibria can exist even in such an environment. It also shows that under certain assumptions about policy behavior, tax disturbances can generate wealth effects in line with the fiscal theory of the price level even if the current regime is one when monetary policy is active and fiscal policy is passive. This suggests that the fiscal theory mechanism may operate more generally than fixed-regime models seem to imply.

Canzoneri, Cumby, and Diba (2001) (CCD) argue that Ricardian equilibria are, in a certain sense, more general than non-Ricardian equilibria. They make this argument by proving a proposition that states that over time the response of the government surplus to total government liabilities merely needs to be bounded away from zero infinitely often for the equilibrium to exhibit Ricardian Equivalence. The key point is that the private sector must expect taxes to adjust “sooner or later,” though the adjustment can be arbitrarily small and infrequent. Because the proposition does not require the fiscal response to be strong enough to make the evolution of government debt stable, the Ricardian equilibria CCD consider are potentially ones with an unbounded debt-output ratio.

Equilibria with unbounded debt-output ratios may not be the most interesting or relevant ones to consider. And they may be misleading if the impacts of taxes hinge on the unboundedness assumption. Unbounded debt-output ratios are well outside any country’s experience, so it is impossible to tell if policy authorities would permit such equilibria to occur. It is quite possible that if a country’s policies made its debt-output ratio appear to grow without limit, the country would undergo fundamental macro policy reforms of the type that neither we nor CCD consider. We assume the political process ensures the debt-output ratio is bounded.

This section presents an analytical example in which policies that satisfy the assumptions of CCD’s proposition deliver a non-Ricardian equilibrium that is unique within the set of equilibria with bounded debt-output ratios. Important conclusions appear to hinge on CCD’s assumption of unboundedness.

Consider a constant endowment version of Sidrauski (1967), modified to include an interest rate rule for monetary policy and a tax rule for fiscal policy. If government consumption is constant, then in equilibrium the representative agent’s consumption, \(c\), is also constant, as is the real interest rate. Preferences over consumption and real money balances are logarithmic. This model implies a Fisher equation

\[
\frac{1}{R_t} = \beta E_t \left[ \frac{1}{\pi_{t+1}} \right],
\]

where \(0 < \beta < 1\) is the discount factor, \(R_t\) is the gross nominal interest rate on one-period nominal government debt, \(\pi_{t+1}\) is the gross inflation rate between \(t\) and \(t+1\), and the expectation is taken with respect to a set \(\Omega_t\) that contains information dated \(t\) and earlier, including the history of regimes up to \(t\). The money demand function is

\[
m_t = \frac{R_t}{R_t - 1} c,
\]

where \(m_t = M_t/P_t\) is the real value of money balances.

Monetary policy adjusts the nominal interest rate in response to inflation according to the rule

\[
R_t = \exp \left( \alpha_0 + \alpha(S_t) \hat{\pi}_t + \theta_t \right),
\]

where \(\hat{\pi}_t \equiv \ln \pi_t\), \(\theta_t\) is an \(i.i.d\). shock, \(S_t\) is the current regime and \(\alpha(S_t)\) is a regime-dependent parameter. Tax policy follows a rule that adjusts lump-sum taxes in response to the real value of total government liabilities:

\[
\tau_t = \gamma_0 + \gamma(S_t)(b_{t-1} + m_{t-1}) + \psi_t,
\]

where \(\tau_t\) is the level of lump-sum taxes, \(b_{t-1} = B_{t-1}/P_{t-1}\) and \(m_{t-1}\) are the real values of debt and money at the beginning of period \(t\), and \(\psi_t\) is an \(i.i.d\). disturbance. The response of taxes to liabilities takes on values that depend on the realization of regime. \(S_t\) obeys an \(N\)-state Markov chain with transition probabilities \(P[S_t = j|S_{t-1} = i] = p_{ij}\), where \(i, j \in \{1, N\}\).

The government’s flow budget identity holds at each date \(t \geq 0\):

\[
\frac{B_t + M_t}{P_t} + \tau_t = g + \frac{M_{t-1} + R_{t-1}B_{t-1}}{P_t}.
\]
given initial nominal liabilities $M_{-1} > 0, R_{-1}B_{-1} > 0$.\(^1\)

Define the expectation error

$$\eta_{t+1} \equiv \frac{R_t}{\pi_{t+1}} - \beta \frac{R_t}{\pi_t}$$

where the equality comes from using the Fisher equation. Combining (1) and (3) and using (6), the inflation process obeys

$$\pi_{t+1} = \alpha(S_t)\pi_t + \alpha_0 + \theta_t - \hat{\eta}_{t+1} + \ln \beta.$$ \hspace{1cm} (7)

Let $l_t = b_t + m_t$. Equations (4) and (5) together with (2) imply that government liabilities evolve according to:

$$l_t = \left( \frac{R_t}{\pi_t} - \gamma(S_t) \right) l_{t-1} - \frac{R_t}{\pi_t} c + D - \psi_t.$$ \hspace{1cm} (8)

where $D = g - \gamma_0$.

Assume that: (i) $E_t[\gamma_{t+1}] = \gamma$; (ii) $\gamma$ satisfies $|1/\beta - \gamma| > 1$; (iii) the inflation process given by (7) is stable in expectation (that is, $E_t\pi_{t+k} < \infty$ for all $k$). Assumptions (i) and (ii) mean that on average fiscal policy is active and assumption (iii) means that on average monetary policy is passive (the Taylor principle does not hold on average).\(^2\)

Iterate forward on (8) to obtain (for $k \geq 0$)

$$l_{t+k} = \prod_{j=0}^{k} \left( \frac{R_{t-1+j}}{\pi_{t+j}} - \gamma_{t+j} \right) l_{t-1} + \sum_{j=0}^{k} \prod_{i=1}^{k-j} \left( \frac{R_{t-1+i+j}}{\pi_{t+i+j}} - \gamma_{t+i+j} \right) (D + \frac{R_{t-1+j}}{\pi_{t+j}} c - \psi_{t+j}).$$ \hspace{1cm} (9)

To solve (9), take expectations as of $t - 1$, apply the law of iterated expectations, and use the Fisher equation. Then we can replace the terms $\frac{R_{t-1+j}}{\pi_{t+j}}$ with $\frac{1}{\beta}$. Under the assumption that $E_t[\gamma_{t+1}] = \gamma$, (9) becomes

$$E_{t-1}[l_{t+k}] = (1/\beta - \gamma)^{k+1} \left[ l_{t-1} - c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right) \right] + c \left( \frac{1/\beta - D/c}{1/\beta - \gamma - 1} \right).$$ \hspace{1cm} (10)

\(^1\)By assuming initial government debt is positive, we do not address the criticism that the fiscal theory of the price level falls apart when $B_{-1} = 0$. The criticism is made in a perfect foresight model by Niepelt (2001) and countered in a stochastic model with incomplete markets by Daniel (2003).

\(^2\)Appendices A and B provide the stability conditions for the inflation process.
Stability in expectation requires that \( l_{t-1} = c \left( \frac{1/\beta - D/c}{1/\beta - \gamma} \right) \), which is positive if \( D/c < 1/\beta \). The value of \( \eta_t \) is obtained from the budget constraint after substituting in the value of \( l \):

\[
\eta_t = \beta \frac{(1 + \gamma(S_t)) (1/\beta - D/c) - (D/c) (1/\beta - \gamma - 1)}{1 + \gamma - D/c} + \frac{\beta}{c} \left( \frac{1/\beta - \gamma - 1}{1 + \gamma - D/c} \right) \psi_t. \tag{11}
\]

Equation (11) is the unique equilibrium mapping from the tax disturbance, \( \psi_t \), and the realization of the tax feedback parameter, \( \gamma(S_t) \), to the forecast error in inflation. The solution for \( \eta \) and the stable inflation process, (7), uniquely determine inflation. For an equilibrium of this type to exist, we restrict the parameters to assure that \( \eta_t \), which is the ratio of two positive numbers, is positive for any realization of \( \psi_t \). A sufficiently small value for \( D/c \), coupled with a sufficiently high bounded negative support for \( \psi \) will do the job.

As a concrete example, suppose there are two regimes, \( N = 2 \), and that the policy parameters take on the values

\[
\alpha(S_t) = \begin{cases} 
\alpha(1) & \text{for } S_t = 1 \\
\alpha(2) & \text{for } S_t = 2
\end{cases} \quad \gamma(S_t) = \begin{cases} 
\gamma(1) & \text{for } S_t = 1 \\
\gamma(2) & \text{for } S_t = 2
\end{cases}
\]

Further suppose that \( \alpha(1) \) and \( \alpha(2) \) are sufficiently small such that, given the transition probabilities, the inflation process, (7), is stable in expectation. The assumption that the tax parameters have constant mean implies

\[
E\left[ \gamma_{t+j} | S_t = 1, \Omega_t \right] = \gamma(1)p_{11} + \gamma(2)p_{21} \\
= E\left[ \gamma_{t+j} | S_t = 2, \Omega_t \right] = \gamma(1)p_{12} + \gamma(2)p_{22} \equiv \gamma, \tag{12}
\]

\( j > 0 \). By assumption \( |\beta^{-1} - \gamma| > 1 \). If either \( \gamma(1) \) or \( \gamma(2) \) is positive and jointly they satisfy (12), then the model satisfies CCD’s premise that taxes adjust to debt infinitely often. But as (11) makes clear, negative tax disturbances generate wealth effects that raise the inflation rate. The only equilibrium with bounded debt is non-Ricardian.

This does not deny the existence of Ricardian equilibria of the kind that CCD emphasize. But if those equilibria do exist, they must imply debt-output ratios that grow without bound.

**Appendix A. Stability Properties of Random-Coefficient Linear Models**

For our purposes, it is sufficient to consider the stability properties of a simple univariate model of the following form:
\[ x_t = a(S_{t-1})x_{t-1} + \xi_{t-1} + \Psi \eta_t, \]

where \( a(S_{t-1}) \) follows an \( M \)-state Markov chain, \( \xi_t \) is an exogenous shock process possibly depending on both the state of the Markov chain and on \( Q \) additional i.i.d. processes \( \Theta_t \) and \( \eta_t \) is an endogenous expectation error satisfying the restriction \( E_t \eta_{t+1} = 0 \). Let the transition matrix of the Markov chain be given by \( \Pi \), where \( \Pi_{ij} = \text{prob}(s_{t+1} = i|s_t = j) \). Finally, the expectation \( E_t \) is taken with respect to the time \( t \) information set \( \{x_{t-j}, S_{t-j}, \xi_{t-j}, \zeta_{t-j} | j \geq 0 \} \), where \( \zeta \) represents a non-fundamental (“sunspot”) shock.

We are interested in the behavior of \( E_t x_{t+T} \) as \( T \) becomes large. Iterating forward on the model shows that

\[
E_t x_{t+T} = E_t \left( \prod_{j=0}^{T-1} a_{t+j} \right) x_t + E_t \sum_{j=1}^{T} \left( \prod_{k=0}^{T-j-1} a_{t+k+j} \right) (\xi_{t+j-1} + \Psi \eta_{t+j}),
\]

where \( a_{t+j} \) denotes \( a(S_{t+j}) \). This expectation can be calculated explicitly using the following recursion relation:

\[
E_t \left[ \prod_{j=0}^{T} a_{t+j} | S_t = k \right] = \sum_{l=1}^{M} E_{t+1} \left[ \prod_{j=0}^{T-1} a_{t+j} | S_{t+1} = l \right] a(S_t = k) \cdot \text{prob}(S_{t+1} = l | S_t = k),
\]

for \( k \in \{1, M\} \).

Define the matrix \( \Gamma(a) \) by \( \Gamma_{ij} \equiv a(j) \cdot \text{prob}(S_{t+1} = i | S_t = j) \) and let the symbol \( a_t^{(l)} \equiv \prod_{j=0}^{l} a_{t+j} \). Now define the vector

\[
E_t(a_t^{(l)} | \bullet) \equiv \left( E_t \left[ \prod_{j=0}^{l} a_{t+j} | S_t = 1 \right], ..., E_t \left[ \prod_{j=0}^{l} a_{t+j} | S_t = M \right] \right),
\]

so that \( \dim(E_t(a_t^{(l)} | \bullet)) = 1 \times M \). With this notation, the previous recursion relation can be written \( E_t(a_t^{(l)} | \bullet) = E_t(a_{t}^{(l-1)} | \bullet) \Gamma(a) \). Accordingly, \( E_t(a_t^{(l)} | \bullet) = \omega \Gamma^l(a) \), where \( \omega \) is a \( 1 \times M \) vector of ones and \( \Gamma^l(a) \) denotes a matrix product.

From this relation, it follows that for \( j > 1 \),

\[
E_t(a_{t+j}^{(l)} \xi_{t+j-1} | S_t = m) = \sum_{n=1}^{M} \text{prob}(S_{t+j-2} = n | S_t = m) \sum_{k=1}^{M} E_t^{(l)}(\xi_{t+j-1} | S_{t+j-1} = k, S_{t+j-2} = n) \cdot \text{prob}(S_{t+j-1} = k | S_{t+j-2} = n) \cdot \sum_{p=1}^{M} E_t(a_{t+j}^{(l)} | S_{t+j} = p) \cdot \text{prob}(S_{t+j-1} = k | S_{t+j} = p)
\]
and therefore

\[ E_t(a_{t+j}^{(l)} \xi_{t+j-1} | \bullet) = \omega \Gamma^l(a)(\Pi \mu \Pi^{-2}), \]

where \( \mu_{ij} \equiv E_t(\xi_{t+1} | S_{t+1} = i, S_{t} = j) \cdot \text{prob}(S_{t+1} = i | S_{t} = j) \).

For \( j = 1 \), \( E_t(a_{t+1}^{(l)} \xi_{t} | S_{t} = m) = \xi_t \omega \Gamma^l(a) \Pi \).

Finally, the endogenous expectation errors can be handled as follows. Consider terms of the form \( d(S_t) \eta_t \). Then define the basis random variables \( \chi_j(S_t) \) associated with the Markov state \( S_t \), which are defined so that \( \chi_j(S_t) = 1 \) if \( S_t = j \) and 0 otherwise.

Project \( d(S_t) \eta_t \) onto the \( \chi_j \) and \( \Theta_t \), the \( Q \) additional \( i.i.d. \) processes:

\[ d(S_t) \eta_t = \sum_{n=1}^{M} a_n \chi_n(S_t) + \sum_{l=1}^{Q} b_l \Theta_{lt} + \varepsilon_t, \]

where \( \varepsilon_t \) is uncorrelated with the \( \chi_j \) and \( \Theta_t \). Note that \( E_t a_{t+j}^{(l)} | S_t = m \) is measurable with respect to \( S_t \), so must be expressible as a linear combination of the \( \chi_j \). It follows that

\[ E_t \left[ a_{t+j+1}^{(l)} \Psi d(S_{t+j}) \eta_{t+j} | S_t \right] = E_t \left[ a_{t+j+1}^{(l)} \Psi d(S_{t+j}) \eta_{t+j} | S_t \right], \]

where \( d(S_{t+j}) \eta_{t+j} \equiv \sum_{n=1}^{M} a_n \chi_n(S_t) + \sum_{l=1}^{Q} b_l \Theta_{lt} \). Ultimately, therefore, we can treat the expectation errors just as we treat the \( \xi \), since any dependence on the sunspot shocks \( \zeta \) drops out of the expectations of interest.

From here on, let us assume that \( \Gamma \) and \( \pi \) have \( M \) distinct non-zero eigenvalues. Further, let \( x(i), i = 1 \ldots M \), be the state-contingent mapping of the \( i.i.d \) shocks into stable solutions of the difference equation, at time \( t \). Finally, define \( X \equiv \text{diag}(x(1) \ldots x(M)) \), a matrix with the \( x(i) \) on its diagonal. With these recursion relations, write

\[ E_t x_{t+k} = \omega \left( \Gamma(a)^k (X + \xi/a(S_t)) + \sum_{j=1}^{k} \Gamma^{k-j}(a)(\eta \Pi^{-1} + \Pi \mu \Pi^{-2}) \Pi \right). \]

Now decompose \( \Gamma \) and \( \Pi \) into linear combinations of projectors onto their eigenvectors: \( \Gamma(a) = \sum_{j=1}^{M} \lambda_j P_j(\Gamma(a)) \) and \( \Pi = \sum_{j=1}^{M} \phi_j P_j(\Pi) \). Then the sum over future shocks can be calculated explicitly to arrive at

\[ E_t x_{t+k} = \omega \sum_{m=1}^{M} P_m(\Gamma(a)) \left[ \lambda_m^k \cdot \left( X + \frac{\xi}{a(S_t)} \right) + \sum_{n=1}^{M} \left( \frac{\lambda_m^k - \phi_n^k}{\lambda_m^k - \phi_n} \right)(\eta \Pi^{-1} + \Pi \mu \Pi^{-2})P_n(\Pi) \right]. \]

Therefore, the long-run expected properties of \( x \) are characterized by the number of explosive roots of \( \Gamma \).
APPENDIX B. SOLVING A RICARDIAN MODEL

From the Fisher equation and the monetary policy rule, we have that
\[ 1/R_t = \beta E_t 1/\pi_{t+1} = \exp(-\alpha_{1t} \pi_t - \alpha_{0t} - \theta_t), \]
where \( \theta_t \) is \( i.i.d. \). Defining
\[ \eta_{t+1} = \frac{1/\pi_{t+1}}{E_t 1/\pi_{t+1}}, \]
we can rewrite these equations in logs as
\[ \widehat{\pi}_{t+1} = \alpha_{1t} \widehat{\pi}_t + \theta_t + \alpha_{0t} + \ln \beta - \widehat{\eta}_{t+1}. \]
In this case, \( \widehat{\eta} \) is not mean-zero, as is apparent from its definition. Therefore, we decompose \( \widehat{\eta} \) into its mean and deviations from the mean:
\[ \widehat{\eta}_{t+1} = \sum_{n=1}^{2} \phi_n \left( \frac{\theta_t}{\lambda_m - \phi_n} \right) P_n(\Pi) = 0, \]
where, as above, \( X \) denotes the initial state-contingent values of the log-inflation process.

The operator \( Q_m \equiv \sum_{n=1}^{2} \xi_n \frac{P_n(\Pi)}{\lambda_n - \phi_n} \) is invertible, so, assuming that both roots of \( \Gamma(\alpha_1) \) are explosive, we can write
\[ \sum_{m=1}^{2} \omega \cdot \sum_{n=1}^{2} \phi_n \left( \frac{\theta_t}{\lambda_m - \phi_n} \right) P_n(\Pi) = 0 \]
since \( \omega \eta = 0 \). This leads to the following sequence of implications:
\[ \Rightarrow \sum_{m=1}^{2} \omega \cdot \sum_{n=1}^{2} \phi_n \left( \frac{\theta_t}{\lambda_m - \phi_n} \right) P_n(\Pi) = 0 \]
\[ \Rightarrow \sum_{m=1}^{2} \omega \cdot \sum_{n=1}^{2} \phi_n \left( \frac{\theta_t}{\lambda_m} \right) P_n(\Pi) = 0 \]
\[ \Rightarrow \sum_{m=1}^{2} \omega P_m(\Gamma(\alpha_1)) \left\{ X \cdot (\Pi^{-1} \lambda_m - I) + \theta_t(I - \Pi/\lambda_m) \right\} = 0 \]

\[ \Rightarrow \omega \left\{ \Gamma(\alpha_1) \cdot X \cdot \Pi^{-1} - X \right\} + \theta_t(I - \Gamma(\alpha_1)^{-1} \Pi) = 0 \]

Now expand this expression to yield

\[ \omega \left( \Pi \left( \begin{array}{cc} \alpha_1(1) \hat{\pi}(1) & 0 \\ 0 & \alpha_1(2) \hat{\pi}(2) \end{array} \right) \Pi^{-1} - X \right) + (1 - 1/\alpha_1(1), 1 - 1/\alpha_1(2)) \theta_t = 0. \]

Finally, after some further algebra with this expression, we can obtain explicit solutions for the inflation function. In this case, the inflation function is very simple:

\[ \hat{\pi}(S_t) = -\theta_t/\alpha_1(S_t). \]

With the intercept term, we would have obtained \( \hat{\pi}(S_t) = -\theta_t/\alpha_1(S_t) + \Delta(S_t), \) where \( \Delta(S_t) \) depends on the still-undetermined \( E_t \eta_{t+1} \). This term can be determined by imposing the condition that \( E_t \eta_{t+1} = 1. \)

Substituting the result for \( \hat{\pi}(S_t), \) we have

\[ \beta E_t \left\{ \exp \left( -\hat{\pi}_{t+1} + \alpha_1(S_t) \pi_t + \theta_t + \alpha_0(S_t) \right) \right\} = 1 \]

\[ \Rightarrow \beta E_t \left\{ \exp \left( \frac{\theta_{t+1}}{\alpha_1(S_{t+1})} - \Delta(S_{t+1}) + \alpha_1(S_t) \Delta(S_t) + \alpha_0(S_t) \right) \right\} = 1 \]

If \( \theta \sim N(0, \sigma) \), then

\[ E_t \left\{ \exp \left( \frac{\theta_{t+1}}{\alpha_1(S_{t+1})} \right) \mid S_{t+1}, S_t \right\} = \exp \left( \frac{\sigma}{2[\alpha_1(S_{t+1})]^2} \right) \]

and therefore

\[ \beta E_t \left\{ \exp \left( \frac{\sigma}{2[\alpha_1(S_{t+1})]^2} - \Delta(S_{t+1}) + \alpha_1(S_t) \Delta(S_t) + \alpha_0(S_t) \right) \right\} = 1. \]

This condition, for each initial \( S_t, \) is sufficient to determine \( \Delta(S_t). \)

Once the expectations errors \( \eta \) have been determined, the long-run properties of the bond dynamics can be derived using a variation of the methods in Appendix A. When steady-state inflation rates are different across regimes, the relevant eigenvalues are those associated with the matrix

\[
\begin{pmatrix}
\left( \frac{\eta(1,1)}{\beta} - \gamma_1(1) \right) & \Pi_{11} & \left( \frac{\eta(1,2)}{\beta} - \gamma_1(1) \right) & \Pi_{12} \\
\left( \frac{\eta(2,1)}{\beta} - \gamma_1(2) \right) & \Pi_{21} & \left( \frac{\eta(2,2)}{\beta} - \gamma_1(2) \right) & \Pi_{22}
\end{pmatrix}
\]
When $\gamma_1$ is independent of regime, however, the law of iterated expectations implies that

$$E_t \prod_{j=1}^{k} \left( \frac{\eta_{t+j}}{\beta} - \gamma_1(S_{t+j}) \right) = \left( \frac{1}{\beta} - \gamma_1 \right)^k.$$

This result holds when the conditional mean of $\gamma_1$ is independent of the initial state. Thus, under these circumstances, the state-contingent linearization scheme will perfectly capture the long-run behavior of the full non-linear system.
Figure 1. Light shading: linear methods and nonlinear model agree; middle shading: linear methods imply existence, nonlinear model implies nonexistence; dark shading: nonlinear model implies non-Ricardian fiscal policy or indeterminacy.
Figure 2. Light shading: linear methods and nonlinear model agree; middle shading: linear methods imply existence, nonlinear model implies nonexistence; dark shading: nonlinear model implies non-Ricardian fiscal policy or indeterminacy.

REFERENCES


Figure 3. Light shading: linear methods and nonlinear model agree; middle shading: linear methods imply existence, nonlinear model implies nonexistence; dark shading: nonlinear model implies non-Ricardian fiscal policy or indeterminacy.