Introduction to Quantum Information Processing

for Cognitive Scientists: Quantum Dynamics

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July 12, 2007

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See References at end of document
Note: this is a very rough draft
There are several related purposes of this article. One is to generate interest in a new and fascinating approach to understanding cognition based on quantum information processing principles. The second is to introduce and provide a tutorial of the basic ideas in a manner that is interesting and easy for cognitive scientists to understand. A third is to review new applications of quantum information processing to several paradoxical cognitive phenomena. The fourth and final goal is to provide directions for future research on this important new approach to cognition.

We should point out from the beginning that we view quantum information processing theory simply as an alternative mathematical/computational approach for generating theoretical ideas. Quantum probability is a generalization of classic probability, quantum dynamics is an alternative to classical dynamics, and quantum computing is a different way to achieve universal computing. We make no assumptions about the biological substrates. Instead this is an exploration into new conceptual tools for constructing cognitive theories.

Why should one even consider this idea? The answer is simply this. The brain is an extremely complex information processing system that has a tremendously large number of unobservable states, and we are severely limited in our ability to measure all of these states. Also the brain is highly sensitive to context and it is easily disturbed and disrupted by our measurements. Finally, the measurements that we obtain from the brain are very noisy and filled with uncertainty. It turns out that classical logic, classic probability, and classic information processing force highly restrictive assumptions on the representation of these complex systems. Quantum information processing theory
provides principles that are more general and powerful for representing and analyzing complex systems of this type.

The article is organized as follows. First we describe a hypothetical yet typical type of cognitive experiment to provide a concrete setting for introducing the basic concepts. Second, we introduce the basic principles of quantum logic and quantum probability theory. Third we discuss basic quantum concepts including compatible and incompatible measurements, superposition, measurement and collapse of state vectors, dynamic evolution by unitary operators, quantum information processing by control U gates, and entangled state vectors. Fourth we review several new applications of quantum information processing to several important areas of cognition including conceptual combinations, probability judgments, and decision making behavior.


Suppose participants are (a) shown pictures of faces, (b) asked to categorize the faces as belonging to either a ‘good guy’ or ‘bad guy’ group, and then (c) decide to act attack or withdraw. Faces are generated from two prototypes, one prototype for the ‘good guy’ group, and another prototype for the ‘bad guy’ group. Payoffs are delivered depending on whether or not the correct action is taken for each face. The experiment involves two critical conditions. In condition CD, participants made an action decision after categorizing a face as either a ‘good guy’ or a ‘bad guy’; and in condition D, participants made an action decision without reporting any categorization.

The idea of the experiment is illustrated in Figure 1 below. Each trial begins with a presentation of a face. This face places the participant in a state denoted z. From this
initial state, the individual has to make a category judgment (good or bad) followed by an action decision (attack or withdraw). The box indicates the categorization question about the face. This question appears in a box because on some trials this question does not apply. The final stage represents the second (or only) action decision. The paths indicted by the arrows indicate all the possible choices for two binary questions.

Figure 1: Illustration of various possible measurement outcomes for condition AB.

2. Basic Principles.

Markov dynamics are derived from three simple but basic rules. Suppose $|x\rangle$, $|y\rangle$, and $|z\rangle$ are mutually exclusive states of a Markov state space.

1. The probability of the transition $|x\rangle \rightarrow |u\rangle$ equals the conditional probability $\Pr(u|x)$. This is a positive real number between zero and one.

2. The path probability for two consecutive transitions $|z\rangle \rightarrow |x\rangle \land |x\rangle \rightarrow |u\rangle$ is equal to $\Pr(u|x) \cdot \Pr(x|z)$.

3. The probability of traveling along either of two possible paths $|z\rangle \rightarrow |x\rangle \rightarrow |u\rangle \lor |z\rangle \rightarrow |y\rangle \rightarrow |u\rangle$ is equal to $\Pr(u|y) \cdot \Pr(y|z) + \Pr(u|x) \cdot \Pr(x|z)$. 
Note that the disjunctive probability defined by assumption 3 must exceed the conjunctive probability defined by step 2. Therefore, the probability of getting from state $|z\rangle$ to state $|u\rangle$ by either of two paths must exceed the probability of following just one path.

Quantum dynamics are derived from a slightly different set of basic rules. Suppose $|x\rangle$, $|y\rangle$, and $|z\rangle$ are distinct quantum states (unit length complex vectors). According to quantum theory, unobserved state transitions obey probability amplitude rules.

1. The probability amplitude of making a transition $|x\rangle \rightarrow |y\rangle$ is equal to the inner product $\langle y|x \rangle$. This is a complex number with magnitude less than or equal to one.

2. The path probability amplitude for two consecutive transitions $|z\rangle \rightarrow |x\rangle \land |x\rangle \rightarrow |u\rangle$ is equal to $\langle u|x \rangle \cdot \langle x|z \rangle$.

3. The probability amplitude of traveling along either of two unobserved possible paths $|z\rangle \rightarrow |x\rangle \rightarrow |u\rangle \lor |z\rangle \rightarrow |y\rangle \rightarrow |u\rangle$ is equal to $\langle u|y \rangle \cdot \langle y|z \rangle + \langle u|x \rangle \cdot \langle x|z \rangle$.

The probability of an observed event is equal to the squared magnitude of the corresponding probability amplitude of the event. For example, in condition CD, we observe the top path $|z\rangle \rightarrow |x\rangle \rightarrow |u\rangle$ with probability

$$\Pr( |z\rangle \rightarrow |x\rangle \rightarrow |u\rangle ) = |\langle u|x \rangle \cdot \langle x|z \rangle|^2 = |\langle u|x \rangle|^2 \cdot |\langle x|z \rangle|^2 ,$$

and we observe the bottom path $|z\rangle \rightarrow |y\rangle \rightarrow |u\rangle$ with probability

$$\Pr( |z\rangle \rightarrow |y\rangle \rightarrow |u\rangle ) = |\langle u|y \rangle \cdot \langle y|z \rangle|^2 = |\langle u|y \rangle|^2 \cdot |\langle y|z \rangle|^2 ,$$

so that the total probability of observing the first path plus the probability of observing the second path equals

$$\Pr( |z\rangle \rightarrow |x\rangle \rightarrow |u\rangle ) + \Pr( |z\rangle \rightarrow |y\rangle \rightarrow |u\rangle ) = |\langle u|x \rangle|^2 \cdot |\langle x|z \rangle|^2 + |\langle u|y \rangle|^2 \cdot |\langle y|z \rangle|^2 ,$$
which exactly matches the Markov model. However, for condition D, we do not observe the path inside the box of Figure 1, and in this case the probability of the two paths is

\[
\Pr(|z\rangle \rightarrow |x\rangle \rightarrow |u\rangle \lor |z\rangle \rightarrow |y\rangle \rightarrow |u\rangle) = |\langle u|y\rangle\langle y|z\rangle + \langle u|x\rangle\langle x|z\rangle|^2
\]

\[
= |\langle u|x\rangle|^2|\langle x|z\rangle|^2 + |\langle u|y\rangle|^2|\langle y|z\rangle|^2 + 2|\langle u|x\rangle\langle x|z\rangle||\langle u|y\rangle\langle y|z\rangle|\cos(\theta),
\]

where \( \theta \) is the angle between \( \langle u|x\rangle\langle x|z\rangle \) and \( \langle u|y\rangle\langle y|z\rangle \) in the complex plane.

The third term in this expansion, called the interference term, can be positive or negative, and this makes quantum theory differ from Markov theory. In particular, if \( \omega = 180 \) degrees, then \( \cos(\omega) = -1 \) and the third term cancels the first two terms so that the probability is reduced to zero. Therefore, the probability of getting from state \( |z\rangle \) to state \( |u\rangle \) by either of two unobserved paths can be less than the probability of following just one path.

3. State Representation.

Both the Markov and the quantum dynamic models postulate a set of basis states. The basis states are used to represent the various mental states that, at least hypothetically, can be measured or reported to occur during an experiment. We will
assume that a person can simultaneously consider beliefs and actions (e.g., these are assumed to be compatible measures), which then produces a set of four basis states denoted \( \Omega = \{ |gw\rangle, |ga\rangle, |bw\rangle, |ba\rangle \}. For example, \( |ga\rangle \) represents the case where you simultaneously believe that the category is ‘good guy’ but you intend to act aggressively. For convenience, we will label these states with indices \( |1\rangle = |gw\rangle, |2\rangle = |ga\rangle, |3\rangle = |bw\rangle, |4\rangle = |ba\rangle \) so that \( |j\rangle \) represents an arbitrary basis state in this system.

**Markov States.** The pure state of the Markov system at time \( t \) can be defined as a convex combination of the basis states:

\[
|P(t)\rangle = \sum_{j \in \Omega} w_j(t) \cdot |j\rangle,
\]

where \( w_j(t) \) is a random indicator variable with values equal to 1 or 0 depending on whether or not the individual is in state \( |j\rangle \) at time \( t \). According to the Markov model, the individual must be in one and only one state at each moment in time so that \( \sum w_j(t) = 1 \). The probability of being in state \( |j\rangle \) at time \( t \) is denoted \( P_j(t) = \Pr[w_j(t) = 1] \).

The mixed state of a Markov system is defined by the column vector, \( P(t) \), where the probability \( P_j(t) \) is the coordinate in the j-th row corresponding to the basis vector \( |j\rangle \).
The probabilities in \( P(t) \) sum to unity, \( \sum P_j(t) = 1 \), so that \( P(t) \) represents the probability distribution across the basis states at time \( t \). The set of all probability distributions defines the (mixed) state space of the Markov system.

**Quantum States.** A superposition state of the quantum system at any moment in time \( t \) is defined as a linear combination of the basis vectors in \( \Omega \):

\[
|\psi(t)\rangle = \mathbf{I} \cdot |\psi(t)\rangle = \sum |j\rangle \langle j| \cdot |\psi(t)\rangle = \sum \langle j|\psi(t)\rangle \cdot |j\rangle = \sum \psi_j(t) \cdot |j\rangle,
\]

where \( \psi_j = \langle j|\psi \rangle \) is the probability amplitude of observing the basis state \( |j\rangle \) at time \( t \). The probability amplitude, \( \psi_j \), is a complex number with a magnitude less than or equal to one. The column vector \( \psi \) represents the quantum state using coordinates defined by the \( \Omega \) basis, where \( \psi_j \) is the coordinate in the \( j \)-th row corresponding to the basis vector \( |j\rangle \).

The squared length of \( \psi \) must be unity, \( |\psi|^2 = 1 \), so that the collection of squared amplitudes produces a probability distribution over the basis states. The state space of the quantum system is defined as the set of all linear combinations of the basis vectors with unit length.
State Interpretations. The superposition state $\psi$ of the quantum model corresponds to the mixed state $P$ of the Markov model. However, these representations of states are mathematically very different: $\psi$ is a complex vector of unit length, whereas $P$ is a non-negative real vector that sums to unity. Conceptually, these states are also radically different. According to the Markov model, for any given realization, the unobserved system occupies exactly one basis state $|j\rangle$ at each moment in time. A sample path of the Markov process is a series of jumps from one basis state to another, which moves like a bouncing particle across time. A different path is randomly sampled for each realization of the Markov process. According to the quantum model, for any given realization, the unobserved system does not occupy any particular basis state at each moment in time. A realization of the quantum process is a fuzzy spread of membership across the basis states, which moves like a traveling wave across time. Each realization of the quantum process is identical, producing the same series of states across time. All of the randomness in the quantum model results from taking a measurement.

4. State Transitions.

Markov Transitions. A mixed Markov state can be transformed into another mixed state by a transition matrix, symbolized as $T$. Each element of the transition matrix, $T_{ij}(t)$, defines the transition probability to state $|i\rangle$ from state $|j\rangle$ during time $t$, for every $(i,j) \in \Omega^2$. In other words, $T_{ij}(t) = \Pr[\text{state } |i\rangle \text{ at time } t \text{ given state } |j\rangle \text{ at time } 0]$. Starting from state $|j\rangle$ the system must land into one of the possible basis states in $\Omega$ at time $t$, and therefore the columns of the transition matrix must sum to one.

According to the basic principles, the total probability of ending at state $|i\rangle$ at time $t$ is obtained from the sum across all the possible starting positions
\[ P(t) = \sum T_{ij}(t) \cdot P_j(0) \]

Thus the probability distribution at time \( t \) is related to the initial distribution by the linear equation

\[ P(t) = T(t) \cdot P(0). \quad (1) \]

Quantum Transitions. A quantum state can be transformed into another state by a unitary operator, symbolized as \( U \), with \( U^\dagger U = I \), which is required to preserve inner products: If we transform \( |z\rangle \) and \( |x\rangle \) to \( U|z\rangle \) and \( U|x\rangle \), then \( \langle z|U^\dagger U|x\rangle = \langle z|x\rangle \). In particular, unitary operators preserve lengths: \( \langle \psi|U^\dagger U|\psi\rangle = \langle \psi|\psi\rangle = 1 \).

The probability amplitude of starting in state \( |z\rangle \) and passing through \( U \) and then observing basis state \( |i\rangle \) can be expressed in terms of the \( \Omega \) basis as:

\[ \langle i|U|z\rangle = \langle i| \cdot U \cdot |z\rangle = \langle i|U \cdot (\sum_{j \in \Omega} |j\rangle \langle j|) \cdot |z\rangle \]

\[ = \sum_{j \in \Omega} \langle i|U|j\rangle \langle j|z\rangle = \sum_{j \in \Omega} u_{ij} \cdot \langle j|z\rangle \]

This follows directly from the basic principles. The transition probability amplitude, \( u_{ij} = \langle i|U|j\rangle \), represents the probability amplitude of transiting to basis state \( |i\rangle \) from basis state \( |j\rangle \) going through the unitary operator \( U \). Thus the right hand side is the sum of all the path probability amplitudes from \( |\psi\rangle \) through \( |j\rangle \) to \( |i\rangle \), and each path probability amplitude is the product of the individual probability amplitudes, \( \langle i|U|j\rangle \langle j|\psi \rangle \).

The above equation implies that the unitary operator is a linear operator. The transition probability amplitudes form a matrix \( U \), with \( u_{ij} \) in row \( i \) column \( j \), representing the coordinates of \( U \) with respect to the \( \Omega \) basis, so that we can express the effect of the unitary operator as the linear transformation: \( \chi = U \cdot \psi \).
If the state $|\psi(0)\rangle$ is processed by the unitary operator $U$ for some period of time $t$, then it produces the new state $|\psi(t)\rangle = U(t)|\psi(0)\rangle$. Using the coordinates defined with respect to the $\Omega$ basis, the state transition rule is expressed by the following linear transition equation:

$$\psi(t) = U(t) \cdot \psi(0).$$  \hspace{1cm} (2)

Note that the transition rule for probability amplitudes in quantum processes (Equation 2) corresponds to the transition rule for probabilities in Markov processes (Equation 1). However the former model operates on complex amplitudes whereas the latter model is restricted to positive real numbers. In terms of this property, the quantum model is more general than the Markov model.

5. Dynamical Equations  

Markov Dynamics. From basic principles of Markov processes, it follows that the transition probabilities satisfy an important property known as the Chapman – Kolmogorov Equation

$$T_{ij}(s+t) = \sum_{k \in \Omega} T_{ik}(s) \cdot T_{kj}(t)$$

or in matrix form

$$T(s+t) = T(s) \cdot T(t).$$  \hspace{1cm} (3)

This is known as the group property of dynamic systems, and from this group property one can derive a differential equation known as the Kolmogorov Forward Equation (Bhattacharya & Waymire, 1990, p. 266):

$$\frac{d}{dt} T(t) = Q \cdot T(t),$$  \hspace{1cm} (4a)

Multiplying both sides of Equation 4a by $P(0)$ and setting $P(t) = T(t) \cdot P(0)$ yields
\[
\frac{d}{dt} P(t) = Q \cdot P(t) .
\]  

(4b)

The \textit{intensity} matrix \( Q \) has elements \( q_{ij} = \lim_{\tau \to 0} \frac{T_{ij}(\tau) - T_{ij}(0)}{\tau} \) in row \( i \) column \( j \) which represents the instantaneous rate of change to \(|i\rangle\) from \(|j\rangle\). These rates control the flow of probability over time.

\textit{Quantum Dynamics.} If the state \(|\psi(0)\rangle\) is processed by the unitary operator \( U \) for some period of time \( t \), and it is immediately processed again by the same unitary operator \( U \) for an additional time \( s \), then it produces the new state

\[
|\psi(s+t)\rangle = U(s+t)|\psi(0)\rangle = U(s)U(t)|\psi(0)\rangle .
\]

Using the matrix coordinates of the operator, this implies

\[
u_{ij}(s+t) = \sum_k u_{ik}(s) \cdot u_{kj}(t)
\]

or more generally,

\[
U(s+t) = U(s)U(t)
\]

(5)

This follows directly from the basic principles of quantum dynamics. Equation 5 for the quantum dynamic model corresponds to Equation 3 of the Markov process model. In other words, this is the quantum analogue of the Chapman-Kolmogorov Equation: the unitary operator for the quantum dynamic model satisfies the same group property as does the transition matrix for the Markov process model (see Hughes, 1989, p. 114; Gardiner, 1991, p. 165).

It follows that the unitary operator satisfies the following differential equation known as the Schrödinger Equation:

\[
\frac{d}{dt} U(t) = -i \cdot H \cdot U(t) .
\]

(6a)
Multiplying both sides of Equation 6a by \(\psi(0)\) and setting \(\psi(t) = U(t) \cdot \psi(0)\) yields

\[
\frac{d}{dt} \psi(t) = -i \cdot H \cdot \psi(t).
\]  

(6b)

The Hamiltonian \(H\) has elements \(h_{ij} = \lim_{\tau \to 0} \frac{U_{ij}(\tau) - U_{ij}(0)}{\tau}\) in row \(i\) column \(j\) representing the instantaneous rate of change to \(|i\rangle\) from \(|j\rangle\). Note that the Schrödinger Equation (Equation 6) corresponds directly to the Kolmogorov Forward Equation (Equation 4). (The inclusion of the factor, \(-i\), is needed to satisfy the unitary property).


For the Markov process, the solution to the forward equation is the matrix exponential

\[
T(t) = e^{Qt}.
\]  

(7a)

Thus the solution for the probability distribution over time is directly obtained from

\[
P(t) = e^{Qt} \cdot P(0).
\]  

(7b)

The solution to the Schrödinger Equation is given by the matrix exponential of the Hamiltonian matrix:

\[
U(t) = e^{-iHt}.
\]  

(8a)

Thus the probability amplitudes evolve across time according to the following equation:

\[
\psi(t) = e^{-iHt} \cdot \psi(0).
\]  

(8b)

Note that the quantum solution given by Equation 8b corresponds directly to the Markov solution given in Equation 7b. However, according to the quantum model, the final probability for each state is obtained by taking the squared magnitude of the
corresponding probability amplitude: \( P_j(t) = |\psi_j|^2 \). Thus the probability distribution for
the Quantum model is a nonlinear transformation of Equation 8b.

7. Model Parameters.

**Intensity Matrix.** The specification of the intensity rates in \( Q \) is critical for Markov
models. The intensity matrix \( Q \) must satisfy the following constraints:

\[ q_{ij} \geq 0 \text{ for } i \neq j \text{ and } \sum_{i \in \Omega} q_{ij} = 0, \]

where the first inequality is required to make the transition probabilities non negative,
and the latter is required because the transition probabilities within a column sum to one.
Note that the intensity matrix \( Q \) of the Markov model is allowed to be asymmetric.

**Hamiltonian Matrix.** The specification of the values in \( H \) is also critical for the
quantum model. The Hamiltonian matrix \( H \) must be Hermitian, \( H = H^\dagger \), to guarantee that
the solution of Equation 8a is a unitary matrix. Note that the Hamiltonian matrix \( H \) must
obey different constraints than the intensity matrix \( Q \), and therefore, the Markov model is
not a special case of the quantum model.

Why must the Hamiltonian be Hermitian? \( U \) is a unitary matrix (ortho-normal
matrix) to guarantee that the state vector remains unit length after transformation, and this
also implies that \( U \) is a normal matrix \( (U^\dagger U = UU^\dagger = I) \). From the theory of spectral
decomposition, it is known that all normal matrices can be diagonalized as follows

\[ U = V \Omega V^\dagger = \sum \phi_j V_j V_j^\dagger, \]

where \( \Omega \) is a diagonal matrix with elements \( \phi_j \) as an eigenvalue, and \( V_j \) is an eigenvector
(column of \( V \)) such that \( V V^\dagger = V^\dagger V = I \). The fact that \( U^\dagger U = I \) implies that \( \phi_j = e^{-i \lambda j} \). This
follows from

\[ U^\dagger U = (\sum \phi_j V_j V_j^\dagger)^\dagger (\sum \phi_j V_j V_j^\dagger) \]
\[\begin{align*}
&= (\sum \phi_j^* \cdot V_j \cdot V_j^\dagger)(\sum \phi_j \cdot V_j \cdot V_j^\dagger) \\
&= \sum |\phi_j|^2 \cdot V_j \cdot V_j^\dagger = I.
\end{align*}\]

All of the eigenvalues of I are one, and so \(|\phi_j|^2 = 1\), which implies that \(\phi_j = e^{-i\cdot\lambda_j}\) for some real valued \(\lambda_j\). Now, for any diagonalizable

\[X = \sum \lambda_j \cdot V_j \cdot V_j^\dagger,\]

then by definition

\[f(X) = \sum f(\lambda_j) \cdot V_j \cdot V_j^\dagger.\]

In particular, \(\exp(X) = \sum \exp(\lambda_j) \cdot V_j \cdot V_j^\dagger\) and \(\ln(X) = \sum \ln(\lambda_j) \cdot V_j \cdot V_j^\dagger\). Therefore \(\exp(\ln(X)) = X\). This implies

\[U = \sum e^{-i\cdot\lambda_j \cdot V_j \cdot V_j^\dagger} = \exp(\ln(U)) = \exp(-i\cdot i \cdot \ln(U)) = \exp(-i \cdot H)\]

where

\[H = i \cdot \ln(U) = \sum i \cdot (-i \cdot \lambda_j \cdot V_j \cdot V_j^\dagger) = \sum \lambda_j \cdot V_j \cdot V_j^\dagger = (\sum \lambda_j \cdot V_j \cdot V_j^\dagger)^\dagger = H^\dagger.\]

From this last expression we see that the Hamiltonian must be Hermitian to guarantee that \(U\) is unitary, and \(U\) must be unitary to keep the state vector at unit length following a transformation of state.

These constraints on the Hamiltonian place strong constraints on the unitary operator. It follows from the spectral decomposition theory that

\[H = \sum \lambda_j \cdot V_j \cdot V_j^\dagger,\]

where \(\lambda_j\) is an eigenvalue and \(V_j\) is an eigenvector of \(H\). The unitary operator can thus be expressed as

\[U(t) = \sum e^{-i \cdot \lambda_j \cdot V_j \cdot V_j^\dagger} = \sum [\cos(\lambda_j \cdot t) - i \cdot \sin(\lambda_j \cdot t)] \cdot (V_j \cdot V_j^\dagger)\]

\[= \sum \cos(\lambda_j \cdot t) \cdot (V_j \cdot V_j^\dagger) - i \sum \sin(\lambda_j \cdot t) \cdot (V_j \cdot V_j^\dagger)\]

\[= \cos(t \cdot H) - i \cdot \sin(t \cdot H).\]
Note that \( U(t)^\dagger = \cos(tH) + i\sin(tH) \) and the difference \( U(t) - U(t)^\dagger = -2i\sin(tH) \). So \( U \) is not Hermitian in general.

This means that \( |\langle i|U(t)|j\rangle|^2 = |\langle i|U(t)|j\rangle^*|^2 = |\langle j|U^\dagger(t)|i\rangle|^2 \neq |\langle j|U(t)|i\rangle|^2 \) so that the transition from basis state \( |j\rangle \) at time zero through a unitary operator to basis state \( |i\rangle \) at time \( t \), is not the same as the transition from basis state \( |i\rangle \) at time zero through a unitary operator to basis state \( |j\rangle \) at time \( t \). At \( t = 0 \), \( U(0) = 1 \), and \( |\langle i|U(0)|j\rangle|^2 = |\langle i|I|j\rangle|^2 = |\langle ij\rangle|^2 = |\langle j|i\rangle|^2 = |\langle j|I|i\rangle|^2 = |\langle j|U(0)|i\rangle|^2 \). Thus when no unitary operator is applied, then the probability of a transition from basis state \( |j\rangle \) to basis state \( |i\rangle \) is the same as the probability of a transition from basis state \( |i\rangle \) to basis state \( |j\rangle \).
References


Andrei Khrennikov Univ of Vaxjo Sweeden

he is doing very important work on interference effects


Diderik Aerts Free Univ of Brussels & Liane Gabora Univ British Columbia

they are working on applications to concepts

see their work on interference effects and the disjunction effect


Riccardo Franco Politecnico di Torino

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she is doing very rigorous work on quantum logic and applying it to a new foundation for decision theory
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% Markov model
% WG WN LG LN

P0 = [1 1 0 0 ]';  P0 = P0./sum(P0);

q12 = .1;  q13 = 0; q14 = .0;
q21 = .01; q23 = 0; q24 = 0;
q31 = 0;  q32 = 0; q34 = .1;
q41 = .02; q42 = 0; q43 = .01;

q11 = -(q21+q31+q41);
q22 = -(q12+q32+q42);
q33 = -(q13+q23+q43);
q44 = -(q14+q24+q34);

Q = [q11 q12 q13 q14; ...
    q21 q22 q23 q24; ...
    q31 q32 q33 q34; ...
    q41 q42 q43 q44];

PM = [];

for i = 1:10000
    t = i/100;
    T = expm(t*Q);
    P1 = T*P0;
    L = [1 0 1 0];
    PM = [PM ; L*P1];
end

plot(PM)
T*P0
% Categorization - Decision
% (Categorize (friendly, enemy) then decide (withdraw, attack)
% {FW PA EW EA}

tt = 10;
a = .5;
b = 1;
mu1 = 0;
mu2 = b;
mu3 = 0;
mu4 = b;
sig1 = 0;
sig2 = a;

PM= [];

H = [ mu1 sig2 0 sig1 ;
     sig2 mu2 0 0 ;
     0 0 mu3 sig2 ;
     sig1 0 sig2 mu4 ];

H = H + 0*eye(4); 

X0 = [1 1 1 1]'; X0 = X0./sqrt((X0'*X0));

for t = 0:.01:tt
    U = expm(-i*t*pi*H);
    PA = U*X0;
    P = abs(PA).^2;
    Pa = [0 0 1 1];
    PM = [PM ; Pa*P];
end

T = 0:.01:tt;
plot(T,PM)
% PD
% Info on Opponent (C,D) then you decide (C,D)
% {DD DC CD CC}

tt = .2;
mu = 5;
mu1 = mu;
mu2 = 0;
mu3 = mu;
mu4 = 0;
sig1 = -1;
sig2 = .2;

PM = [];

H = [ mu1    sig2   0      sig1 ;
     sig2   mu2    0      0    ;
     0      0      mu3    sig2 ;
     sig1   0      sig2   mu4 ];

H = H + 2*eye(4);

M = [1 0 0 0;
     0 0 1 0;
     0 0 0 0;
     0 0 0 0];

a1 = sqrt(.5); b1 = sqrt(1-a1^2);
a2 = sqrt(.5); b2 = sqrt(1-a2^2);
X1 = [ a1 b1 0 0]'; X1 = X1./sqrt(X1'*X1);
X2 = [ 0 0 a2 b2]'; X2 = X2./sqrt(X2'*X2);
X3 = (X1 + X2); X3 = X3./sqrt(X3'*X3);

for t = 0:.01:tt
    U = expm(-i*t*pi*H);
    S1 = U*X1;
    S2 = U*X2;
    S3 = U*X3;
    P1 = M*S1; P1 = P1'*P1;
    P2 = M*S2; P2 = P2'*P2;
    P3 = M*S3; P3 = P3'*P3;
    P = [ P1 P2 P3 (.5*P1 + .5*P2) ];
    PM = [PM ; P];
End

T = 0:.01:tt;
plot(T,PM(:,1),':o',T,PM(:,2),':*',T,PM(:,3),':^',T,PM(:,4),'-+')
legend('KD','KC','U','AVG')
\text{sig} \, 1 = 0

\begin{align*}
\text{sig} \, 1 &= 0 \\
\text{KD} &= \text{KC} = \text{U} = \text{AVG}
\end{align*}
$\text{Sig1} = -1$

![Graph with multiple lines representing different functions and values at various points on the x-axis.]
Three different Initial States

Position

Initial
Final

Probability

Position

Probability

Position