Problem 1: (done in class)

Problem 2: \( \frac{5}{2} \cdot (\ln 6) \cdot 6^{\sqrt{x+1}}(x+1)^{-1/2} \)

Problem 3: Let \( f(x) = \sqrt[3]{x} \). The local linear approximation near \( x = 1000 \) is

\[
f(1000 + \Delta x) \approx f(1000) + f'(1000) \cdot \Delta x.
\]

To estimate \( f(994) \) we take \( \Delta x = -6 \). We know that \( f(1000) = \sqrt[3]{1000} = 10 \). To compute \( f'(1000) \), first evaluate \( f'(x) \); then plug in 1000:

\[
f'(x) = \frac{d}{dx} x^{1/3} = \frac{1}{3} x^{-2/3};
\]

\[
f'(1000) = \frac{1}{3} 1000^{-2/3} = \frac{1}{3} \left(1000^{1/3}\right)^{-2} = \frac{1}{3} \cdot 10^{-2} = \frac{1}{3} \cdot \frac{1}{10^2} = \frac{1}{300}.
\]

Finally,

\[
\sqrt[3]{994} \approx 10 + \frac{1}{300} \cdot (-6) = 10 - \frac{1}{50} = 10 - 0.02 = 9.98.
\]

(Note that a calculator gives \( \sqrt[3]{994} \approx 9.97996 \).)

Problem 4: Point of tangency is \( \left(1, \frac{1}{2}\right) \). The slope is \( f'(1) \), which is computed using the quotient rule:

\[
f'(x) = \frac{d}{dx} \frac{x^3}{1+x} = \frac{(1+x) \cdot 3x^2 - x^3 \cdot 1}{(1+x)^2} = \frac{3x^2 + 3x^3 - x^3}{(1+x)^2} = \frac{3x^2 + 2x^3}{(1+x)^2};
\]

\[
f'(1) = \frac{5}{4}.
\]

So the equation of the tangent line is

\[
y - \frac{1}{2} = \frac{5}{4}(x - 1).
\]

The \( x \)-intercept is found by setting \( y = 0 \) and solving for \( x \) to obtain \( x = \frac{3}{5} \).
Problem 5: \( A(r) = \pi r^2 \). \( A'(r) = 2\pi r \). \( A'(r) = \frac{dA}{dr} \) is measured in units of \( \frac{\text{square feet}}{\text{feet}} = \text{feet} \).

\[ A'(100) = 2\pi \cdot 100 \approx 628.3. \]

Interpretation: when the radius is 100 feet, each increase of 1 foot in the radius results in an increase of 628.3 square feet in the area.

Remark: This is not literally true. The area of a circle of radius 101 feet is 32047.4 square feet, while the area of a circle of radius 100 feet is 31415.9 square feet. The difference is 631.5 square feet. The reason for the discrepancy is that the number 628.3 is the \textit{instantaneous} rate of change at a radius of exactly 100 feet. The instantaneous rate of change will be slightly higher when the radius is 100.1 feet or 100.7 feet, for example. This is why the actual increase in area for a one foot increase in radius is slightly higher than 628.3.

Observe that the formula for \( A'(r) \) is the same as the formula for circumference. This is no coincidence. The additional area of a circle of radius 101 feet, compared with a circle of radius 100 feet is a ring of width 1 foot and circumference \( 2\pi \cdot 100 \) feet.

Problem 6. Find the best estimate of \( h''(5.5) \) from the tabulated data for \( h(x) \) by using the average of left and right estimates for both first and second derivatives.

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<th>4.5</th>
<th>5.0</th>
<th>5.5</th>
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<td>( h''(x) )</td>
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<td>4</td>
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</table>

Problem 7: E is not true, since water is flowing into the tank during the entire time interval between \( t = 1 \) and \( t = 2.5 \).

Problem 8: \( A'(3) = f'(g(3)) \cdot g'(3) = f'(4) \cdot 3 = -3 \cdot 3 = -9 \).

\[ B'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{[g(3)]^2} = \frac{4 \cdot (-8) - (-5) \cdot 3}{4^2} = -\frac{17}{16}. \]

Problem 9: Doubling time: \( \frac{\ln 2}{\ln 5} = 4.3068 \) days.

Problem 10: Relative rate of change equals 8, computed using \( \frac{f'(4)}{f(4)} \) or \( \frac{d}{dt} \ln f(t) \bigg|_{t=4} \)

Problem 11: \( x = \frac{\ln 5 - \ln 8}{(\ln 10) - 3} \)

Problem 12: \( P = 100 \cdot 1.04^3 + 200 \cdot 1.04^2. \)