The Price Level, the Quantity Theory of Money,  
and the Fiscal Theory of the Price Level  
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Abstract: This paper examines price level determination from the perspective of portfolio choice. Arbitrages among money balances, bonds, and investment goods determine their relative demands. Returns to real balance holdings and after-tax returns to investment goods determine the relative values of nominal and real assets. Because expectations of government policies ultimately determine the expected returns to both nominal and real assets, the price level depends on interactions among current and expected future monetary and fiscal policies. The quantity theory and the fiscal theory emerge as special cases produced by restricting both the margins and the policies considered.

Keywords: Monetary Policy, Fiscal Policy, Quantity Theory, Fiscal Theory, Price Level Determination, Money Demand

JEL classification: E31, E41, E52, E62

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The Price Level, the Quantity Theory of Money, and the Fiscal Theory of the Price Level

David B. Gordon and Eric M. Leeper*

1. Introduction

Two largely independent views of price level determination currently coexist. The first view stems from the venerable quantity theory of money.1 The second view is the fiscal theory of the price level.2 In their starkest forms, the quantity and fiscal theories are distinguished by which nominal government liability is sufficient to determine the price level: in the quantity theory it is the money supply and in the fiscal theory it is government debt.

Despite their very different views on price level determination, there are similarities in the standard descriptions of the quantity and fiscal theories. In his classic description of the quantity theory, Friedman (1956) whittles a general money demand specification down to a very simple one by assuming stable substitutions exist among a wide range of assets. He also makes little, if any, mention of the fiscal policies consistent with his analysis, for which Brunner and Meltzer (1974) take him to task. Leeper (1991), Sims (1994), and Woodford (1995) make analogous simplifications to expound the fiscal theory in models with only nominal assets and non-distorting fiscal policies. Other versions of the fiscal theory consider “cashless” economies in which monetary policy is characterized entirely as controlling the nominal interest rate [Woodford (1998a) and Cochrane (1999)].

The simplified theoretical environments used to describe the two theories have another striking similarity. As popularly presented, the quantity and fiscal theories employ identical money demand functions of the form:

\[ \frac{M^d}{P} = h(i, y), \]  

where \( P \) is the price level, \( i \), the nominal interest rate, is the opportunity cost of money, and \( y \) is a scale variable. While empirical specifications like (1) are often justified on grounds of long-run stability [see, for example, Friedman (1959), Meltzer (1963), and Lucas (1988)], theoretical justifications are more involved and constitute a central theme of this paper.

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1 Among the classic works on the quantity theory are Fisher (1913), Friedman (1956), and Patinkin (1965).

This paper examines price level determination from the perspective of portfolio choice. Arbitrages among money balances, bonds, and investment goods determine their relative demands. Returns to real balance holdings and after-tax returns to investment goods determine the relative values of nominal and real assets. Because expectations of government policies ultimately determine the expected returns to both nominal and real assets, monetary and fiscal policies jointly determine the price level. These mechanisms imply that the price level depends on dynamic interactions among current and expected future macro policies. The quantity and fiscal theories turn out to emerge as limiting cases produced by restricting both the margins and the policies considered.

The basis for our analysis is not new. Hicks (1939) embeds the determination of the overall level of prices in the general problem of portfolio choice and asset valuation. His perspective found formal voice in the work of Friedman (1956), Tobin (1961) and Brunner and Meltzer (1972). Because those authors treat the demand for money symmetrically with the demands for other assets, the equilibrium price level is determined by the valuation of all assets jointly. That treatment suggests that in an economy with a wide array of real and nominal assets it is futile to try to value one type of asset separately from the rest of the portfolio. Nonetheless, the portfolio choice problems emphasized in earlier analyses often receive scant attention. Asset substitutions are frequently omitted or ignored in both monetary theory and empirical estimates of money demand. Work on the fiscal theory of the price level has generally followed suit by making simplifying assumptions about asset substitutions.

Expression (1) springs from the arbitrage between money and nominal bonds when the nominal interest rate suffices to capture the substitutions between these assets. Missing from (1) is the substitution between real and nominal assets—a margin that the Tobin (1965) effect highlights—that expected monetary and fiscal policies might induce. If those substitutions are also important for determining the demand for money, the nominal interest rate does not adequately summarize the opportunity cost of money and (1) does not adequately summarize the determinants of the price level. This paper derives a more general money demand specification and obtains sets of conditions—corresponding to the quantity theory or the fiscal theory—under which the general money demand function collapses to (1).

In our analysis, the quantity theory and the fiscal theory emerge from a common theoretical background under assumptions that differ in the degree to which monetary policy is constrained to satisfy fiscal financing needs. The quantity theory completely describes the path of the price level only when monetary policy and government debt play no role in financing government expenditures. The price level is then independent of fiscal policy. In contrast, the benchmark exposition of the fiscal theory of the price level is based on an extension of Sargent and Wallace’s unpleasant monetarist arithmetic and arises when fiscal obligations constrain monetary policy.

Combining a portfolio choice perspective with rational expectations complicates the analysis of price level determination. The price level depends fundamentally on jointly consistent

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3 Other important analyses include Hicks (1935), Friedman (1948), Tobin (1969,1980), and Brunner and Meltzer (1993).

4 Important exceptions that do adopt the broad asset substitution perspective of Hicks are Bryant and Wallace (1979,1984) and Wallace (1981,1989).
combinations of current and expected future policies. Current policies directly affect prices and they indirectly affect prices through changes in expectations of future policies. Similarly, expected future policies feed back to constrain current policy options. These complications imply that the impacts of open-market operations or fiscal policy changes depend on how agents expect current and future policies to adjust to restore equilibrium.

Lost in many expositions of the quantity and fiscal theories is a straightforward discussion of price level determination in an environment with a rich array of assets and with largely unrestricted policies. This paper presents such a setting and argues that the combination of a minimal set of assets and mutually consistent current and future policies produces an understanding of price determination and its dependence on macroeconomic policies that differs dramatically from those emphasized in existing literature.

2. A Simple Model of Portfolio Choice

We consider a standard Ramsey-Cass-Koopmans growth model combined with a transactions sector that provides a substitute for the transactions services of money. Private agents may hold two nominal assets, money and government bonds, and a real asset, capital. This range of substitutions suffices to provide a richer perspective on price level determination than appears in much of the recent literature. Direct taxes are put on an equal footing with inflation taxes because both distort portfolio choices. Equilibrium can be characterized as a mapping from current and expected future policy variables to portfolio choices and, consequently, equilibrium asset values and the price level.

2.1 The Model

The model consists of a representative household, two firms—one producing goods and one producing transactions services—and a government.

The gross physical assets of the economy at date $t$, $f(k_{t-1})$, are allocated to consumption, $c_t$, capital, $k_t$, or government purchases, $g_t$. The aggregate resource constraint each period is

$$ f(k_{t-1}) \geq c_t + k_t + g_t, \tag{2} $$

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5 Current policies may constrain future policy options, if current policies imply debt obligations or future expenditure commitments.
with $c_t \geq 0$, $k_t \geq 0$, and $g_t \geq 0$. The function $f(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable.

Two types of representative firms rent factors of production from households and then sell their outputs back to households. Goods producing firms rent $k$ from households at rental rate $r$ and pay taxes levied against sales of goods. Firms choose $k$ to solve

$$\max \ D_{Gt} = (1 - \tau_t) f(k_{t-1}) - (1 + r_t) k_{t-1},$$

taking the tax rate, $\tau$, and $r$ as given.

Transactions service producing firms rent labor, $l$, from households at wage rate $w$ and sell transactions services, $T(l)$, to households at price $P_T$. The function $T(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable. Firms choose $l$ to solve

$$\max \ D_{Tt} = P_T T(l_t) - w_l,$$

taking $P_T$ and $w$ as given.

The household owns the firms and receives factor payments, so its income at the beginning of period $t$ is

$$I_t = (1 + r_t) k_{t-1} + D_{Gt} + w_l l_t + D_{Tt},$$

where $D_G$ and $D_T$ are dividends received from the goods-producing and transactions-producing firms.

Households use money balances and transactions services to acquire goods. Transactions services demanded from the financial sector at time $t$ execute the fraction $T_t^d \in [0,1]$ of private expenditures on goods. Choices of money and services must satisfy the finance constraint:

$$\frac{M_{t-1}}{P_t} + T_t^d (c_t + k_t) \geq c_t + k_t,$$

where $M_{t-1}$ is nominal money balances carried into period $t$ and $P_t$ is the price level at $t$.

Transactions services may be thought of as a clearinghouse, money market mutual funds, or credit cards, although our specification abstracts from any institutional details. In advanced economies, where most transactions involve the financial sector but do not involve cash directly, $T_t^d$ may be close to unity on average. Holding resources devoted to the financial sector fixed, the constraint implies that doubling the value of transactions doubles the value of transactions.

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6 Complete depreciation of capital simplifies many of the expressions without affecting the characteristics of the equilibrium that concern us. For a version of the model with partial depreciation of capital, see Gordon, Leeper and Zha (1998).

7 Including investment goods in the finance constraint, as in Stockman (1981), is substantive. Excluding investment goods implies that the acts of investing or reallocating investments do not generate any demand for money or for transactions services.
performed with services by doubling the size of each transaction. It also implies that the marginal product of transactions services increases with the value of transactions performed.\(^8\)

Preferences are defined over consumption and leisure. The current period utility function, \(U(\cdot)\), is time-separable, strictly increasing in both arguments, strictly concave, and continuously differentiable. Households are endowed with one unit of time each period and choose \(c, k, l, T^d, M,\) and \(B,\) nominal bonds, to solve

\[
\max E \sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t), \quad 0 < \beta < 1, \tag{7}
\]

subject to the budget constraint

\[
c_t + k_t + \frac{M_t + B_t}{P_t} + P_t T^d_t \leq l_t + \frac{M_{t-1} + (1+i_{t-1})B_{t-1}}{P_t}, \tag{8}
\]

and the finance constraint, (6), with \(0 \leq l, 1, P_t, P_{t+1}, i_t\) and \(\tau_t\) taken as given, and the initial conditions \((k_{-1}, M_{-1}, (1+i_{-1})B_{-1})\).\(^9\) Total government expenditures, \(g\), are financed by printing money, \(M\), and selling nominal bonds, \(B\), paying a net nominal interest rate of \(i\), and levying a proportional tax rate, \(\tau\), against net output. The government’s budget constraint is

\[
\tau_t f(k_{t-1}) + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1+i_{t-1})B_{t-1}}{P_t} = g_t, \tag{9}
\]

Let the state of the economy at the beginning of each period be given by the resources available to the private sector and expected sequences of future policies. At time \(t\) the state is \(z_t = (k_{t-1}, M_{t-1}, (1+i_{t-1})B_{t-1}, \{E_t \rho, E_t \tau, E_t s_j\}_{j=t}^{\infty})\), where \(\rho\) is money growth, \(\rho_t = M_t/M_{t-1}\), and \(s^g\) is the government-spending share, \(s^g_t = g_t/f(k_{t-1})\). Rational expectations require that expected policies are consistent with equilibrium. A perfect foresight competitive equilibrium is a set of sequences \(\{c_t, k_t, l_t, T^d_t, M_t, B_t, P_t, P_{t+1}, i_t, \tau_t, g_t\}_{t=0}^{\infty}\) such that given the \(\{P_t, P_{t+1}, i_t, \tau_t, g_t\}\) sequences, the \(\{c_t, k_t, l_t, T^d_t, M_t, B_t\}\) sequences solve the firms’ and the household’s optimum problems, clear markets, and satisfy the government’s budget constraint at each date.

### 2.2 Functional Forms

To obtain an explicit characterization of the model’s equilibrium, we assume the following functional forms for the production functions and for preferences:

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\(^8\) The specification is in the spirit of Baumol’s (1952) and Tobin’s (1956) theories of cash inventories, where economies of scale “result from costs that are as large for hundred-dollar transactions as for million-dollar transactions” [Tobin and Golub (1998), p. 49]. The specification is also in keeping with Friedman’s (1956) view of the myriad ways that households and firms may create substitutes for money in transactions.

\(^9\) The operator \(E\) in (7) denotes equilibrium expectations of private agents over future policy. Although we focus on perfect foresight equilibria, we employ the notation \(E_t\) to distinguish between current and past policies (dated \(t\) and earlier) and future policies (dated \(s > t\)).
\[ f(k_{t-1}) = k_{t-1}^{\sigma}, \quad 0 < \sigma \leq 1 \]  
\[ T(l_t) = 1 - (1 - l_t)^\alpha, \quad \alpha > 1 \]  
\[ U(c_t, 1 - l_t) = \ln(c_t) + \gamma \ln(1 - l_t), \quad 0 < \gamma < \alpha. \]  

2.3 First-Order Conditions

The firms’ first-order conditions are:

\[ 1 + r_t = \sigma(1 - r_t)k_{t-1}^{\sigma-1}, \]  
\[ w_t = \alpha(1 - l_t)^{\alpha-1}P_{tT}. \]  

Letting \( \varphi \) be the lagrange multiplier for the budget constraint and \( \lambda \) be the multiplier for the finance constraint, the household’s first-order conditions are:

\[ \varphi_t + \lambda_t = \frac{1}{c_t} + \lambda T_t^d, \]  
\[ \frac{\gamma}{1 - l_t} = w_t \varphi_t, \]  
\[ \varphi_t P_{tT} = \lambda_t(c_t + k_t), \]  
\[ \frac{\varphi_t}{P_t} = \beta E_t \left[ \frac{\varphi_{t+1} + \lambda_{t+1}}{P_{t+1}} \right], \]  
\[ \frac{\varphi_t}{P_t} = \beta(1 + i_t)E_t \left[ \frac{\varphi_{t+1}}{P_{t+1}} \right], \]  
\[ \varphi_t + \lambda_t = \lambda T_t^d + \beta E_t(1 + r_t)\varphi_{t+1}. \]  

Expressions (18)-(20) price assets and (17) prices transactions services. Interconnections among the asset pricing equations make it clear that price level determination is only one component of a general portfolio choice problem. Only under special circumstances that effectively decouple (18) and (19) from (20) can nominal assets be priced independently of the real asset.\(^{10}\)

2.4 The Equilibrium

We characterize the equilibrium at time \( t \) in terms of two policy expectations functions, \((\mu_t, \eta_t)\), current government claims to goods, \( s_t^g \), and beginning-of-period assets, \((k_{t-1}, M_{t-1}, (1 + i_{t-1})B_{t-1})\). A solution maps expectations of policy into portfolio choices, where those expectations are restricted to policy paths consistent with equilibrium in each future period. The functions \((\mu_t, \eta_t)\) summarize everything agents need to know to form rational expectations about the equilibrium of the economy.

\(^{10}\) Necessary and sufficient conditions for an optimum are (6), (8), (9), (13)-(20) plus transversality conditions for \( M_t/P_t, B_t/P_t \), and \( k_t \).
As detailed below, \( \eta \) and \( \mu \) capture the portfolio balance effects of expected policies. \( \eta \), a product of solving the first-order condition for capital, measures the direct tax distortion on investment as well as the extent to which government expenditures are financed by taxing output. A rise in expected future taxes lowers \( \eta \) and lowers investment. \( \mu \), a product of solving the first-order condition for money, reflects expected inflation and thus the expected return on nominal assets. Higher expected inflation lowers \( \mu \) and lowers the demand for money.

Although in general the equilibrium depends on the entire expected paths of future policies, it is convenient to focus on circumstances in which the economy is in a stationary equilibrium for dates \( s > t \) but starts from some other equilibrium at \( t \). Assume:

\[
\begin{align*}
\rho_{t+j} &= \rho_F, \quad \forall j > 0 \\
\tau_{t+j} &= \tau_F, \quad \forall j > 0 \\
s^g_{t+j} &= s^g_F, \quad \forall j > 0.
\end{align*}
\] (21)

The model exhibits a type of dichotomy that allows the real and nominal portfolio choices to be solved recursively. To solve the real side, first write the aggregate resource constraint as

\[
c_i + k_i = (1 - s^g_i) f(k_{t-1}),
\] (22)

where \( s^g_i = g_i / f(k_{t-1}) \). Using the functional forms and first-order conditions (14)-(17), after imposing equilibrium the lagrange multipliers are

\[
\varphi_i = \frac{1}{c_i} - \frac{\gamma/\alpha}{c_i + k_i},
\] (23)

\[
\lambda_i = \frac{\gamma/\alpha}{(1-T_i)(c_i + k_i)}.
\] (24)

Combining these expressions with the firms’ first-order condition for capital and the aggregate resource constraint the Euler equation for capital, (20), becomes:

\[
\frac{1}{1-s_i} = \sigma \beta E_t \left[ \frac{1 - \tau_{t+1}}{1-s^g_{t+1}} \left( \frac{1}{1-s_{t+1}} \right) \right] + E_t \left[ 1 - \sigma \beta \frac{\gamma}{\alpha} \frac{1 - \tau_{t+1}}{1-s^g_{t+1}} \right].
\] (25)

The solution to (25) is

\[
\frac{1}{1-s_i} = \eta_i,
\] (26)

where

\[
\eta_i = E_t \sum_{j=0}^{\infty} (\sigma \beta)^j d_i^\eta \left[ \frac{1 - \sigma \beta \frac{\gamma}{\alpha} \frac{1 - \tau_{t+j+1}}{1-s^g_{t+j+1}}}{1-s_{t+j+1}} \right], \quad d_i^\eta = \prod_{j=0}^{i-1} \left( \frac{1 - \tau_{t+j+1}}{1-s^g_{t+j+1}} \right), \quad d_0^\eta = 1.
\]

A second dynamic relation arises from substitution between money and transactions services. Combining the expressions for \( \varphi \) and \( \lambda \) with the finance constraint at equality and the first-order condition for money, (18), we obtain a difference equation in velocity, \( 1-T_i \):
\[
(1-T_i) \left[ \frac{1}{1-s_i} - \frac{\gamma}{\alpha} \right] = \beta \frac{1}{\rho_i} E_t \left( (1-T_{i+1}) \left[ \frac{1}{1-s_{i+1}} - \frac{\gamma}{\alpha} \right] \right) + \frac{\gamma}{\alpha}.
\]

The solution is:

\[
(1-T_i) \left[ \frac{1}{1-s_i} - \frac{\gamma}{\alpha} \right] = \frac{\mu_i}{\rho_i},
\]

where \( \rho_i = \frac{M_i}{M_{i-1}} \) and \( \mu_i \equiv \beta \frac{\gamma}{\alpha} E_t \sum_{i=0}^{\infty} \beta^i d_i^\mu \), \( d_i^\mu \equiv \prod_{j=0}^{\infty} \frac{1}{\rho_{i+j+1}} \), \( d_0^\mu \equiv 1 \).

Given the policy assumptions in (21), the expectations functions are

\[
\eta_i(\tau_F, s_F) = \frac{1-\sigma \beta \frac{\gamma}{\alpha} \left( 1-\tau_F \right)}{1-\sigma \beta \left( 1-\frac{\tau_F}{s_F} \right)},
\]

and

\[
\mu_i(\rho_F) = \left\{ \frac{\beta \frac{\gamma}{\alpha}}{1- \beta / \rho_F} \right\}.
\]

Signs appearing in parentheses above the function arguments in (29) and (30) denote partial derivatives of the functions with respect to expected future values of the policy variables.

The equilibrium capital stock is

\[
k_t = \left( 1 - \frac{1}{\eta_t} \right) (1-s_t^F) f(k_{t-1}),
\]

and equilibrium real money balances are

\[
\frac{M_t}{P_t} = \left( \frac{\mu_t}{\eta_t - \gamma / \alpha} \right) (1-s_t^F) f(k_{t-1}).
\]

In (32), \( \mu \) substitutes for the nominal interest rate, as arbitrage between nominal assets implies

\[
\mu_t = \beta \frac{\gamma}{\alpha} \left( 1 + \frac{1}{i_t} \right).
\]

Equilibrium nominal interest rates depend only on expected future money growth.

**3. Price Level Determination**

In the equilibrium outlined above expectations of government policies determine the returns on money holdings, bonds, and real investments, and therefore determine the price level. This
section derives the implications for the price level of various policies and explores the tradeoffs between current and expected policies that are consistent with equilibrium.

3.1 Policy Expectations and the Price Level

The velocity equation, (32), determines the price level, given the policy expectations functions \( \mu_t \) and \( \eta_t \). Equilibrium real balances are

\[
\frac{M_t}{P_t} = \Delta_t (1 - s_t^g) f(k_{t-1}),
\]

where

\[
\Delta_t = \frac{\mu_t}{\eta_t - \gamma/\alpha},
\]

with

\[
\Delta_t(\rho_F, \tau_F, s_F^g) = \frac{\beta^{\gamma/\alpha}}{1 - \gamma/\alpha} \left[ \frac{1 - \sigma \beta}{1 - s_F^g} \right].
\]

\( \mu \) and \( \eta \) capture three distinct aspects of the influence of policy expectations on the price level. The first works through \( \mu \), the marginal value of end-of-period real money balances, which is ubiquitous in dynamic monetary models. All else equal, changes in \( \mu \) imply changes in expected inflation and the rate of return on money holdings, producing the direct effects of monetary policy. Expectation of a higher rate of money growth depreciates the real value of money, lowers \( \mu \), induces substitution away from money, and raises the equilibrium price level.

\( \eta \) captures a Tobin effect through two interdependent impacts of expected policies. One impact is a direct tax distortion, which alters the private return on real assets. To isolate this effect, consider the impact of higher expected future taxes, holding future money growth and government-spending shares fixed. Further suppose that debt is identically zero and, in order to focus on substitution effects, that the revenues collected through higher distorting taxes are rebated as a lump sum. Higher future tax rates reduce the expected return on investment and induce agents to substitute from capital into consumption. A lower expected return on capital also induces substitutions into nominal assets, including money, and produces the Tobin effect. With the current money stock fixed, higher money demand drives down the price level today.

A second impact comes from \( \eta \)'s summary of the composition of expected fiscal financing in terms of the relative sizes of the real and the inflation tax bases. Higher \( \eta \) reflects an increase in expected nominal liability creation, a rise in the inflation tax base, and a reduction in the role of real taxation in financing government expenditures. This tradeoff can be seen heuristically from an alternative expression for the terms \( (1 - \tau)/(1 - s^g) \) that appear in the definition of \( \eta \) in (29). A transformation of the government budget constraint yields:

\[
\frac{1 - \tau_t}{1 - s_t^g} = 1 + \frac{(M_t - M_{t-1} + B_t - (1 + i_{t-1})B_{t-1})/P_t}{(1 - s_t^g)f(k_{t-1})}, \quad t \geq 0.
\]
Terms in \((1 - \tau)/(1 - s^g)\) reflect the fraction of private resources absorbed by the acquisition of new nominal liabilities issued by the government. Higher \(\eta\) indicates an expected shift in future financing that expands the inflation tax base and contracts the real tax base. By reflecting the relative sizes of the two tax bases, changes in \(\eta\) generate an expected inflation effect that is not embedded in the nominal interest rate. Whenever policies change \(\eta\), the conventional money demand expression in \((1)\) will mispredict the impacts of future policies on the price level.

3.2 Equilibrium Policies

The preceding discussion focused on how changes in rates of return on real and nominal assets affect the price level and how the rates of return relate to government policies. Well-posed policy questions require a complete specification of current and future policies. Even standard questions like “What are the impacts of an increase in the expected growth rate of money?” or “What are the effects of an increase in the expected tax rate?” must be linked to other policies to satisfy the equilibrium conditions and clear the government budget constraint.

Suppose expectations are initially consistent with equilibrium, and that agents expect future government expenditures to be financed by money creation or tax receipts. Absent some other change in government expenditures or taxes, an increase in the growth rate of money (a decline in \(\mu\)) increases seigniorage in the future, and neither current nor future government budget constraints are satisfied. Similarly, a rise in the expected tax rate (a fall in \(\eta\)) is consistent with equilibrium only if it is associated with an appropriate expected change in future money growth, government spending, or debt creation.

Not only do current policies constrain future policy options, but variations in fiscal financing in the future can also require changes in current policy. Any change in expected future policy that affects the current price level will alter the real values of current changes in money and debt. As a result, the government budget constraint will continue to be satisfied only under specific circumstances.

To analyze how jointly consistent combinations of changes in current and future policy affect the price level, we must consider (i) which policy combinations are consistent with equilibrium given current expectations (\(\mu\) and \(\eta\)) and (ii) how current policy changes affect the set of future policies that are consistent with equilibrium—that is, which policy expectations are rational.

The government budget constraint in period \(t\) yields policy combinations that satisfy (i) given current expectations:

\[
\rho_t - 1 + \left( \frac{B_t}{M_t} \right) - \frac{(1+i_{t-1})}{\rho_t} \left( \frac{B_{t-1}}{M_{t-1}} \right) \Delta_t = \frac{s_t^g - \tau_t}{1-s_t^g},
\]

where \((B/M)_t = B_t/M_t\) and \(\Delta_t\), defined in \((35)\), summarizes the given expected policies.\(^{11}\)

We now characterize which future (date \(F\)) policies satisfy (ii), given the state of government indebtedness at date \(t\), as captured by the bond-to-money ratio \((B/M)_t\). Assume that future interest liabilities are correctly anticipated at time \(t\) and that the bond-to-money ratio is constant.

\(^{11}\) Details of the derivations underlying \((38)\) and \((39)\) appear in Appendix A.
at \((B/M)_F\) in the future stationary equilibrium. Re-labeling variables dated \(t+1\) with an “F” subscript and imposing a stationary equilibrium yields:

\[
\Delta_t = \left( \frac{s^g_t - \tau^g_t}{1 - s^g_t} \right) \left[ \frac{1}{\frac{B}{M}_F} - \frac{1}{\beta} \frac{B}{M}_t + \frac{\rho_F - 1}{\rho_F} \right].
\]  

(39)

Given government indebtedness at the end of period \(t\), expression (39) describes the tradeoffs among future policies consistent with equilibrium and a given set of current policies.\(^{12}\)

To deduce the impacts on the price level of changes in monetary and fiscal policy, the contemplated policies must satisfy relationships (38) and (39). Those relationships imply that standard questions about the impacts of policy changes or changes in expectations may not be consistent with equilibrium now or in the future, and must be reinterpreted after being coupled with policies that jointly determine the equilibrium.

3.3 The Effects of Changes in Expected Policies

3.3.1 Expected Inflation

We have already considered the usual description of the effects of an increase in expected inflation due to an increase in expected money growth. The expected inflation rate rises, the real return on money holdings falls (\(\mu\) falls), and the current price level rises. If we start from an initial equilibrium in which the government has a positive return from net money and bond growth, this cannot be the total adjustment because the real value of current nominal liability creation has fallen and neither current nor future government budget constraints are satisfied.

The full effect of the change in expected monetary policy depends on the changes in other policy expectations that produce the new equilibrium. A decrease in future taxes or an increase in future government spending generates a substitution away from money and towards real saving. This substitution further increases the current price level. Higher expected money growth directly raises the price level and reduces the real return to current nominal liability creation. If current taxes rise to offset the loss in revenue, there is no additional impact on the price level because private real resources are unchanged. If government spending falls instead, private resources increase, raising the demand for money and moderating the effect on the price level.

3.3.2 Tax Policy Expectations

In section 3.1 we considered the direct effects of changes in expected taxes (through \(\eta\)). A fall in the expected future tax rate produces an increase in current capital investment, substitution away from nominal assets, and a rise in the current price level. Again, the resulting fall in government revenues is not consistent with equilibrium, in either the current period or in the future, without some further adjustment in policy. If the fall in expected tax receipts is compensated for by an increase in expected money growth, expected inflation rises and the rise

\(^{12}\) We focus on government fiscal obligations in the form of debt, but any expenditure commitments, such as spending programs, Medicare, or Social Security, that constrain future policy options are subject to the same analysis.
in the current price level is exacerbated. The result is identical to the analysis above of a change in expected money growth with the tax rate adjusting to maintain the same level of government spending.

In contrast, suppose the fall in the expected tax rate is associated with an equal decline in expected government spending and no change in money growth. Under the assumption that there is an expected government deficit ($s^* > \tau_e$), $\eta$ falls. This change in expectations directly increases the demand for money and decreases the price level. Neither current nor future government budget constraints clear with this policy change alone. A lower current price level increases the real return to money creation. Similarly, the rise in the government-spending share in the future implies that the price level rises in the future, decreasing expected seigniorage (a decline in the seigniorage tax base with no change in the stationary inflation rate reduces seigniorage revenues). With unchanged monetary policy, future taxes must rise more than government spending, moderating the impact on the current price level.

4. The Quantity Theory of Money

Our perspective on price determination emphasizes that the price level depends on jointly consistent current and expected policies. This analysis does not conflict with either the quantity theory of money or the fiscal theory of the price level: each theory arises from particular sets of restrictions on policies.

4.1 The Quantity Theory of Money

The quantity theory is sometimes described as positing strict proportionality between the supply of money and the aggregate price level, and delivering the corresponding neutrality of money. Friedman (1956) rejects this depiction as too extreme and provides a short description of the quantity theoretic perspective. His description includes (a) a demand for money relation that depends on a restricted set of variables and (b) an assumption that the money demand function is stable.\(^\text{13}\)

Even in the simple model considered here, (a) is satisfied only if the restricted set of variables includes the parameters of the processes determining policy. In addition, those parameters must be observable. To satisfy (b) we must add the strong assumption that the policy process is “stable” in some well-defined sense.

The velocity equation is often expressed in the form of an equilibrium “money demand” function. While not formally a demand function, it has the familiar form of including an index of expenditures and a measure of opportunity cost. Although Friedman’s (1956) general demand function includes a role for expected inflation that is independent of the nominal interest rate, most monetary analyses include only this rate-of-return influence of expected inflation [for example, Laidler (1985)].

Expected inflation affects the demand for money by changing the own return on nominal money balances, which is embedded in the nominal interest rate, $i$. A second expected inflation effect operates through the composition of future fiscal financing, which $\eta$ summarizes. Changes

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\(^{13}\) Friedman also requires a degree of independence between the factors determining the supply and demand for money. We have nothing to add on this point.
in \( \eta \) trigger the asset substitutions that generate the Tobin effect. The velocity equation, expressed in terms of the nominal interest rate, is:

\[
\frac{M_t}{P_t} = \frac{\beta \gamma \left( \frac{1 + \i_t}{\i_t} \right)}{\alpha} \frac{1}{\eta_t - \gamma / \alpha} (1 - s^e_t) f(k_{t-1}).
\]

(40)

This expression underscores that the nominal interest rate is generally an inadequate measure of the opportunity cost of money balances. Without auxiliary assumptions that eliminate the Tobin effect and the associated substitutions between real and nominal assets, the nominal rate incompletely summarizes the opportunity cost.

Special policy assumptions are necessary for the price level to be determined by monetary policy alone. Those assumptions result in a dichotomy between the nominal and real sides of the economy, but not an independence of the price level from debt. Consider a policy that sets the net-of-interest budget surplus to zero each period, so \( \tau_t = s^e_t \) for all \( t \). In this special case, the model delivers a simple quantity theory because \( \eta_t = (1 - \sigma \beta \gamma / \alpha) / (1 - \sigma \beta) \) for all \( t \). This policy removes substitutions between nominal and real assets, so the demand for money stems entirely from arbitrage between bonds and money. Equilibrium real balances reduce to

\[
\frac{M_t}{P_t} = h(i_t, \epsilon_t + k_t),
\]

(41)

which is analogous to the specification in (1). Now the price level is independent of fiscal policy, but it is not independent of debt unless debt is zero: money growth must finance the interest obligations, so the level of debt matters for the level of real money balances.\(^{14}\)

4.2 Representations of Money Demand

Friedman (1956) presents the demand for money as a function of a vector of rates of return on money and alternative forms of wealth. It is evident that stability of the simple money demand relation in (41) requires special assumptions about fiscal policy. We now relax those assumptions to ask whether expectations of fiscal policy summarized by \( \eta \) in (40) can be replaced by a judicious choice of rates of return.

Can (40) be converted into a simple and stable money demand relation if it is expressed in terms of a real return and expected inflation, as Friedman argues? Because the real interest rate described by \( r \) includes transactions costs, no simple relation links it to the nominal rate and the expected inflation rate. A simple Fisher relation can be obtained by adjusting the real rate for transactions costs. Let \( r^*_{t+1} \) denote the net real rate of return on capital between \( t \) and \( t + 1 \) after adjusting for transactions costs. The net-of-transactions-costs real intertemporal marginal rate of transformation is given by

\[
\varphi_t = \beta E_t (1 + r^*_{t+1}) \varphi_{t+1}.
\]

(42)

\(^{14}\) With lump-sum taxes, the quantity theory and Ricardian equivalence hold under the wider range of policies described in Leeper (1991) as “active monetary” and “passive fiscal” policies [see also Woodford (1999) or Cochrane (2001b)]. The convention of using lump-sum taxes in association with the quantity theory is consistent with Friedman’s (1976) “Marshallian approach to theory,” as opposed to “Tobin’s Walrasian approach.”
Simplification yields the Fisher equation

\[ 1 + i_t = E_t(1 + \pi_{t+1})(1 + r^*_{t+1}). \]  

Combining this simple Fisher equation with (40) yields an equivalent version of (40) in terms of expected inflation and the real return on capital adjusted for transactions costs:

\[ \frac{M_t}{P_t} = \frac{\beta \gamma}{\alpha} \left( \frac{1}{E_t(1 + \pi_{t+1})(1 + r^*_{t+1})} + 1 \right) \frac{1}{\eta_i - \gamma/\alpha} (1 - s^e) f(k_{t+1}). \]  

This money demand relation is stable only under certain assumptions about fiscal policy.

An alternative expression involving the real rate of return, \( r \), may be obtained using the capital Euler equation, (20):  

\[ 1 + i_t = E_t(1 + \pi_{t+1})(1 + r_{t+1}) \left( \frac{\eta_i - \gamma/\alpha}{\eta_i} \right). \]  

Using (45) yields another equivalent, although somewhat more complicated equilibrium money demand relation in terms of expected inflation, the expected return on capital, and expected fiscal variables:

\[ \frac{M_t}{P_t} = \frac{\beta \gamma}{\alpha} \left( \frac{1}{E_t(1 + \pi_{t+1})(1 + r_{t+1})} - 1 \right) \frac{1}{\eta_i - \gamma/\alpha} (1 - s^e) f(k_{t+1}). \]  

Effects of expected fiscal policies continue to appear separately through \( \eta_i \).\(^{15}\) Brunner and Meltzer (1972) specify a money demand function that includes the expected real return on real assets, an argument closely related to \( \eta_i \).

On a theoretical level there is no basis for distinguishing among these various representations of equilibrium \( M/P \). They may have some descriptive value in terms of the nature of the substitutions at work, but they are not structural relationships, invariant to policy interventions. Moreover, because expectations of policy enter as an argument in all these relationships, strong assumptions are necessary for these relationships to be stable over time.\(^{16}\)

Friedman argues that if asset substitutions are very stable, the price level can be determined without analyzing all the margins relevant in the general specification. This argument may be without fault on a theoretical level. It may also be consistent with some broad empirical regularities. But our analysis is another example of circumstances under which it is misguided to treat an empirically stable relationship involving real money balances as identifying structural elements of the economy.

\(^{15}\) It is worth point out that adding a long-term nominal interest rate, as Meltzer (1963) and Lucas (1988) do, does not eliminate the Tobin effect from the money demand function. Arbitrage between the one-period bond and the long-term bond implies that the equilibrium long rate also depends only on expected money creation. Similarly, considering alternative scale variables, such as consumption, also does not eliminate the Tobin effect.

\(^{16}\) Lucas (1988) makes a similar observation about the money demand function he derives by presenting agents with a portfolio problem.
5. The Fiscal Theory of the Price Level

Under policy assumptions that effectively suppress substitutions between real and nominal assets, a version of the conventional fiscal theory emerges: a current fiscal expansion, supported by appropriate monetary policy, increases current demand for goods and drives up the price level. Under different, equally plausible policy assumptions, fiscal policy continues to influence the price level, but with very different implications—a current fiscal expansion, holding the money stock fixed, increases the demand for money and reduces the price level.

5.1 Expositions of the Fiscal Theory

Many expositions of the fiscal theory spring from the following equilibrium condition:

\[
\frac{W_{t-1}}{P_t} = E_t [\text{Discounted Present Value Surpluses Plus Seigniorage}] .
\]  

(47)

This equilibrium condition holds across economic models, independently of the monetary and fiscal policies in place. Cochrane (1999) and Woodford (2001), in expounding the fiscal theory, imagine this thought experiment: news arrives leading agents to expect lower future surpluses, while all other future policies are held fixed. \(W_{t-1}\), as the total nominal government liabilities at the beginning of period \(t\), is predetermined. To ensure that (47) is satisfied, \(P_t\) must rise in response to the lower expected surpluses.

Several questions arise from this thought experiment. First, are there reasonable conditions under which this experiment can occur in equilibrium? Woodford and Cochrane—and many others—assume that monetary policy pegs the nominal interest rate and the fiscal surpluses are exogenous. Second, if we introduce a margin between real and nominal assets by adding capital and modeling tax distortions, then are there reasonable conditions under which the thought experiment is well posed? Under those conditions do we obtain the fiscal theory conclusion that the price level rises?

These are fundamental questions that underscore the confusion surrounding the fiscal theory of the price level. Confusion stems from several factors: (i) efforts to draw causal inferences from an equilibrium condition like (47); (ii) the assumption that there is no substitution between real and nominal assets; (iii) in some papers, the absence of a completely solved model that produces a complete characterization of the equilibrium.

The confusion has spawned a variety of criticisms of the fiscal theory. Buiter (2002) accuses the literature of being logically inconsistent. Kocherlakota and Phelan (1999) suggest the literature inappropriately treats the public and the government asymmetrically in their perceptions of their respective budget constraints. Bassetto (2002) calls for and provides an explicit characterization of how policy authorities behave off of equilibrium paths. McCallum (2001, 2003) argues that a particular formulation of the fiscal theory produces multiple equilibria and the one selected by the fiscal theory literature is unappealing because it is not E-stable. But as Evans and Honkapohja (2002) show, conclusions about the E-stability of the fiscal theory depend on which version of the theory one considers. The conventional theory presented in

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\[17\] Condition (47) stems from imposing transversality conditions on the government’s intertemporal budget constraint.
Leeper (1991) and Woodford (2001) and in section 5.3 below, for example, does not hold fixed the path of the nominal money stock and is E-stable, according to Evans and Honkapohja.

It is our view that these criticisms, though understandable in light of the many expositions of the fiscal theory, are distractions. Something like the fiscal theory can emerge from a completely characterized equilibrium under certain restrictive conditions on monetary and fiscal policy behavior. Under alternative assumptions, however, a rather different version of the fiscal theory emerges. That version is quite non-Keynesian, as section 5.4 shows.

5.2 Unpleasant Monetarist Arithmetic as a Fiscal Theory

Sargent and Wallace (1981) develop an environment in which fiscal policy drives inflation. In their example, limitations on tax receipts result in a fiscal responsibility for the monetary authority. As Sargent (1986) emphasizes, this perspective places responsibility for controlling inflation equally on monetary and fiscal authorities.

The potential for unpleasant arithmetic arises in the present model when considering an open market operation. Consider a bond sale with $M_t + B_t$ fixed. In addition, fix the sequence $\{s_t^g, s_t^F\}$ and the current value of $τ_s$. Because the monetary contraction, engineered through an open-market sale, raises future government indebtedness, it is clear that some policy must change in the future. With government-spending shares fixed exogenously, either future taxes, $τ_s^F$, or future money growth, $ρ_s^F$, must rise to accommodate the increase in debt service. Unpleasant monetarist arithmetic arises when individuals expect higher money growth in the future, which lowers the expected return on money (lower $µ$), decreases money demand, and raises the price level. This effect is counteracted by the direct negative effect on the price level from the decrease in $M_t$ due to the open-market sale.

The ultimate effect on the price level can go either way, as Sargent and Wallace show. In this model, it depends on how much future money growth must adjust following an open-market operation to be consistent with equilibrium. The change in future money growth, in turn, depends on the bond-money ratio at the time of the nominal asset exchange. To see this, note that the proportion by which $(B/M)_t$ changes from a given open-market operation varies with the initial bond-money ratio. And the larger the change in $B/M$, the greater is the change in debt service and, therefore, money growth and inflation in the future.

Price level changes from open-market operations depend on the corresponding changes in policy expectations, and on the state of government indebtedness. State-dependence of the price effects of changes in money supply produced by normal central bank procedures arises from fiscal considerations frequently overlooked in conventional monetary analyses.

5.3 The Canonical Fiscal Theory Exercise

Like the quantity theory, the fiscal theory can be understood as eliminating all substitutions between real and nominal assets, and emerges as a special case of the general analysis above. What we term the “canonical” fiscal theory exercise stems from the policy assumptions in Leeper (1991), Sims (1994), and Woodford (1995), among many others. This exercise restricts asset substitution to occur only between current nominal assets.

Assume all future policies, $(ρ_s^F, τ_s^F, s_t^F)$, and $s_t^g$ are fixed. These policies make $η_s$ constant and equilibrium real money balances again reduce to (1). They also peg the nominal interest rate.
Under these assumptions, consider the impact of a bond-financed tax cut, so $\tau_t$ falls and $B_t$ rises. Are fixed expected future policies consistent with equilibrium? If current money growth is unchanged, then future government liabilities, summarized by $(B/M)_t$, rise. Government budget constraint (39) implies that some future policy must change. There is only one equilibrium consistent with unchanged future policies; that equilibrium arises when current money growth expands to prevent $(B/M)_t$ from rising when nominal debt expands. Expression (38) yields the monetary adjustment required when $\tau_t$ is reduced, if future government liabilities are to remain unchanged. Note that future money growth is held fixed. The current monetary expansion required to maintain equilibrium is exactly enough so that the increase in future seigniorage (because the level of money supplied is now higher) at the fixed rate of monetary growth just suffices to pay for the increased debt service. To determine the effect on the equilibrium price level, note that fixing $(s^g, \rho_F, \tau_F, s^g_F)$ pegs the nominal interest rate and determines a constant level of real money balances. The new higher level of $M_t$, together with (32), yield the new higher equilibrium price level. As in unpleasant arithmetic, monetary policy is constrained by the government’s fiscal obligations.

5.4 Pure Fiscal Effects

Fiscal policy may affect the price level independently of monetary policy. Consider the impact on the price level of the substitution between current and future taxes that a debt-financed tax cut induces. If the sequences of money growth rates and government-spending shares, $\{\rho_t, \rho_F, s^g_t, s^g_F\}$, are fixed exogenously, a bond-financed tax cut at $t$ increases government liabilities in the future and necessitates some change in future policy, as (39) indicates. In the spirit of a Ricardian experiment, suppose that future direct taxes rise. Higher $\tau_F$ reduces the expected return on physical capital. A lower return on the real asset relative to nominal assets increases the value of the current stock of money and bonds and reduces velocity. With the money stock fixed, the price level at time $t$ falls. A Tobin effect gives debt a natural role in determining the price level, and a role that is independent of the stock of money. This pure fiscal effect from a tax cut reduces nominal demand and the price level, producing non-Keynesian impacts from a debt-financed fiscal expansion.

One consequence of the policy change is that even though money growth is constant, seigniorage revenues rise with the lower price level, due to the impacts of future taxes on the demand for money and equilibrium real money balances. Of course, the change in seigniorage is a consequence of the fiscally induced change in the price level. Designating either the fiscal policy change or the response of monetary policy as the source of the price level change is completely arbitrary.

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18 Again, with lump-sum taxes, a range of policies is consistent with the fiscal theory. Leeper (1991) labels them “active fiscal” and “passive monetary” policies.
19 Models with a single asset make the logical point that the fiscal theory can be independent of money: Woodford’s (1998a, 1998b) “cashless limit”; Sims’s (1997) model with nominal bonds and no money; Cochrane’s (2001b) model with zero overnight money demand.
6. Concluding Remarks

Thirty years ago the profession was unclear about the precise implications of the quantity theory of money. Today confusion centers on the fiscal theory of the price level. This paper takes a portfolio choice perspective in which the price level depends generically on both current and expected future monetary and fiscal policies. From this perspective the quantity theory of money and the fiscal theory of the price level are highly restricted special cases.

The quantity and fiscal theories emerge under restrictions on policy behavior that eliminate substitutions between real and nominal assets. The conventional quantity theory of money emerges only when neither money creation nor debt expansion finances fiscal expenditures. If all future policies are fixed, the current money stock must adjust passively to satisfy money demand at a pegged nominal interest rate. This produces the canonical fiscal theory of the price level example but attribution of the resulting price change to one policy or the other is arbitrary.

Although the required policy restrictions are severe, they are revealing about conditions consistent with either the quantity theory or the fiscal theory is likely to prevail. During periods when direct tax revenues approximately cover government spending and monetary policy is conducting routine open-market operations, as an empirical regularity the quantity theory should hold well. On the other hand, periods when the central bank supports bond prices (as it might during wars), we should find that fiscal policy plays a larger role in determining the price level.

These restrictions also point toward circumstances when the quantity theory and the fiscal theory are likely to breakdown. Those circumstances include periods when expectations of fiscal policy change in important ways, triggering substitutions between real and nominal assets. The Tobin effect should be important for large changes in macro policies, including tax reforms, big new spending initiatives, or changes in the inflation target pursued by the central bank. These circumstances call for a more general money demand specification than commonly appears in macro models.

The quantity theory and the fiscal theory approaches to price determination are powerful in their simplicity but require assumptions on policy behavior that are unlikely to hold generally. We have offered a broader perspective that admits the two theories as special cases. The combination of portfolio choice and mutually consistent current and expected macro policies applies generally, with only a small sacrifice in simplicity.
Appendix A: The Government Budget Constraint

This appendix uses the model’s equilibrium together with the government budget constraint to obtain the equilibrium connections among current and future policies. Those connections are used in section 3.2 where particular policy rules are studied.

Using expression (32) for equilibrium real balances, the government budget constraint in period $t$ can be written as

$$1 + \left( \frac{M_t - M_{t+1} + B_t - (1 + i_t)B_{t+1}}{M_t} \right) \Delta_t = \frac{1 - \tau_t}{1 - s^e_t}.$$  \hspace{1cm} (A.1)

where

$$\Delta_t = \frac{\mu_t}{\eta_t - \gamma/\alpha}. \hspace{1cm} (A.2)$$

Equivalently,

$$\left[ \frac{\rho_{t+1}}{\rho_t} - 1 \right] + \frac{B_t}{M_t} - \left( 1 + i_{t+1} \right) \frac{B_{t+1}}{M_{t+1}} \Delta_t = \frac{s^e_t - \tau_t}{1 - s^e_t}. \hspace{1cm} (A.3)$$

Given expectations of policy, as embodied in $\Delta_t$, (A.3) reports the tradeoffs that exist in equilibrium among current policies, given the initial conditions summarized by $(1 + i_{t+1}) (B_{t+1}/M_{t+1})$.

We now seek to characterize the tradeoffs that exist among future policies given the current state of government indebtedness. Combine the equilibrium nominal interest obtained from the arbitrage between money and bonds,

$$\mu_t = \beta \frac{\gamma}{\alpha} \left( 1 + \frac{1}{i_t} \right), \hspace{1cm} (A.4)$$

with the recursive representation of $\mu$,

$$\mu_t = \beta \left[ \frac{\gamma}{\alpha} + E_t \frac{\mu_{t+1}}{\rho_{t+1}} \right], \hspace{1cm} (A.5)$$

to obtain

$$1 + i_{t+1} = 1 + \frac{\gamma/\alpha}{E_t(\mu_t/\rho_t)}. \hspace{1cm} (A.6)$$

Substitute (A.6) into (A.3) and push the dating forward one period to obtain

$$\frac{1}{\rho_{t+1}} \left[ \frac{B_{t+1}}{M_t} - \left( 1 + \frac{\gamma/\alpha}{E_t(\mu_{t+1}/\rho_{t+1})} \right) \frac{B_t}{M_t} - 1 \right] + \frac{s^e_t - \tau_{t+1}}{\Delta_{t+1}(1 - s^e_{t+1})}. \hspace{1cm} (A.7)$$

Under the assumption of perfect foresight, policy choices at $t+1$ are known at $t$, so the equilibrium nominal interest rate adjusts to those choices. Then (A.7) may be rewritten as
\[
\begin{align*}
\left[ \frac{B_{t+1}}{M_{t+1}} - \frac{1}{\rho_{t+1}} \left( 1 + \frac{(\gamma/\alpha)\rho_{t+1}}{\mu_{t+1}} \right) B_t - \frac{1}{\rho_{t+1}} + 1 \right] \Delta_{t+1} &= \frac{s_{t+1}^g - \tau_{t+1}}{1 - s_{t+1}^g}.
\end{align*}
\tag{A.8}
\]

Denote future values by an “F” subscript, consistent with the assumptions about policy in (21), and denote current values by a “t” subscript. If \(B/M\) is constant in the future at the ratio \((B/M)_F\), so nominal debt grows at the same rate, \(\rho_F\), as money, then substituting for \(\Delta_{t+1}\) and imposing future values,

\[
\mu_{t+1} \left[ \left( \frac{B}{M} \right)_F - \frac{1}{\beta} \left( \frac{B}{M} \right)_t \right] + \left( 1 - \frac{1}{\rho_F} \right) \left( \frac{B}{M} \right)_t = \frac{s_{t+1}^g - \tau_{t+1}}{1 - s_{t+1}^g} \left( \eta_{t+1} - \gamma/\alpha \right),
\]

which reduces to

\[
\mu_{t+1} \left[ \left( \frac{B}{M} \right)_F - \frac{1}{\beta} \left( \frac{B}{M} \right)_t \right] + \left( 1 - \frac{1}{\rho_F} \right) \left( \frac{B}{M} \right)_t = \frac{s_{t+1}^g - \tau_{t+1}}{1 - s_{t+1}^g} \left( \eta_{t+1} - \gamma/\alpha \right).
\]

(A.9)

Substitute for \(\mu_{t+1}\) and \(\eta_{t+1}\) from (30) and (29) to obtain

\[
\frac{\beta^{\gamma} \alpha}{1 - \gamma/\alpha} \left[ \frac{1}{\rho_F - \beta \left( \frac{B}{M} \right)_F} \right] \left( \rho_F \left( \frac{B}{M} \right)_F - \frac{1}{\beta} \left( \frac{B}{M} \right)_t \right) + \left( \rho_F - 1 \right) = \frac{s_{t+1}^g - \tau_{t+1}}{1 - s_{t+1}^g - \sigma \beta (1 - \tau_F)}.
\]

(A.10)

Expression (A.11) can be rewritten in the form

\[
\frac{\beta^{\gamma}}{1 - \gamma/\alpha} \left[ \frac{1}{\rho_F - \beta \left( \frac{B}{M} \right)_F} \right] \left[ \left( 1 - s_{t+1}^g \right) - \sigma \beta (1 - \tau_F) \right] = \left( s_{t+1}^g - \tau_{t+1} \right) \left[ \rho_F \left( \frac{B}{M} \right)_F - \frac{1}{\beta} \left( \frac{B}{M} \right)_t \right] + \left( \rho_F - 1 \right) \left[ \frac{1}{\rho_F} \right].
\]

(A.12)

Given the state of government indebtedness at \(t\), as summarized by \((B/M)_t\), with which the economy enters the stationary equilibrium in periods \(s > t\), (A.12) characterizes the tradeoffs among future policies in equilibrium.

Using (A.2), together with (30), (29), and (A.12),

\[
\Delta_s = \left( \frac{s_{t+1}^g - \tau_{t+1}}{1 - s_{t+1}^g} \right) \left( \frac{B}{M} \right)_F - \frac{1}{\beta} \left( \frac{B}{M} \right)_t \left( \rho_F - 1 \right) \left( \frac{1}{\rho_F} \right).
\]

(A.13)

(A.3) and (A.13) deliver expressions (38) and (39) in the text.
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