Does Protective Measurement Tell Us Anything about Quantum Reality?

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Abstract
An analysis of the two routes through which one may disentangle a quantum system from a measuring apparatus, hence protect the state vector of a single quantum system from being disturbed by the measurement, reveals several loopholes in the argument from protective measurement to the reality of the state vector of a single quantum system.

1 Introduction

In the late 1920s von Neumann axiomatized non relativistic quantum mechanics (QM henceforth) with the Hilbert space, and ever since then generations of physicists have been accustomed to view this space as indispensable to the description of quantum phenomena. The Hilbert space with its inner product and the accompanying non commutative algebra of its orthomodular lattice of closed subspaces have also given rise to a manifold of “interpretations” of non relativistic QM, and to a subsequent plethora of puzzles and metaphysical speculations about the inhabitant of this space, the “state vector” (Nye & Albert 2013), its “branching” (Saunders et al. 2010), and its “collapse” (Ghirardi et al. 1986).

Such puzzles are not surprising. Faithful as they are to the Galilean imperative (“The book [of Nature] is written in mathematical language” Galilei 1623/1960)), theoretical physicists often interpret the mathematical constructs that they use in the description of physical reality as des-
ignating pieces of that reality. In classical mechanics such a correspon-
dence is relatively straightforward: the dynamical variables of the theory
can be assigned values, and so can the attributes of the physical objects
they represent. In QM, however, things are more complicated. To be-
gin with, the mathematical structure can support two types for dynamical
variables, the operators and the vectors they operate on (the historical rea-
son for this duality can be traced back to the birth of quantum theory; see,
e.g., Beller (1999, pp. 67–78)). Next, this mathematical structure doesn’t
yield itself to correspondence with physical reality as easily as in classical
mechanics. One candidate for the dynamical variables, the operator, can
be assigned (eigen)values only in specific circumstances (namely, when it
operates on eigenvectors), and, furthermore, not all operators can be as-
signed (eigen)values simultaneously (Kochen & Specker 1967); the other
candidate, the state vector, can be assigned values (amplitudes), but in-
stead of actual attributes of a physical object, these (or rather their mod
square) represent the probabilities of the object’s having these attributes as
an object represented by the respective operator.

A possible reaction to the probabilistic interpretation of the state vector
of a single quantum system is to regard the state vector as a mathematical
object which does not represent pieces of reality, but rather our knowledge
of that reality (Unruh 1994), knowledge which is inherently incomplete
(Caves et al. 2002). Those who insist on interpreting the state vector of a
single quantum system as designating a real physical object can do so by
(i) altering QM, namely by introducing a physical collapse of the state vec-
tor as a genuine physical process (Ghirardi et al. 1986), or by (ii) replacing
QM with a deterministic and non–local alternative (Goldstein et al. 1992),
or by (iii) keeping QM intact but committing to a metaphysical picture of
multiple worlds (Saunders et al. 2010).

20 years ago another argument for the reality of the state vector of a sin-
gle quantum system was put forward with the notion of protective mea-
surement (Aharonov & Vaidman 1993; Aharonov et al. 1993; 1996). The
basic idea behind this argument is that if, in some hypothetical situations,
one could weakly interact with a single quantum system, one would be
able to extract information from that system without disturbing its state.
In particular, one would be able to extract the expectation values of some
observable by repeatedly measuring (and without disturbing) the state vec-
tor of that same single quantum system.\(^1\)

To give a concrete example, think of the proverbial Stern Gerlach set-up that is supposed to measure the spin of a spin-half particle along some axis. In standard QM, the interaction between the particle and the apparatus is described with Schrödinger’s equation as an impulse interaction, so that the particle and the apparatus become entangled. After the measurement one can read the result from the apparatus, but the state vector of the composed system has suffered a “collapse” (epistemically interpreted as an adjustment of knowledge and not as a physical process). For this reason one requires (and in fact actually one does measure) an ensemble of particles whose spin statistics conforms with the Born rule.

The protective measurement scenarios differ from this standard measurement set-up in that in them, the state vector is “protected” and is disentangled from the apparatus, and therefore doesn’t “collapse” when one measures the state of the latter. Since the state vector of the system is not disturbed by the (protective) measurement, several expectation values of the measured observable can be read repeatedly, leading to a complete characterization of the state vector even in the case of a single system. This characterization, according to Aharonov et al. (1993), supports interpreting the state vector of a single system as representing a piece of reality and not just as representing the observer’s state of (incomplete) knowledge.

How can one achieve such a disentanglement between the state and the apparatus? In other words, how can one acquire information about the single system without disturbing its state? Aharonov et al. (1993) propose two possible mechanisms. The first relies on the quantum adiabatic theorem; the second on the quantum Zeno effect.

In this paper I shall argue that the argument from protective measurement to the reality of the state vector of a single quantum system is unconvincing. Drawing on the two processes suggested by Aharonov et al. (1993) as possible mechanisms for realizing protective measurement scenarios, in sections (2) and (3) I will show that in both scenarios there is no way to know when to halt the “protection process” so that the state one intends to measure in each scenario is indeed protected. One must “consult

\(^1\)Note that one can already extract these expectation values from measurements performed on an ensemble of quantum systems whose state vectors are disturbed by each of the measurements; such a protocol, however, doesn’t allow one to assign physical reality to the state vector of a single system, and is the very type of interaction the argument from protective measurement would like to avoid.
an oracle”, as it were, who knows when to halt the process by knowing in advance either the state (by preparing it via standard, non–protective measurement), or the Hamiltonian of the system (which then allows one to calculate the expectation values without measuring the state). This feature, namely, that the question whether the state vector of a single quantum system is “protected” is undecidable, severely weakens the argument from protective measurement to the reality of the state vector of a single quantum system, or so I shall claim in section (4).

Before we start, two disclaimers are in place. First, this paper has a sole purpose, namely, to demonstrate that protective measurement cannot be used convincingly to argue for the reality of the state vector of a single quantum system. Whether or not the state vector of a single quantum system is indeed “real” is a question I will not discuss here. Consequently, I will say nothing about other possible and more recent suggestions that purport to establish with arguments from first principles that the state vector is “real” (e.g., Pusey et al. 2012).

Second, this paper addresses only the conceptual implications of protective measurements on the reality of the state vector, and says nothing about the implementation of the mechanisms that may result in a state being probabilistically disentangled from an ancilla, e.g., quantum error correction via dynamical decoupling. Apart from the fact that these quantum information processing protocols are realistic, i.e., involve finite precision and no infrared or ultraviolet divergences, they also do not purport to fully characterize the quantum state of a single system, but only to extract information about the correlations that may exist between the eigenvectors of the computational basis that span the state. Consequently, these implementations are immaterial to the question of using protective measurement as an argument for the reality of the state vector of a single quantum system.

2 How Slow is Slow Enough

A crucial step in the argument from protective measurement to the reality of the state vector is the construction of the specific Hamiltonian through which the system and the apparatus interact in such a way that the state of the system remains undisturbed by the measurement. Such a construction is non trivial given the constraint (Aharonov et al. 1993, 47) that the
state vector of the system and its Hamiltonian are unknown; all that is known is that in order for the state vector to be protected the state vector must be an eigenstate of the Hamiltonian.

Now we certainly know how to prepare a state to be an eigenvector of a known operator – i.e., we measure the state in a certain orthonormal basis and "collapse" it on one of the eigenvectors of the operator – but if this collapse is not a physical process, as Aharonov et al. (1993) assume, then how does one prepare an operator so that a quantum state is its eigenstate?

One way Aharonov et al. (1993) suggest such a preparation could take place is via the quantum adiabatic theorem. According to this theorem (Messiah 1961, pp. 739–746), an eigenstate (and in particular a ground state) of an initial time–dependent Hamiltonian of a quantum system can remain unperturbed in the process of deforming that Hamiltonian, if several conditions are met: (i) the initial and final Hamiltonians, as well as the ones the system is deformed through, are all non–degenerate, and their eigenvalues are piecewise differentiable in time (ii) there is no level–crossing (i.e., there always exists an energy gap between the ground state and the next excited state), and (iii) the deformation is adiabatic, i.e., infinitely slow.2

More precisely, consider a quantum system described in a Hilbert space $\mathcal{H}$ by a smoothly time–dependent Hamiltonian, that varies between $H_I$ and $H_F$, for $t$ ranging over $[t_0, t_1]$, with a total evolution time given by $T = t_1 - t_0$, and with the following convolution:

$$H = H(t) = \left(1 - \frac{t}{T}\right)H_I + \frac{t}{T}H_F.$$ (1)

When the above conditions (i–iii) of the adiabatic theorem are satisfied, then if the system is initially (in $t_0$) in an eigenstate of the initial Hamiltonian $H_I$, it will evolve at $t_1$ (up to a phase) to an eigenstate of $H_F$. In the special case where the eigenstate is the ground state, the adiabatic theorem ensures that in the limit $T \to \infty$ the system will remain in the ground state throughout its time–evolution.

Now, although in practice $T$ is always finite, the more it satisfies the minimum energy gap condition (condition (ii) above), the smaller will be

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2To take an everyday analogy, if your sound sleeper baby is sound asleep in her cradle at the living room, then moving the cradle as gently as possible from that room to the bedroom will not wake her up.
the probability that the system will deviate from the ground state. This means that the gap condition controls the evolution time of the process, in the exact following way:

\[ g_{\text{min}} = \min_{0 \leq t \leq T} (E_1(t) - E_0(t)). \]  
\[ T \gg \frac{1}{g_{\text{min}}^2}. \]

where \( E_0 \) and \( E_1 \) are the ground state and the first excited state, respectively.

*Prima facie* the adiabatic theorem seems highly suitable for a protective measurement scenario, as it appears to allow one to remain agnostic about both the final state vector at \( t_1 \) and the final Hamiltonian \( H_F \), while ensuring that the former is an eigenstate (and, in particular, a ground state, in the case one starts at \( t_0 \) from a state one knows to be the ground state of \( H_I \) of the latter.

Several commentators, however, have already pointed out (e.g., Alter & Yamamoto 1996; Dass & Quershi 1999) that the adiabatic theorem is correct only at the limit \( T \to \infty \). In actual physical scenarios, \( T \) is always finite, and so there is always a finite probability that the state will not be protected after all. In those cases, repeated measurements of the same state would send it to a state orthogonal thereto, and so one would not be able to fully characterize it as envisioned by Aharonov *et al.* (1993). To quantify this possibility and calculate its probability one requires an ensemble of systems, but then, one could not argue for a correspondence between the state vector of a single system and reality.

Another common objection (e.g., Rovelli 1994; Alter & Yamamoto 1997) is that one can disentangle the system from the apparatus and “protect” it only if one knows in advance the Hamiltonian, in contrast to what Aharonov *et al.* (1993) claim.

In response, Aharonov *et al.* (1996, 121) have argued that they were misunderstood:

The main point of the majority of works criticizing our proposal was that we cannot measure an unknown quantum state of a single system since we cannot protect unknown quantum states. But we never claimed otherwise. More than this, we have claimed the opposite.
But in Aharonov et al. (1993, 47) one finds the following, which is hard to reconcile:

We emphasize that we do not know what $|\psi\rangle$ is before the measurement and need not know what $H$ is. We need to know only that $|\psi\rangle$ is protected in order to determine $|\psi\rangle$.

This debate, and the confusion it has generated, call for a deeper analysis, which I shall offer next. As we shall see, that $T$ is always finite and that prior knowledge of the Hamiltonian is required are two facets of the same fact, namely, that the question whether a state is actually protected as a result of the protection process of the kind described above is an undecidable question. Indeed, in response to these objections Aharonov et al. (1996) have argued that all they require for their conclusion is that there "exists" such a situation in which the state is protected, and that they can make the probability of failure of the adiabatic process as small as they want by simply enlarging the finite evolution time $T$. The problem, however, is that even in the realistic case of a finite $T$, without prior knowledge of the Hamiltonian, the question what $T$ is, i.e., when to halt the protection process so that the state would be fully protected, is undecidable; it is simply impossible to compute $T$ in advance, unless one has prior knowledge of the Hamiltonian (from which the state can be calculated).

To see why, let us focus on the gap condition (eq. 3). This condition is in the heart of the adiabatic theorem, as it ensures its validity, and, more important, as it governs the evolution time of the entire process. This spectral gap $g = E_1 - E_0$ may be calculated for $H_1$ at $t_0$, since we ourselves construct this Hamiltonian and prepare the system to be in its ground state. But how does this gap behave along the convolution? Note that the question here is not whether the gap exists along the convolution. The objections presented above all assume that at some point a level–crossing would occur with a finite probability, and so the process would not be protective. Given that Aharonov et al. (1996) can presumably rebut this objection, I would like to dig deeper, and so I concede that a gap (hence a protected state) do "exist". Conceding this, the question I ask now is how small is this gap in all instances other than $t_0$. For in the absence of a detailed spectral analysis, in general nobody knows what $g$ is, how it behaves, or how to compute it!

This question of the gap behavior is, however, of crucial importance. Recall that the gap controls the velocity of the process with which one would like to "protect" a state vector. To achieve "protection", one needs
to deform the initial known Hamiltonian $H_I$ slow enough, until one arrives at the final unknown Hamiltonian $H_F$. But how slow is slow enough? How slow should one go, and, more important, when should one stop the process if one doesn’t know in advance either (a) the spectral gap or (b) the final Hamiltonian $H_F$? The problem, therefore, is not that the evolution time $T$ is finite, but that since $g$ or $H_F$ are unknown, $T$ (and hence the ”speed” $T^{-1}$), are finite alright, but also unboundedly large (slow).

Can we estimate $T$ in advance? For each given ”protection” process that ends with $H_F$ we have to come up with a process whose rate of change is $\sim T_f^{-1}$. How do we know that we are implementing the correct rate of change while $H(t)$ is evolving? Apparently, by being able to measure differences of order $T_f^{-1}$, that is, having the sensitive ”speedometer” at our disposal. But when the going gets tough, we approach very slow speeds of the order of $\sim T_f^{-1}$, which begs the question, since we can then compute $T_f$ using our ”speedometer”, and hence compute $H_F$ and determine its ground state without even running the ”protection” process.

Now if we don’t have a ”speedometer”, then even if we decided to increase the running–time from the order of $T_f$ to, say, the order of $T_f + 7$, we will have no clue that the machine is indeed doing $(T_f + 7)^{-1}$ and not $T_f^{-1}$, and that the state is protected. And so in the absence of knowledge in advance of $g$ or $H_F$, we will never know how slow should we evolve our physical system so that the final state is protected. But then we will also fail to fulfill the adiabatic condition which ensures us that in any given moment the system remains in its ground state.

I therefore conclude, along with other commentators, but from a different reason, a reason that has to do with the undecidability of the question whether a state is protected via an adiabatic process, that a protective measurement that relies on the adiabatic theorem can be realized with certainty only if one knows in advance the desired Hamiltonian; without such knowledge, one would never know when to halt the protection process. As I shall argue in section (4), this negative result rules out the adiabatic theorem as a means to establishing the conclusion that Aharonov et al. (1993) would like to establish, namely, that a state vector of a single quantum system represents a piece of reality. This negative result doesn’t mean that there are no other ways to realize protective measurements, but as we shall see, the alternative mechanism suggested by Aharonov et al. (1993) is no more plausible than the one we have just analyzed.
3 How Fast is Fast Enough

Another route to protective measurement that Aharonov et al. (1993) point at is the well–known quantum Zeno effect (Degasperis et al. 1974; Peres 1980), also known as “the watched pot” effect. Here the idea is that the survival probability of an evolving quantum state is predicted to be altered by a process of repeated observations with a macroscopic apparatus. In the ideal case of continuous measurement process, the state remains “frozen” and doesn’t change in time. In the more realistic case of a finite measurement process, the probability that the state changes vanishes as the frequency of the repeated measurements increases. In effect we have here a mirror image of the quantum adiabatic theorem: there, one could keep the (ground) state intact by interacting with it infinitely slow; here one can keep the (excited) state intact by interacting with it infinitely fast.

More precisely, consider a quantum system composed of an unknown state $|\psi\rangle$ which interacts with a measuring apparatus, and evolves according to the total time–independent Hamiltonian $H$ (units are chosen so that $\hbar = 1$). The probability of the ”survival” of the initial state is

$$|\langle \psi | e^{-iHt} |\psi\rangle|^2 \approx 1 - (\Delta H)^2 t^2 - \ldots,$$

where $\Delta H$ is the standard deviation defined as

$$(\Delta H)^2 = \langle H\psi \mid H\psi \rangle - (\langle \psi \mid H\psi \rangle)^2.$$  

(5)

It follows that if the projection operator on the initial state $\psi$ is measured after a short time $t$, the probability of finding the state unchanged is

$$p = 1 - (\Delta H)^2 t^2,$$

(6)

but if the measurement is performed $n$ times, at intervals $t/n$, there is a probability

$$p = [1 - (\Delta H)^2 (t/n)^2]^n > 1 - (\Delta H)^2 t^2,$$

(7)

that in all measurements one will find the state unchanged. In fact, for $n \to \infty$, the left hand side in (7) tends to 1, and so in the limit when the frequency of measurements tends to infinity, the state will not change at all, and will remain “frozen”.

From a mathematical perspective, the situation can be explained as a case in which, given a certain orthonormal basis for $H = H_0 + V$, all the
diagonal terms reside in $H_0$, and $V$ contains only off–diagonal elements. The eigenspace of $H_0$ is thus a "decoherence free subspace" (Lidar & Whaley 2003), a sector of the system’s Hilbert space where the system is decoupled from the measuring apparatus (or more generally, from the environment) and thus its evolution is completely unitary. Physically this means that the additional term $V$ in the Hamiltonian $H$ rapidly alters the phases of the eigenvalues, and the decay products (responsible for the decaying of the state) acquire energies different from those of $H$, as to make the decay process more difficult, or, in the limit, impossible. Now if one could locate one’s state vector in the eigenspace of $H_0$, then consecutive measurements would not change the state vector, and one could acquire information about its expectation values without disturbing it, thereby establishing Aharonov et al. (1993) conclusion about the reality of the state vector of a single quantum system.

The quantum Zeno effect was originally presented in terms of a paradox (Misra & Sudarshan 1977), the idea being that without any restrictions on the frequency of measurements (time being what it is, namely, a continuous parameter), QM fails to supply us predictions for the decay of a state which is monitored by a continuous measurement process, and so appears to be incomplete. Aharonov et al. (1993) suggest harnessing this situation as a procedure that would allow one to protect a quantum state from being disturbed by a measurement.

But as in the earlier case of the adiabatic theorem, the state can be guaranteed to be protected by the quantum Zeno effect only at the limit $n \to \infty$ (i.e., $T \to 0$). In actual physical scenarios, $T \neq 0$ hence the frequency of the measurement is always finite, and so there is always a possibility that the state will not be protected after all. We have already seen that this limitation, all by itself, is not sufficient to demolish the argument from protective measurement to the reality of the state vector, as one could just replace actual infinity with potential unboundedness, and argue that a decoherence free subspace simply "exists", and, moreover, that one can make the probability of decay as small as one wants (Dickson 1995).

What is thus needed is another argument that shows, as in the case of the adiabatic theorem, that such a decoherence free subspace – a subspace in which one can measure a quantum state without disturbing it – can be constructed with certainty only if one knows in advance the state one wants to protect. Indeed, further analysis has revealed (Ghirardi et al. 1979) that a correct interpretation of the time–energy uncertainty relation (i) relates
the measurement frequency to the uncertainty in energy of the state:

\[ T\Delta E \geq 1 \]  

(8)

where \( T \) is the time duration of the measurement and \( \Delta E \) is the energy uncertainty, or spread, of the state vector one is measuring, and (ii) controls, via this uncertainty, the rate of decay of the state:

\[ |\langle \psi_0 | \psi_t \rangle|^2 \geq \cos^2(\Delta Et), \quad \text{for} \quad 0 \leq t \leq \frac{\pi}{2\Delta E}. \]  

(9)

Before we go on to analyze these results, two remarks are in place. First, inequality (9) was first derived by Mandelstam and Tamm in 1945 for \( \Delta E = \Delta H \), i.e., for the case where the uncertainty (spread) in energy is given in terms of the standard deviation, under the assumption that the latter is finite. Recently it was shown that since \( \Delta H \) can diverge, a more suitable measure of uncertainty is required (Uffink 1993). To our purpose here this caveat is immaterial; even after a correct measure of uncertainty is chosen, the probability of decay of the state still depends on this measure of uncertainty. Second, while the duration of the measurement is a parameter and not a quantum operator, the application of the time–energy uncertainty relation is legitimate here because we are dealing with what may be called a static experiment (Aharonov & Bohm 1961): in this case \( T \) is not a preassigned or externally imposed parameter of the apparatus, but is on the contrary determined dynamically through the workings of the Schrödinger equation which describes the evolution of the wave packet through the measuring system. The wave packet may perhaps be said to interact with the apparatus during the time interval \( T \) and in this sense the time–energy uncertainty relation is valid.\(^3\)

With this preliminaries we can now state our negative result, which is similar to the one we have established in the case of the adiabatic theorem. For even in the case where the frequency \( T^{-1} \) is finite but unbounded (hence \( T \) is not exactly 0 but arbitrarily close to 0), one still needs to know the state in advance in order to know how fast one needs to repeat the measurements that project the state onto its protected subspace and thereby disentangle it from the measuring apparatus.

\(^3\)Note that the time–energy uncertainty relation all by itself doesn’t constrain \( T \) to be finite. It is the additional assumption that in realistic scenarios energy is finite and bounded from above that does the trick. See, e.g., Hilgevoord (1998, p. 399).
The reason is that also here, for each given "protection" process $k$, we have to come up with a repeated measurement process whose frequency is $\sim T_k^{-1}$. But how do we know that we are implementing the correct frequency? Apparently, by being able to measure differences of order $T_k^{-1}$, that is, having the sensitive "speedometer" at our disposal. But when the going gets tough we approach very fast speeds of the order of $\sim T_k^{-1}$, which begs the question, since we can then compute $T_k$ using our "speedometer".

Now if we don't have a "speedometer", then even if we decided to increase the frequency of the repeated measurements from the order of $T_k^{-1}$ to, say, the order of $(T_k - 7)^{-1}$, we will have no clue that the apparatus is indeed working at $(T_k - 7)^{-1}$ and not $T_k^{-1}$. And so also here, in the absence of knowledge in advance of the energy spread $\Delta E$ or the desired state (from which, along with the initial state, the energy spread can be calculated), we will never know how fast should we measure our physical system so that this state is protected. But then we will also fail to fulfill the conditions for the quantum Zeno effect which ensure us that the system remains in its initial state.

4 Why Certainty is Not a Bliss

Taking stock, in both protection scenarios analyzed above we know the initial state or the initial Hamiltonian, and our target is a final (protected) state or a final Hamiltonian. But in both protection scenarios we do not know when to halt the protection process, that is, we do not know how fast or how slow the protection process – whose end result is the protected state – should be implemented, unless we have prior knowledge of the final (protected) state or the final Hamiltonian. In this final section I shall offer several reasons as to why this undecidability severely weakens the argument from protective measurement to the reality of the state vector of a single quantum system.

First let us set aside what the problem is not. The problem is not that actual infinity is not realizable in practice; rather, it is the undecidability of the protection process itself that is at stake here. Indeed, that we cannot realistically wait forever (or measure that quick) only means that we will never be certain that the state we are trying to characterize via the protective measurement process is actually protected. Defenders of the pro-
tective measurement argument may reply that such an uncertainty is not uncommon in physics. Many experiments do not give us certainty about their final outcome, and yet experimentalists believe these outcomes nevertheless, after repeating them several times. Why should we treat protective measurement scenarios differently?

What defenders of the argument from protective measurement to the reality of the state vector of a single quantum system are ignoring in this putative objection, is that the undecidability of the protection process we have just exposed not only prevents them from quantifying this uncertainty, but also blunts their argument: contrary to the mundane case of uncertainty in probabilistic experimental results, in the protective measurement scenarios certainty is a necessary condition for the argument’s validity. For suppose we run an adiabatic process for a period of time, say, an hour, and measure the state thereafter. What would that measurement tell us? Well, if the state is protected, we would have extracted one expectation value of the observable, and we can now reuse the system and measure it’s state again, thus concluding, along with Aharonov et al. (1993), that the state is “real”. But if the state is unprotected, we have just projected it onto some orthogonal subspace. Not only the value we have extracted from the measurement is the wrong one, but now we cannot reuse the same physical system; we must start the protection process again with another system and wait another period of time, presumably longer than the earlier period, and thus cannot conclude that the state of a single quantum system is “real”.

The upshot is that since the question whether the state is protected is undecidable, without prior knowledge of the state or the Hamiltonian there is no physical way we can distinguish between these two possible procedures. Of course we can just ignore the undecidability, chose an arbitrary period of time for the adiabatic process, and run the experiment repeatedly after this period has lapsed, reusing the same physical system over and over again, but then the results of this procedure would not convert the heretic, for what at stake here is not the result of the experiment, but the reality of the state vector of a single quantum system, and this reality can be established, according to the proponents of the argument from protective measurement, only under the assumption that the state is protected. Yet this assumption, namely, that the state is protected, remains forever unjustifiable unless one possesses prior knowledge of the state or the Hamiltonian.
"Fine, so we must know the state or the Hamiltonian in advance. What’s wrong with that?” proponents of protective measurement may ask. The answer is that this knowledge of either the state or the Hamiltonian weakens again their argument for the reality of the state vector of a single quantum system, rendering as it does the notion of protective measurement unnecessary to this argument.

Start with the state. What does it mean physically “to know the state”? It means that someone has prepared the physical system in a certain quantum state with a standard non–protective measurement, thereby ”collapsing” it onto a specific eigensapce of an observable (or, if you are a Bohmian, realizing some hefty dynamical process whereby the position of the particles that constitute the system is guided by the wave function to the desired result). But now what reasons do proponents of the protective measurement argument have to convince us that it is the protective measurement that ensures us that the state of a single quantum system is real, and not the physical collapse (or the hefty Bohmian dynamics) that presumably the state has gone through during the state preparation sometimes before this (protective) measurement process? In other words, allowing someone to prepare the state via standard measurement renders the notion of protective measurement unnecessary to the argument for the reality of the state vector of a single quantum system.

Now move to the Hamiltonian. Knowledge of the Hamiltonian also blunts the argument from protective measurement, because such a knowledge allows one to calculate the expectation values without any measurement, protective or otherwise. But if one could characterize the state without measuring it, then what reason does one have for insisting that it is the protective measurement, and not the knowledge of the Hamiltonian, that allow us to extract these values and declare the state ”real”? In other words, allowing knowledge of the Hamiltonian renders again the notion of protective measurement unnecessary to the argument for the reality of the state vector of a single quantum system.

I thus conclude that the argument from protective measurement to the reality of the state vector of a single quantum system (“if the state vector is protected, we can perform protective measurement repeatedly without disturbing it, hence reuse the same physical system and not an ensemble”) remains unconvincing. Even if we grant that the argument is valid, its premise (that the state is protected) is unjustifiable, since the question whether an unknown state of a single quantum system is protected, is
still undecidable. Worse, once we try justifying this premise by allowing prior knowledge of the state (or the Hamiltonian), we render the argument moot, as now protective measurement – sufficient as it may be for establishing the reality of the state vector of a single quantum system – becomes unnecessary, shadowed as it is by this prior knowledge of the state (or the Hamiltonian).

References


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