A GAME THEORETICAL ANALYSIS OF SEXUALLY TRANSMITTED DISEASE EPIDEMICS

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ABSTRACT

Social scientists who study sexual behavior have consistently found that some HIV-infected individuals continue to have unprotected sex with uninfected partners. In this essay, we address three questions that stem from this empirical finding. First, what can game theory contribute to the study of sexual behavior? Second, can a person deduce the HIV status of a potential sexual partner from observed behavior? Third, what are the implications of a game theoretic analysis for infection rate dynamics? Briefly, we argue that game theory models capture the interactive aspects of sexual behavior such as signaling status through behavior. We then go on to show that some simple signaling game models predict that behavior does not distinguish infected from uninfected partners. In those models, uninfected individuals will engage in high-risk sex with potentially infected partners if the perceived rate of infection is sufficiently low. Otherwise, they engage in low-risk sex. The final section of the paper analyzes an epidemiological model where individuals play the signaling game in discrete time periods. Simulations of the model under varying conditions show that the most virulent epidemics occur when individuals randomly select partners but base their estimate of the infection rate on the percentage of socially close individuals who are infected. Simulations also show that epidemics where individuals restrict partners slow the spread of disease more than other models but that the epidemic lasts longer. Another implication is that with replacement, infection rates will oscillate because individuals will switch from low to high-risk sex as the percentage of infected individuals changes. We conclude by discussing empirically testable predictions of the model.

KEY WORDS • epidemic models • game theory • HIV and computational model • signaling games

Social scientists who study sexual behavior have found that some individuals who contract HIV and other sexually transmitted diseases sometimes continue to put others at risk by engaging in risky sex (Wenger et al. 1994; Kalichman et al. 1997; Eich-Hochli et al. 1998). Such findings are troubling: in addition to exposing HIV individuals to potentially lethal illness, this behavior demonstrates that some individuals place self-gratification above public health. The purpose of this article is to model the sexual behavior of these individuals and study the effects of such behavior on the spread of HIV through a population.

Our model analyzes situations where HIV+ and HIV− individuals interact and where individuals know if they are infected but do not share this information with potential sexual partners. Coordination under conditions of limited information is the central issue: uninfected individuals try to ‘match’ with other uninfected individuals, and HIV+ individuals attempt to engage in unprotected sex with any partner regardless of that partner’s HIV status. When potential sexual partners can’t definitively determine each other’s HIV status, the best that they can do is attempt to determine each other’s status through observing his/her behavior. One goal in this article is to argue that the failure to correctly match potential sexual partners is a basic feature of casual sexual contacts. Our model predicts that infected and uninfected individuals will behave in the same way when perceived infection rates are sufficiently low.

The second goal of our article is to construct and analyze a discrete time epidemic model that is based on the micro-level theory presented in the article. In our model, individuals at a given time period will be randomly matched and they will try to deduce their potential partner’s status. Their ability or inability to interpret behavior will determine the type of sex they have, which may result in the transmission of disease. Infection rates will change and future behavior will change in response to new conditions. This endogenous process model, though simple, yields a number of interesting results from an epidemiological and public policy perspective.

We employ a game theoretical framework for this article because it is a natural setting for investigating such coordination issues and for extending micro-level theories to macro-level processes (Coleman 1994). Game theoretic models capture several important features of sexual behavior, such as signaling HIV status, through behavior. Game theory models allow for the interdependence of action. Furthermore, signaling game theory, which forms the basis
for our analysis, provides the tools for connecting micro-level analysis with macro-level processes because the interpretation of observed behavior, ‘signals’, depends on prior beliefs which themselves may be based on potentially erroneous perceptions of global processes. The use of game theory in conjunction with epidemiological models allows for a rich analysis of the relationship between rational action, individual beliefs, social structure, and the spread of disease.

In this article, we discuss the nature of the HIV problem and present our game theoretical analysis by describing the HIV problem, discussing alternative approaches to this problem, reviewing the empirical literature that motivates our model, presenting the model and focusing on situations where behavioral signals fail, study the implications of the game theoretical model for the analysis of epidemics through a series of computer simulations, and conclude by discussing testable hypotheses derived from the analysis.

A note of caution is in order at this point. The reader should understand that our model is not intended to describe every possible sexual encounter that might result in the transmission of disease. For example, we do not consider the situation where an individual transmits HIV because he or she is an unknowing carrier (though we acknowledge that this is an important source of HIV transmission). Since we discuss in some detail the situation where an HIV+ individual desires to have unprotected sex with an HIV− one, the reader might infer that the authors believe that HIV+ individuals recklessly put others at risk. This is not the case. We recognize that many individuals who are afflicted with HIV behave responsibly and that to suggest otherwise would be irresponsible and insulting to those who suffer from this illness. Nevertheless, a small but significant number of HIV+ individuals do behave this way, and this poses not only a sociological problem but also a continuing threat to public health. It is the presence and interactions of these individuals with the larger HIV− population which are central to our analysis.

The HIV Problem: The Decline of Mortality, the Stabilization of Infection Rates, and the Value of Condom Use

Recent advances in medical treatment for people infected with HIV have dramatically reduced the annual rate of AIDS cases and
AIDS-related deaths in the United States since late 1996. According to the Center for Disease Control, however, the rate of new incidents of HIV infection has not dropped significantly over that same period of time (New York Times, 31 August 1999). This, coupled with recent HIV booms in African-American populations, suggests that many people continue to engage in HIV-transmitting behaviors, including unprotected (high-risk) sex, despite the widely accepted fact that condoms greatly reduce the likelihood of HIV transmission. Much previous research and theorizing has attempted to understand and account for the factors that influence people's decision to use condoms during sexual activity, but such work has tended to focus on the psychological attributes of a single individual or has placed the decision within the context of on-going and socially or culturally embedded sexual relations. While these perspectives may provide some accurate description of sexual behavior, they are less able to offer a meaningful explanation of that behavior.

STD transmission research has shown that the number of sexual partners is positively correlated with the incidence of HIV and other STDs (Pandian et al. 1990; Laumann et al. 1994). Because vaginal/anal intercourse without the use of condoms remains a relatively efficient and common means of transmitting HIV from one person to another, high rates of sexual partners pose a significant threat to members of a population engaging in this pattern of sex where any members of that population are carriers of HIV. Such a pattern undoubtedly played a large factor in the rate of HIV infection among gay men in the U.S. in the 1980s (see Shilts 1993).

Given a high rate of sexual activity for at least some of its members, the gay community has responded to the presence of HIV by campaigning for the modification of sexual practices, the increased use of condoms, and the encouragement of celibacy and of monogamous coupling. However, for the substantial portion of the population which fails to adopt a celibate lifestyle as a disease-prevention technique (a pattern noted in Anderson et al. 1990; DiClemente 1990; Kann et al. 1991; Catania et al. 1992; Hein 1992; Peterson et al. 1992), and which is not in monogamous relationships, changes in sexual behavior and the use of condoms stand as the principal means of preventing the sexual transmission of HIV (Van de Perre et al. 1987; Cates 1990; Cates and Stone 1992; DiClemente and Peterson 1994). Because HIV can be transmitted through a single instance of unprotected high-risk sex, and because of the seriousness of the disease, the need for the regular
use of condoms as a preventive measure against HIV remains high; according to Reiss and Leik (1989), condom use has been shown to be a ‘far more effective method of reducing HIV infection’ (p. 411) than simply reducing the number of sexual partners.

How effective are condoms in preventing HIV transmission? A recent review of 25 published studies of condom use and HIV transmission among heterosexuals (Davis and Weller 1999) suggests that the overall effectiveness of condoms for preventing HIV transmission from HIV+ to HIV− partners is approximately 87 percent, though the 95 percent confidence interval ranges from 60 percent to 96 percent. This study concludes that condom efficacy as an HIV prevention technique is similar to that for preventing pregnancy, that is, condoms are effective though somewhat imperfect. We assume in this article that the effectiveness of condoms is comparable (if not in fact identical) for heterosexual and male homosexual couples.

The AIDS epidemic has started its third decade in the Western world, and as a result of national and international health campaigns public awareness of HIV is high: most sexually active individuals in Western countries, at least, have some sense about what HIV is and how it is acquired. But the simple dissemination of this information has not led to the eradication of new cases of HIV infection. Although any HIV individual having sex with an unfamiliar partner would appear to have a strong incentive to protect him- or herself from the possibility of acquiring HIV, many such individuals still choose not to take this action (and may do so regardless of the availability of condoms). Conversely, an HIV+ individual having sex with an unfamiliar partner may have significant incentives to put that partner at risk.1

These patterns of behavior can be examined using a number of theoretical models, including the ‘Health Belief Model’, the ‘Communication Perspective’, and the ‘Theory of Reasoned Action’. Each of these represents an attempt to understand high-risk behavior, but each also fails to address the fundamental problems of trust, deception, and behavioral incentives that are resolved in the game theoretic model.

The Health Belief Model

The fear of transmission alone does not prevent people from engaging in high-risk behaviors. A group of social psychologists in
the U.S. Public Health Service developed the Health Belief Model (HBM) in the 1950s in an effort to explain the widespread failure of people to participate in various disease protection and prevention programs (Hochbaum 1958; Rosenstock 1960, 1966, 1974). This model has also been used to evaluate people’s decisions about utilizing condoms to prevent HIV transmission during sexual activity (Dey 1993; Rosenstock et al. 1994; Blevins 1998). As a ‘value-expectancy’ theory of behavior, the HBM is based on cognitive psychological theory: the perceived value of a particular outcome is combined with a subjective expectation that this outcome will occur to create a rational description of a particular person’s actions.

Briefly, the HBM predicts that individuals will modify their behavior when they feel threatened by their current behavior, believe that the consequences for failure to change are high, believe that a particular change will significantly alter the existing threat, believe that the necessary changes are reasonable, and/or believe that they are capable of making these changes. The model has received substantial empirical support, with ‘perceived barriers’ as the single most powerful predictor of behavior (Janz and Becker 1984).

However, the model’s weakness in its particular attempt to describe sexual behavior lies in that it fails to consider the two-person nature of sexual activity. While health-related considerations doubtless play a considerable role in an individual’s decision to use condoms, they only describe a portion of the decisions that two actors make in the moment of deciding to engage in sex. An individual may decide to risk contracting disease in order to have sex, but he/she must find a partner who is willing to bear some risk as well. The HBM does not address this fact. It is a useful tool for understanding individual decision-making processes, but is less useful for understanding the interpersonal processes of the exact moment when rational health-related decisions must be made in a sexual context, since any consensual sexual activity requires some degree of negotiation between two actors.

The Communication Perspective

Among other methods used to describe decisions concerning safer sex is the Communication Perspective, a direction in modeling which seeks to incorporate the effects of both public and private level messages about safer-sex practices to describe the decisions individuals make about sex. This perspective examines the concepts
of sexual scripting, the influence of social norms, and the role that interpersonal communication plays in the negotiation of condom use.

Simon and Gagnon (1987) identify a number of script types that individuals draw upon for sexual decision-making, while acknowledging that these scripts ‘provide little instruction . . . for appropriate behavioral sequences in novel, ambiguous, and emergent situations—precisely the kind of situation in which sexually active people find themselves when they want to have sex with an unfamiliar partner’ (Metts and Fitzpatrick 1992: 5). These are the precise conditions under which the practice of safer sex for the prevention HIV transmission is most crucial.

The influence of social norms in the promotion of safer sex practices is another aspect of the Communication Perspective. As a variation on this perspective, the Morin model (Morin and Batchelor 1984; Puckett and Bye 1987) suggests that the promotion of safer sex as a satisfying activity and the belief that one has the encouragement of one’s peers in adopting safer sex practices are important variables in changing sexual behaviors, and Fisher (1988) emphasizes the role of peers and the creation of safe-sex norms in the prevention of HIV transmission. The power of norms in the evolution and maintenance of particular patterns of behavior is clear: norms can reduce (though probably never eliminate) the perceived costs of condom use, can stigmatize unsafe-sex practices, and can work to create social environments which are hostile to HIV transmission.

A central part of the interpersonal aspect of the Communication Perspective relies on the role of the exchange of historical information between sex partners. Before sex takes place, two individuals frequently have the opportunity to verbally ‘feel each other out’, to discuss their likes and dislikes, and to exchange information about their personal histories. Public policy advocates have encouraged such behavior in the past as a method for controlling the spread of HIV—‘knowing your partner’ appears initially as an important method for individuals to use when screening out potential HIV carriers or for ensuring the use of safer-sex practices with those partners who are in fact HIV+.

The Communication Perspective, however, has serious limitations. What this model fails to consider is that when the participants are not well acquainted or are not firmly embedded in overlapping social networks, one partner or the other may have an incentive to
exercise deception in order to engage in risky sexual behavior. Because it is often difficult for two individuals to know with certainty how honest the other person is being, attempts at communication can actually backfire: a person’s claims of being HIV—may be false (‘I was just tested a month ago and I assure you I am negative’), his or her historical preferences for safer-sex practices may be expressed inaccurately (‘I am one of those people who always uses condoms with their partners, so if you are negative then I won’t need to use one with you’), and relevant messages may be misleading or misunderstood (‘I am healthy’). Under such conditions, the role of communication is not only limited but is actually dangerous because of the significant incentives and opportunities for distortion and misinformation.

Theory of Reasoned Action

Of the three perspectives we discuss in this article, the Theory of Reasoned Action (TRA) (Fishbein and Ajzen 1975; Ajzen and Fishbein 1980) comes closest to considering the social characteristics which are relevant to a game theoretic model of understanding sexual decision-making, though it still depends on the effects of personal and behavioral norms to explain behavior (Budd and Spencer 1985; Grube et al. 1986). The TRA represents an application of the tenets of utility theory to public health concerns by treating actors as rational decision-makers who must weigh the value of expected outcomes of health-related behaviors to the probability that a given outcome will follow from any particular behavior. The TRA includes many of the variables of the HBM, including the idea of perceived costs and benefits of particular courses of action, but also acknowledges that sexual behavior is a two-person decision-making process. It accounts for the fact that sex is a cooperative behavior and that activities like condom use require negotiation and agreement or at least consent. In addition to these factors, and in order for condoms to be used consistently, the TRA considers the fact that condoms must not detract significantly from the satisfaction of the sexual experience in order for condom use to be ‘rational’, something the HBM and Communication Perspective largely ignore. By arguing that attitudes and social norms have an interactive effect on intentions and actions, the TRA demonstrates how pressure from friends and peers can be a significant factor in the decision to use condoms: if condoms are not costly to use, if
the public consequences of engaging in risky sex are sufficiently high, and if sexual partners are willing to cooperate on the use of condoms, then we can reach an outcome where risky sexual activity does not occur.

This theory, however, is weakened by the very factors that make it a more accurate model for predicting behavior. The reliance on an individual’s adherence to social norms of behavior, while increasing the predictive power of the theory, adds nothing to its basic argument, namely that individual actors tend to behave rationally: when given circumstances present an opportunity for a rational action which is in direct contradiction to some socially accepted norm, actors no longer have a consistent plan for behavior. Sexually active individuals must consistently engage in low-risk sex in order to prevent the transmission of HIV, and such consistency is heavily dependent upon an agreement between the forces of social norms and those of rational incentives. To consider the possibility that norms can actually operate as preventive measures against rational action decisions is something that the TRA does not do.

Because game theory does not presuppose the existence of norms or cultural scripts and instead considers only the perceived costs and benefits of actors’ decisions, game theoretic models are capable of describing the interactions of two or more people in an interdependent way and can offer greater insight into the mechanics of people’s decisions to engage in protected or unprotected sex more so than in any of the previously mentioned perspectives. Rather than seeing our model as a radical departure from previous theories of sexual behavior, we see the game theoretical approach as an extension/evolution of these theories. We continue in the tradition of the health belief model and the theory of reasoned action by focusing on how individuals perceive the world and how this affects the decision to engage in risky behavior, and we draw on the communications perspective by incorporating interaction into our model.

Relevant Empirical Findings and the Game Theoretical Model

Game theory models start with a description of preferences and then move to a description of what individuals may do. The strength of game theoretical modeling is that preferences can be concisely described and individuals are predicted to maximize their expected utility given what others are likely to do. In our model, we assume
that some HIV+ individuals prefer to have unprotected sex over protected sex, even if they believe that their partner is uninfected. This feature of the model is motivated by the empirical finding that some infected individuals continue to have risky sex with uninfected individuals. Muñoz-Perez et al. (1998) report that some individuals who knowingly carry HIV choose to engage in unprotected intercourse, thereby exposing others to risk. Eich-Hochli et al. (1998; N = 117), Kalichman et al. (1997; N = 86), Wenger et al. (1994; N = 227), and O’Mahony and Barry (1992; N = 38) all show that those who have contracted HIV express diminished concern with infecting their partners, and Bedimo et al. (1998; N = 15) found that women who are HIV+ often choose not to negotiate safer-sex practices with long-term partners. Likewise, the sample of HIV+ homosexual men in Nadeau et al.’s study (1993; N = 65) shows that ‘70.6% of subjects who had a regular partner had, with them, non-risky sex. In contrast, only 39.3% of subjects who had a [some] casual partner(s) had non-risky sex’ (p. 231, our translation). These findings demonstrate that some HIV+ individuals knowingly expose their partners to HIV because of their preference for unprotected sex.

The Game Theoretic Model

In this section, we present and analyze the ‘risky sex game’. The game involves two individuals who are randomly selected from a population where some individuals are infected with HIV. There are three possible sets of actors: a set of two HIV− actors, a set of two HIV+ actors, or a ‘mismatched’ HIV−/HIV+ set of actors. In the model, HIV− actors will always prefer to have protected sex with HIV+ partners, will prefer unprotected sex with other HIV− partners, and will prefer no sex at all to unprotected sex with HIV+ partners; HIV+ actors will prefer unprotected sex to protected sex under all circumstances, and will prefer any type of sex to no sex at all. Again, let us emphasize that the preferences of actors in the model do not constitute a blanket statement about all HIV+ individuals in the real world, but are features of a model designed to capture the behavior of some people as documented in the sexual behavior literature.

The model allows each individual to make an offer of sex and then to reject or accept the final sexual offer. These offers are presented
sequentially and because infected actors have different preferences than uninfected actors it may be possible for actors to guess whether their potential partner is infected. It also allows for the possibility of bargaining—an infected actor may offer the riskiest kind of sex but may have to settle for protected sex in order to avoid rejection by an uninfected partner. The focus of the analysis is on the kinds of situations where disease transmission occurs, situations where signaling fails to distinguish infected from uninfected individuals.

It is important to discuss the scope of our model. As we stated at the beginning of the essay, we do not claim that this model captures all the relevant aspects of risky sexual behavior, or that it even catches all the kinds of encounters that might lead to HIV transmission. For example, we assume that an individual will know with some accuracy that they have HIV. This cannot be the case given that years can pass before an infected person shows any AIDS symptoms. Because HIV infection correlates with other STD’s as well as the number of sexual partners (Pandian et al. 1990, Laumann et al. 1994), it is reasonable to believe that individuals will know that they are at risk for HIV because they know whether they have had many past sexual partners or have contracted diseases whose symptoms are more visible. Another limitation of our model is that we do not consider cultural differences in sexual behavior or the possibility that heterosexual couplings may differ in substantial ways from homosexual couplings. Further research in this direction must investigate how culture or sexual orientations interact with the behavioral incentives that are the focus of our paper.

Notation

Our model uses the following notation. $U_i(x_j, s_1, s_2)$ is defined to be the subjective real value for actor $i$ (1 or 2) of an outcome $x_j$; $s_1$ and $s_2$ are the HIV statuses of actor #1 and actor #2, each denoted by $+$ (infected) or $-$ (not infected). $EU_i(y, s_1, s_2)$ is the expected value of an action $y$ given the statuses $s_1$ and $s_2$. This means that if there is a probability $p$ that $x_1$ will be the outcome of taking action $y$ and $x_2$ occurs with a probability $1 - p$, then $EU_i(y, s_1, s_2) = pU(x_1, s_1, s_2) + (1 - p)U(x_2, s_1, s_2)$. The expected utility is the sum of the values of each possible outcome of the action weighted by the probability that the outcome will occur. $A >_i B$ means that actor $i$ prefers outcome A over B or that the utility of A for $i$ is greater...
than B. When the context is clear, we will drop notation to facilitate exposition.

**Modeling the Risky-sex Game**

This section describes the model by employing the standard tools of rational choice theory: we specify a set of actors as well as the interactions between them and how the actors value the various possible outcomes. It is assumed that actors will ‘maximize their personal expected utility’ (i.e. attempt to get the best outcome they possibly can, given their beliefs about the context in which they are acting), and predictions about behavior will be deduced from this premise.

We analyze a population of agents who wish to have sex with unfamiliar partners and who are willing to bear some risk while doing so. This population contains two kinds of individuals: infected and uninfected. Every individual has a prior belief about the frequency of infection in the entire population, and every individual knows the preferences of all HIV+ and HIV− actors: HIV− actors will not have unprotected sex with those that they can identify as HIV+, and HIV+ actors generally prefer to have unprotected sex regardless of the HIV status of their partners. Altruistic HIV+ agents would attempt to sort HIV− and HIV+ partners and to engage only in protected sex with the former—for simplicity’s sake, we exclude them from the model. If agents cannot identify the status of their partners with certainty, then they will decide to have either protected or unprotected sex based on the expected utility of their actions.

The key feature of our model is that sexual decision-making takes place in an environment with *limited information*. Although both individuals are treated as being knowledgeable of their own HIV status (either + or −), they do not (in fact, for the purposes of the model, cannot) know with certainty the HIV status of the other person. The game is described in the following manner, which we call the ‘risky-sex game’:

(a) the game contains two actors: #1 and #2;
(b) each actor is either HIV+ or HIV− (denoted + or −);
(c) both actors have a commonly shared belief that the other actor may be HIV+ but they do not know with certainty and do not find out the truth, even if they eventually have sex. Call this probability ‘p’;
(d) actor #1 offers to have either Protected Sex (PS) or Risky Sex (RS);
(e) actor #2 makes a counter offer of PS or RS;
(f) actor #1 either accepts the final offer or ends the interaction with No Sex (NS);
(g) actor #2 confirms, and the pair have the type of sex offered by #2; otherwise #2 ends the interaction;
(h) at every point in the game, each actor updates his or her belief that the other actor has HIV. This means that each actor uses Bayes’s rule\(^2\) to calculate their posterior belief that the other actor has HIV.
(i) the preferences of the actors depend on whether they are HIV+ or HIV−.

The game tree shown below illustrates the game under conditions of perfect information. Under imperfect information, there would be four copies of the tree in diagram 1, one for each possible pairing of infected and uninfected individuals. The terminal nodes of the tree are labeled with the outcome.

The preferences of each actor are described in the following table:

<table>
<thead>
<tr>
<th>Actor #1 Status</th>
<th>Actor #2 Status</th>
<th>Preferences of Actor #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIV +</td>
<td>HIV+</td>
<td>RS &gt; PS &gt; NS</td>
</tr>
<tr>
<td>HIV+</td>
<td>HIV−</td>
<td>RS &gt; PS &gt; NS</td>
</tr>
<tr>
<td>HIV−</td>
<td>HIV+</td>
<td>PS &gt; NS &gt; RS</td>
</tr>
<tr>
<td>HIV−</td>
<td>HIV−</td>
<td>RS &gt; PS &gt; NS</td>
</tr>
</tbody>
</table>

RS = Risky Sex; PS = Protected Sex; NS = No Sex.

Table 1 illustrates the idea that if #1 and #2 have the same HIV status (i.e. both + or both −) then they both prefer unprotected sex, and if one is HIV+ and the other is HIV− then the HIV+ individual always prefers unprotected sex and the HIV− individual prefers protected sex over no sex and no sex over unprotected sex. This captures the empirical result that HIV+ individuals will sometimes attempt to have unprotected sex with casual HIV− partners while HIV− individuals prefer to avoid having unprotected sex with casual HIV+ partners.
Figure 1. HIV Game Tree. Moves After Nature Determines the HIV Stats of Actors 
#1 and #2. Each Actor is HIV+ with Probability $0 < p < 1$. 

1=Actor #1; 2=Actor #2; PS=Protected Sex; RS=Risky Sex; A=Accept Player #2's Offer; 
R=Reject Player #2's Offer; NS=No Sex.
The model described above captures some important features of sexual behavior: it is a model of negotiation, it allows for individuals to keep their actual HIV statuses hidden from each other, and at most, actors must attempt to deduce information about each other from their partners’ observed behavior, i.e. on what that partner has done in the game. Such games where actors infer the nature of otherwise hidden information about each other are called signaling games and actions which reveal hidden information or appear to reduce uncertainty are called signals. In the risky-sex game, the proposal of protected or risky sex and the acceptance or rejection of a partner’s sexual offer are all signals. Games of this type have been used for a number of years in economics and political science to understand bargaining and signaling in uncertain environments (Banks 1991; Osborne and Rubenstein 1994).

The Notion of the Bayesian Equilibrium

Nash Equilibrium is a set of actions such that the actions taken by each individual maximize that individual’s utility given the actions of the other individual(s). A Bayesian equilibrium, on the other hand, is a set of actions and beliefs that maximizes each individual’s utility given the behavior of the other individual(s) and these actions are consistent with rationally updating beliefs about the world (Osborne and Rubenstein 1994). This more sophisticated notion of equilibrium is needed when analyzing situations where actors operate in an environment with limited information.

It is well known that fairly simple games with signals can have multiple equilibria (Banks 1991). In this essay, we discuss two kinds of simple equilibria. The first kind is called a ‘separating equilibrium’; this means that HIV− and HIV+ individuals act in different ways and can be clearly identified as such given their actions. The second kind is called a ‘pooling equilibrium’: in a pooling equilibrium, HIV+ and HIV− individuals act in the same way during the course of the game; if this occurs, either the HIV+ or HIV− individual is lying about his or her HIV status, i.e. is engaging in ‘mimicry’. Other sorts of complicated equilibria can occur, but our focus will be on pooling because this equilibrium results in risky behavior for uninfected individuals.
**Separating Equilibria**

In a separating equilibrium, HIV+ and HIV− individuals behave differently and can be distinguished by their actions. In probabilistic terms, the initial belief that the other person is HIV+ changes during the interaction from \( p < 1 \) to \( p^* = 1 \). In a separating Bayesian equilibrium, no person has an incentive to deviate from his or her course of action given the action of the other person and this action is compatible with the posterior belief \( p^* = 1 \).

Proposition: The Risky Sex Game has a separating equilibrium.

Proof: If actor #1 is HIV+, he will always offer RS (Risky Sex); then, an HIV+ actor #2 will counter-offer RS and an HIV− actor #2 will offer PS (Protected Sex) and actor #1 will accept either way. An HIV− actor #1 will offer PS to actor #2 no matter what he believes actor #2’s status to ultimately be. Actor #2, regardless of status, will counter offer and accept PS. Deviations will result in what at least one actor considers a suboptimal outcome. QED.

While this is not strictly separating because actor #2 of any type offers the same response, the signal separates actor #1. If it is the case that HIV+ actors are a small minority of the population, or as \( p \to 0 \) but \( p > 0 \), then one arrives at the result that many 1-shot sexual interactions result in protected sex. This may seem like a counterintuitive result, since one would expect that in a world with little disease, unprotected sex would be common. One must remember that this particular outcome occurs because the status of actor #2 remains unknown to actor #1 even after actor #1 has signaled his status. The existence of disease in the separating equilibrium is enough to deter an HIV− actor from having risky sex. In a game where both actors’ true statuses are revealed before engaging in intercourse, the situation would be reversed—actors of similar status would engage in unprotected sex while partners with different statuses would have protected sex. The perfect information version of this game results in a world of completely unprotected sex because HIV− actors would simply refuse to have sex with HIV+ actors; the remaining sexual contacts would be between partners of similar status who would prefer unprotected sex.
Pooling Equilibrium

A pooling equilibrium is defined by the fact that the action of an individual does not reveal what kind of actor he or she is. In a risky-sex game pooling equilibrium, actor #2 knows nothing about the status of actor #1 after #1 has made an initial offer. It is as if actor #1 had not made an offer at all and the entire interaction focuses on the counteroffer made by actor #2 because the second actor can make any counteroffer—the first move does not provide any information for actor #2. The offer made by #2 is based only on the belief that #1 is infected. Thus, the Bayesian equilibrium is one where #1 can make any offer, #2 makes an offer that depends on whether he is infected and both players accept or reject the offer after updating their beliefs. In order to simplify the analysis, we further assume that both players can pool—the offer by #2 does not reveal any further information to player #1.

Proposition: The Risky Sex Game has a pooling equilibrium where player #2, infected or uninfected, makes an offer of RS and player #1 accepts.

Proof: In a pooling equilibrium, #2 will offer RS (risky sex) under the following conditions: the expected utility for risky sex is higher than for protected sex and given the belief that player #1 is HIV+ with probability , #2 believes risky sex will be preferred to no sex by player #1.

These conditions are expressed by the following equations, for $j = +$ or $-$:

$$EU_1(\text{RS}, +, j) > EU_1(\text{NS}, +, j) \quad \text{and}$$

$$EU_1(\text{RS}, -, j) > EU_1(\text{NS}, -, j),$$

(1)

$$EU_2(\text{RS}, j, +) > EU_2(\text{RS}, j, +) \quad \text{and}$$

$$EU_2(\text{RS}, j, -) > EU_2(\text{PS}, j, -),$$

(2)

Using the definition of expected utility, we can expand equation (1), #1’s preferences:

$$EU_1(\text{RS}, +, j) > EU_1(\text{NS}, +, j),$$

(3)
\[ pU_1(\text{RS,} - , +) + (1 - p)U_1(\text{RS,} - , -) > pU_1(\text{NS,} - , +) + (1 - p)U_1(\text{NS,} - , -). \]  

Equation 3 means that the HIV+ actor #1 will always accept any offer of sex while above some cut-off point \( p \), and an HIV– actor will choose Risky Sex over No Sex. Equation 4, which describes the preferences of an HIV– actor, can be solved:

\[ p < \frac{[U_1(\text{NS,} - , -)U_1(\text{RS,} - , -)]/[U_1(\text{RS,} - , +) - U_1(\text{NS,} - , +) + U_1(\text{NS,} - , -)] = p^*. \]  

This fraction can be negative or positive. For some assignments of utilities, the fraction will be between zero and one. For example, assigning 3 to the most preferred outcome, 2 to the next highest ranked outcome and 1 to the least preferred yields \( p^* = 1/3 \).

The interesting implication of the model is not that it captures a common sense intuition about the ability of HIV+ individuals to couple with HIV– individuals, or that it suggests that individuals who may be dealing with an infected partner will insist on using a condom, but that it can make predictions about infection rates over time. If the pooling equilibrium is an accurate description of what happens when individuals seek out sexual partners, then we would expect that as infection rates rise, uninfected individuals would change their behavior and make less-risky sexual offers. This would lead to a decrease in the overall infection rate. This observation motivates the final section of the paper in which we examine epidemics where actors engage in the risky sex game.

**The Pairs at a Party Model**

The purpose of this section is to analyze the relationship between the risky sex game and population HIV infection rates. Whereas the discussion above implies that high infection rates will start to decline once uninfected individuals rely on protected sex, the present section studies the timing of this reversal under varying conditions. The decline of infection rates is analyzed via agent simulation.
The dynamic model that we present is a modification of the ‘pairs at a party’ model (Daley and Gani 1999). In that epidemiological model, hereafter referred to as the PaP model, individuals are randomly paired and infected individuals transmit a disease with a fixed probability $\beta$. Starting with one infected individual, this process is repeated until the entire population has become infected.

The 1-shot risky sex game model and the PaP model complement each other. The risky sex game provides a story of how individuals choose to engage in hazardous sexual practices and the PaP model provides a way to relate micro-level processes to the spread of a disease though a population. The PaP model is a particularly appropriate extension of the risky sex game because sexually transmitted disease occurs within pairs of individuals, unlike other diseases that may be transmitted through common contact with a contagion. Another useful feature of this model is that it can be easily modified. For example, coupling can last through multiple time periods or ‘rounds’, so individuals can have multiple partners. Daley and Gani (1999) present other extensions of this model.

We start with a basic model in which random pairings of individuals (‘pairs’) occur in discrete time periods (‘parties’) and then modify it in three ways. First, instead of having a fixed probability of disease transmission from infected to uninfected individuals, the probability of transmission depends on whether the two matched individuals agree to have risky or protected sex. Second, we allow selection of partners to be non-random—individuals can choose partners who are socially close to them as defined by location in a larger social network. Third, the prior belief of the rate of infection can vary from one time period to another. We allow individuals to draw their prior beliefs from two possible sources: either the ‘true’ rate of infection or an estimate based on socially close individuals. We then compare the effects of these different modifications on the timing of the reversal of infection rates by estimating the mean infection rate for each model for 500 time periods.

Previous sections have offered a justification for making the probability of transmission vary according the equilibrium behavior of the two randomly matched individuals—prior beliefs about the occurrence of disease dictate different strategies that carry different risks of contracting disease. We now discuss the motivations for allowing actors to choose partners from a network and for allowing actors to draw information from their network.
A substantial literature in sociology deals with diffusion through social networks (see Strang and Soule (1998) and Valente (1995) for reviews). Recently, network analysis has been applied to the study of how diseases spread through such networks. A thorough empirical analysis of this can be found in Laumann et al. (1999) and Laumann and Youm (1999), who found that sexual contacts were not random and that differences in patterns of sexual contacts could account for some of the variation in infection rates among different ethnic groups. Other researchers have offered theoretical and empirical evidence that the transmission of sexually transmitted disease can be slowed by focusing on central individuals rather than randomly selected individuals (Deszo and Barbasi 2001), that network bridges accelerate disease by transmitting disease from one densely connected region of a network to another (Morris et al. 1996; Morris and Kretzschmar 1995, 1997) and that tie strength affects the probability of engaging in risky behavior (Valente and Vlahov 2001).

Research has also shown that individuals can acquire information from public sources of knowledge such as the media and public health campaigns as well as from private sources of information, i.e. social networks. Sociologists who study networks often find that individuals who are close to each other frequently share information (see Ingram and Roberts (2000) and Hepworth and Ryan (2000) for recent results). Because of these findings, we study the effects of allowing agents in our model to base their prior beliefs on their social network.

We study infection rates in four different models. Each model is a modification of the basic PaP model with players who engage in a risky sex game. Disease in all models is transmitted with probability \( \beta_i \), which depends on whether sex is ‘risky’ or ‘protected’. The model is varied in two ways: partners may be drawn from a pre-existing social network or randomly from the population at large, and the prior belief may be either the true rate of infection (public disclosure) or be determined from the actor’s social network. Table 2 summarizes the four different models:
Table 2. Features of the Simulated Models

<table>
<thead>
<tr>
<th>Features of All Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete Time Periods</td>
</tr>
<tr>
<td>Transmission Probability Varies According to Type of Sex</td>
</tr>
<tr>
<td>Every individual finds a partner</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Features of Specific Models</th>
<th>Prior Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partners</td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>Whole population</td>
</tr>
<tr>
<td>Model 2</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>From Ego Neighborhood</td>
</tr>
<tr>
<td>Model 3</td>
<td>From Ego Neighborhood</td>
</tr>
<tr>
<td>Model 4</td>
<td>From Ego Neighborhood</td>
</tr>
</tbody>
</table>

Specification of Simulation

We analyze the different models via agent simulation. In each model, there are 100 actors who belong to a randomly generated social network, which remains the same in Models 2 and 4. Since individuals usually vary in the number of ties that they have, for each actor, we draw an integer $X$ from a Poisson distribution with mean $\lambda = 2$ and randomly select $X$ other individuals who will be tied to that actor. We further impose the condition that every actor must have at least one tie to another actor. This allows for all actors to have at least one tie, prohibits social isolates, and allows for the existence of a few high centrality actors.

Before the first time period, a single randomly selected individual has the disease and no one else in the population is infected. In each subsequent time period, individuals are matched and engage in the risky sex game. Individuals in models 3–4 randomly select partners from adjacent individuals; otherwise they are randomly selected. In models 2 and 4, the prior belief is the fraction of adjacent individuals who are infected; otherwise the prior belief is the fraction of all individuals who are infected. In all models, prior beliefs and statuses dictate equilibrium behavior and one of three behaviors occurs with concurring rates of infection: No Sex, Protected Sex, or Risky Sex ($0 < \beta_{\text{protect-sex}} < \beta_{\text{risky-sex}}$).

The mechanism of the models with network-based prior beliefs is slightly different than that with public beliefs. In models 2 and 4, actors do not always share the same prior as they do in the risky
sex game because they draw information from different ego-neighborhoods. Rather than develop a more complicated model where individuals are hyper-Bayesian and have beliefs about other people’s beliefs, we impose a simplifying condition: each actor assumes that the other actor has the same prior as they do. Therefore, it is possible that one player will engage in a strategy that the other player perceives to be a strategy off the equilibrium path. If that occurs, then the player will respond with the non-equilibrium path strategy. The implication for models 2 and 4 is that infected actors will accept any offer of sex while uninfected ones will opt for No Sex if offered Risky Sex when Protected Sex is the predicted behavior.

We numerically estimate infection rates because the explicit computation of infection rates can be computationally intensive and somewhat intractable. In the unmodified PaP model, the expected value for the infection rate for time \( T + 1 \) in each model is computed by first calculating the probability that \( j \) individuals will contract disease, conditional on the fact that \( i \) is the number of uninfected persons at time \( T \). When simplified, this yields the following:

\[
\frac{2^i Z_T! (2M - Z_T)! M!}{j! (1/2(Z_T j))! (M - 1/2(Z_T + j))! (2M)!}
\]

if \( (j = Z_T, Z_T - 1, \ldots, 0) \) or \( 0 \) otherwise,

where \( M \) is one half the population and \( Z_T \) is the number of ways that mixed HIV−/HIV+ couples can form. To derive the expected rate of infection, average these for all \( i + j = 2M \) while taking into account that the number of the number of transmissions is binomially distributed. The introduction of networks only serves to further complicate this calculation. However, in situations where it is difficult to directly compute \( E[f(X)] \), it is possible to arrive at an approximation, \( E[f(X)] \approx 1/n \sum_{i=1}^{n} f(X_i) \), which is guaranteed by the ergodic theorem as long as the \( X_i \) are independent (see Gilks et al. (1995) for discussion of such procedures for the analysis of distributions).

We arrived at our estimates of the infection rates in models 1–4 by executing 180 simulations of each model in MATLAB 5.3.1. Using the methods we described above, we started by generating a network where each actor’s ties were drawn from a Poisson distribution.
truncated at 1 and a network that remains constant over the simulations of Models 2, 3 and 4. For each simulation of each model actors are paired off 500 times, randomly or according to their network ties, and actors in all models follow the strategies dictated by the risky sex game. The prior that will trigger a switch from Risky Sex to Protected Sex will be set to 1/3, the $p^*$ predicted when the utility function assigns 1 to the least preferred outcome, 2 to the middle outcome, and 3 to the most preferred outcome. The probability of contracting disease during risky sex is 0.1 and the probability of contracting disease during protected sex is 0.01, an assignment motivated by the previous discussion on the efficacy of condom usage—condom usage reduces but does not eliminate the risk of contracting HIV. For each simulation of the model, infection rates were recorded for each of the 500 time periods. We estimated the mean rate of infection for each time period for the four models by taking the average of the recorded infection rates. Diagram 2 shows the estimated infection rates for Models 1–4 for the first 150 periods:

**Diagram 2.** Estimated Infection Rates for Models 1–4 for the First 150 Periods
The most striking feature of Diagram 2 is that the epidemic represented by Model 2 has the highest infection rate, almost 25 cases per thousand. The peaks for Models 1, 3, and 4 are substantially less approximately 15 cases per thousand in each of the others. It is in Model 2 that individuals rely on their ego-neighborhood for information but randomly select partners from the population. Actors in Model 2 put themselves at more risk than in other models because the incidence of disease in an ego-neighborhood may not be an accurate reflection of the infection rate in the entire population. The estimate of infection rates for this model suggest that once most ego-neighborhoods contain at least a few infected individuals and uninfected individuals assume a corresponding behavior change, then rates of disease infection drop and will begin to resemble those of the other models. The analysis of Model 2 indicates why the spread of disease may not be slowed even when individuals know about safe sex practices; dependence on social networks for information causes individuals to engage in high-risk behavior in high-risk environments rather than low-risk environments.

Another important finding of the simulation is that in early time periods, the infection rates for Model 1 are between Model 2 and Models 3–4. In Model 1, individuals randomly choose partners and accurately know the probability that they are dealing with an infected partner. As the model with the second highest rate of infection, this suggests that restricting the choice of partners has a stronger effect on reducing infection rates than having an accurate knowledge of infection rates; Models 3–4, where individuals choose partners from their network, both have lower infection rates at the end of the simulation than the models where actors choose partners randomly. The models where individuals have correct knowledge of infection rates do not have consistently higher or lower infection rates than the models where information is drawn from networks—this suggests that the restriction of sex to network partners may be more important in limiting the spread of disease than accurate knowledge about disease rates in the population.

Interesting implications for these models include the timing of the reversal—what kinds of behavior will trigger faster reversals in infection rates? According to the simulations, the epidemic that will reach its zenith first is Model 1—public information and random coupling—although fewer individuals are infected than in Model 2. The epidemics reach their heights last in Models 3 and 4,
those where actors select their partners from networks, and infection rates in Models 3 and 4 are still near their maximum when infection rates in other models fall. The conclusion is that when individuals restrict their partners, maximum infection rates are reduced but the reversal of rates is slow when compared to models where individuals randomly choose partners. This would suggest that reducing promiscuity would reduce the size of an epidemic, but make it last longer.

**Conclusion**

The risky sex model that we have presented does not assume that any norm governs behavior, but that individuals have goals and that these goals must sometimes be achieved through consensus under uncertain conditions. Game theoretic analysis shows that offers of sexual acts with varying degrees of risk by themselves may not allow for the distinction between HIV+ and HIV− individuals, and this allows disease transmission—in fact, the presence of the pooling equilibrium shows that attempting to rely on such signals (as many individuals negotiating a sexual interaction with a relative stranger might be tempted to do) can actually result in an *increase* in the likelihood of disease transmission and therefore in the population rate of infection. As the risk of finding HIV− partners decreases over time, most HIV− individuals will switch to less risky behaviors because they cannot determine if a potential partner is an HIV carrier—this eventually results in the reversal of infection rates in the general population.

In the models we have presented, there is no replacement; the population is stable. If we allow for replacement, then we arrive at a novel prediction: as uninfected individuals enter the population (through birth, migration, etc.) and HIV+ individuals leave (through illness), the proportion of infected individuals will decrease. Once this proportion falls, prior beliefs about the proportion of the infected population will fall, and if this new prior belief is low enough, then HIV− individuals will switch from protected to unprotected sex. The long-term effect of replacement in our model, then, is an oscillation in infection rates. The peaks depicted in Diagram 2 will repeat themselves as uninfected individuals depress the incidence of disease; rates will increase and then will
decrease as individuals switch once again from high-risk to low-risk activities. There is some evidence that oscillations in infection rates do occur, and the current leveling-out in HIV rates among whites in the U.S. (New York Times, 25 April 1998) may be the first sign of such a population rate reversal in the industrialized Western world (Centers for Disease Control 2001a: 437); the dramatic recent rise in HIV rates among gay African-American men (Centers for Disease Control 2001b), however, represents a subpopulation which has yet to reach its ‘acceptable’ epidemiological crest. An intriguing avenue for further research would be to link these patterns in infection rates to the behavior depicted in our model.

Aside from offering an interesting perspective on disease transmission, the game theoretic analysis provides a flexible starting point for modeling the spread of disease through populations. Epidemiological modeling often focuses on entire populations, while relatively little has been done to describe personal interactions. Models with behavioral assumptions tend to simply categorize individuals as ‘high-risk’ or ‘low-risk’ without connecting the micro-level models to general social science theories (O’Neill 1995). The game theoretical model we have presented can help to address this problem and to increase the theoretical reach of epidemiological models by making the probability of disease transmission a function of micro-behavior. A family of models can then be developed that varies according to the model of signaling that individuals use as well as the conditions under which individuals find sexual partners and acquire knowledge.

Finally, this analysis can be used to generate hypotheses about sexually transmitted diseases in specific populations. One of the key findings of the simulation was that sexually transmitted diseases spread quickly through populations where individuals randomly couple, but whose estimate of the rate of infection comes from their personal network. A population where norms of casual sex predominate, but where individuals are poorly informed about public health issues, fits this description. Because knowledge of sexual health issues can vary according to SES, race, and educational attainment, the analysis presented in this article allows for the linking of social stratification to health outcomes via the game theoretic analysis.
NOTE

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NOTES

1. Note that our model does not deal directly with cases where one individual or the other is unaware of his or her HIV status.

2. Bayes’s rule is: \( \text{prob}(X|Y) = \text{prob}(X) \times \text{prob}(Y|X) / \text{prob}(Y) \). \( \text{prob}(X) \) is often interpreted as the probability that some model of the world is true and \( \text{prob}(Y) \) is the probability that \( Y \) (the data) occurred regardless of the actual state of the world. Bayes’s rule is a formula for recalculating the subjective probability that \( X \) is true given the fact that \( Y \) was observed. In the case at hand, \( X \) is ‘the other actor has HIV’ and \( Y \) is what sort of sex the other actor proposed. For the sake of clarity, we have not developed the model with the exact definition of Bayes’s rule since it is not necessary for the model.

3. The traditional definition of a Nash equilibrium is a set of actions for every possible sequence of events for every actor such that each actor has no incentive to change their own set of actions (called a strategy) given the actions of the other actors. This is a more precise definition and we omit its use for the sake of clarity. Interested readers can read a fully developed presentation of all the technical concepts in this paper in Osborne and Rubinstein (1994).

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ROJAS & SCHROEDER: A GAME THEORETICAL ANALYSIS


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